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Dynamic Behavior for a Model of Coupled Limit Cycle Oscillators with Delays

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---***ABSTRACT***---
In this paper, the stability and oscillatory behavior of the solutions for a model of coupled limit cycle oscillators with delays are investigated. By means of mathematical analysis method, some sufficient conditions to guarantee the stability and oscillation of the solutions are obtained. Computer simulations are provided to demonstrate our results.
KEYWORDS: *a coupled limit cycle system, delay, stability, oscillation*

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I. INTRODUCTION

It is well known that the coupled system is a research topic of science studying how parts of a system lead to the collective behavior. Since last decade, a lot of researches about coupled systems have appeared in a wide areas, such as engineering, biology, social science, neural networks and so on [1-12]. The following delay-coupled limit cycle oscillators have been investigated by many researchers [13-19]:

$$\begin{cases} Z_1'(t) = (1 + i\omega_1 - |Z_1(t)|^2)Z_1(t) + k[Z_2(t-\tau) - Z_1(t)], \\ Z_2'(t) = (1 + i\omega_2 - |Z_2(t)|^2)Z_2(t) + k[Z_1(t-\tau) - Z_2(t)]. \end{cases} \tag{1}$$

where $Z_1(t)$ and $Z_2(t)$are the complex amplitude of the oscillator 1 and oscillator 2, respectively. Each oscillator has a stable limit cycle of unit amplitude $|Z_i(t)| = 1$ with angular frequencies ω_1 and $\omega_2, k > 0$ is the coupling strength and $\tau > 0$ is a measure of time delay. In [13], Li et. al have studied the double Hopf bifurcation at zero equilibrium point for system (1). The authors gave not only the critical values of Hopf and double Hopf bifurcations, but also derive the universal unfolding and a complete bifurcation diagram of the system. The normal forms of several strong resonant cases were listed. Recently, Thounaojam and Shrimali have considered the following coupled system [20]:

$$\begin{cases} Z_{1,3}'(t) = \left(A + i\omega - \left|Z_{1,3}(t)\right|^2\right)Z_{1,3}(t) + \varepsilon\left[Z_2(t-\tau) - Z_{1,3}(t)\right], \\ Z_2'(t) = (A + i\omega - |Z_2(t)|^2)Z_2(t) + \varepsilon[Z_1(t-\tau) + Z_3(t-\tau) - Z_2(t)] + \varepsilon_U U, \\ U'(t) = -kU - \varepsilon_U Re(Z_2). \end{cases} \tag{2}$$

where A is a parameter, $Re(Z_2)$ is the real part of $Z_2(t)$, and k is the rate of decay of the linear system U. System (2) has been employed in a system of Hindmarsh-Rose neurons to incorporate an interaction with external environment. For the cases $Z_1(t) = Z_3(t)$and $Z_1(t) = -Z_3(t)$, the authors have demonstrated that a relay system of time delay coupled limit cycle oscillators supports a dynamical regime where two spatially separated oscillators operate in anti-phase when middle oscillator is driven to its fixed point. However, the authors did not provide the theoretical analysis. Motivated by the above models, in this paper we consider the following general coupled time delay model:

$$\begin{cases} Z_1'(t) = (a + i\omega_1 - |Z_1(t)|^2)Z_1(t) - k_{11}[Z_1(t-\tau_1) - Z_1(t)] \\ \qquad + k_{12}[Z_2(t-\tau_2) - Z_1(t)] + k_{13}[Z_3(t-\tau_3) - Z_1(t)], \\ Z_2'(t) = (b + i\omega_2 - |Z_2(t)|^2)Z_2(t) + k_{21}[Z_1(t-\tau_1) - Z_2(t)] \\ \qquad - k_{22}[Z_2(t-\tau_2) - Z_2(t)] + k_{23}[Z_3(t-\tau_3) - Z_2(t)], \\ Z_3'(t) = (c + i\omega_3 - |Z_3(t)|^2)Z_3(t) + k_{31}[Z_1(t-\tau_1) - Z_3(t)] \\ \qquad + k_{32}[Z_2(t-\tau_2) - Z_3(t)] - k_{33}[Z_3(t-\tau_3) - Z_3(t)]. \end{cases} \tag{3}$$

Where $0 < a, b, c, k_{ij}, \omega_i$; $0 \le \tau_i$ $(1 \le i, j \le 3)$ are parameters. By means of mathematical analysis method, the dynamical behavior of system (3) has been discussed.

II. PRELIMINARIES

For convenience, writing $Z_j(t) = x_j(t) + iy_j(t)$ $(j = 1, 2, 3)$,then system (3) can be written as a real form of six dimensional system:

$$\begin{cases} x_1'(t) = (a + k_{11} - k_{12} - k_{13})x_1(t) - \omega_1 y_1(t) - k_{11}x_1(t-\tau_1) \\ \quad +k_{12}x_2(t-\tau_2) + k_{13}x_3(t-\tau_3) - x_1^3(t) - x_1(t)y_1^2(t), \\ y_1'(t) = \omega_1 x_1(t) + (a + k_{11} - k_{12} - k_{13})y_1(t) - k_{11}y_1(t-\tau_1) \\ \quad +k_{12}y_2(t-\tau_2) + k_{13}y_3(t-\tau_3) - x_1^2(t)y_1(t) - y_1^3(t), \\ x_2'(t) = (b + k_{22} - k_{21} - k_{23})x_2(t) - \omega_2 y_2(t) + k_{21}x_1(t-\tau_1) \\ \quad -k_{22}x_2(t-\tau_2) + k_{23}x_3(t-\tau_3) - x_2^3(t) - x_2(t)y_2^2(t), \\ y_2'(t) = \omega_2 x_2(t) + (b + k_{22} - k_{21} - k_{23})y_2(t) + k_{21}y_1(t-\tau_1) \\ \quad -k_{22}y_2(t-\tau_2) + k_{23}y_3(t-\tau_3) - x_2^2(t)y_2(t) - y_2^3(t), \\ x_3'(t) = (c + k_{33} - k_{31} - k_{32})x_3(t) - \omega_3 y_3(t) + k_{31}x_1(t-\tau_1) \\ \quad +k_{32}x_2(t-\tau_2) - k_{33}x_3(t-\tau_3) - x_3^3(t) - x_3(t)y_3^2(t), \\ y_3'(t) = \omega_3 x_3(t) + (c + k_{33} - k_{31} - k_{32})y_3(t) + k_{31}y_1(t-\tau_1) \\ \quad +k_{32}y_2(t-\tau_2) - k_{33}y_3(t-\tau_3) - x_3^2(t)y_3(t) - y_3^3(t). \end{cases} \tag{4}$$

The system (4) can be expressed in the following matrix form:

$$u'(t) = Au(t) + Bu(t-\tau) + f\big(u(t)\big) \tag{5}$$

where $u(t) = [x_1(t), y_1(t), x_2(t), y_2(t), x_3(t), y_3(t)]^T, u(t-\tau) = [x_1(t-\tau_1), y_1(t-\tau_1), x_2(t-\tau_2), y_2(t-$ $\tau 2, x3t-\tau 3, y3t-\tau 3T, A$ and Bboth are six by six matrices, and $f(u(t))$ is a six by one vector as follows:

$$A = (a_{ij})_{6\times 6} = \begin{pmatrix} a_{11} & -\omega_1 & 0 & 0 & 0 & 0 \\ \omega_1 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & -\omega_2 & 0 & 0 \\ 0 & 0 & \omega_2 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & -\omega_3 \\ 0 & 0 & 0 & 0 & \omega_3 & a_{66} \end{pmatrix},$$

where $a_{11} = a_{22} = a + k_{11} - k_{12} - k_{13}, a_{33} = a_{44} = b + k_{22} - k_{21} - k_{23}, a_{55} = a_{66} = c + k_{33} - k_{31} - k_{32}$.

$$B = (b_{ij})_{6\times 6} = \begin{pmatrix} -k_{11} & 0 & k_{12} & 0 & k_{13} & 0 \\ 0 & -k_{11} & 0 & k_{12} & 0 & k_{13} \\ k_{21} & 0 & -k_{22} & 0 & k_{23} & 0 \\ 0 & k_{21} & 0 & -k_{22} & 0 & k_{23} \\ k_{31} & 0 & k_{32} & 0 & -k_{33} & 0 \\ 0 & k_{31} & 0 & k_{32} & 0 & -k_{33} \end{pmatrix},$$

$$f(u(t)) = [-x_1^3(t) - x_1(t)y_1^2(t), \quad -x_1^2(t)y_1(t) - y_1^3(t), \quad -x_2^3(t) - x_2(t)y_2^2(t), \\ -x_2^2(t)y_2(t) - y_2^3(t), \quad -x_3^3(t) - x_3(t)y_3^2(t), \quad -x_3^2(t)y_3(t) - y_3^3(t)\ ]^T.$$

The linearized system of (5) is

$$u'(t) = Au(t) + Bu(t-\tau). \tag{6}$$

Lemma 1IfR (=A+B) is a nonsingular matrix, but isnot a positive definite matrix, then there exists a unique equilibrium point for system (4) (or (5)).

Proof Noting that $f(u(t))$ is a continuous function and only $f(0) = 0$. Obviously, zero is an equilibrium point of system (4) (or (5)). Now we prove that zero is a unique equilibrium point of system (4) (or (5)). Assume that $u^* = [x_1^*,\ y_1^*,\ x_2^*,\ y_2^*,\ x_3^*,\ y_3^*\ ]^T \ne 0$ is an equilibrium point of system (5), then we have the following algebraic equation:

$$Au^* + Bu^* + f(u^*) = (A+B)u^* + f(u^*) = Ru^* + f(u^*) = 0. \tag{7}$$

From (7) we get

$$Ru^* = -f(u^*). \tag{8}$$

Let M_i be the matrix obtained from R by replacing column i of R by $-f(u^*)$. Since R is a nonsingular matrix, then the determinant of R is not equal zero. By the Cramer's Rule of linear algebra, $x_1^* = \frac{\det M_1}{\det R}$, $y_1^* = \frac{\det M_2}{\det R}$, $x_2^* = \frac{\det M_3}{\det R}$, $y_2^* = \frac{\det M_4}{\det R}$, $x_3^* = \frac{\det M_5}{\det R}$, $y_3^* = \frac{\det M_6}{\det R}$.

This implies that we must have $x_1^* = y_1^* = x_2^* = y_2^* = x_3^* = y_3^*$=0, and zero is a unique equilibrium point of system (4) (or (5)). The proof is completed.

Lemma 2 All solutions of system (4) are bounded.
Proof To prove the boundedness of the solutions in system (4), we construct a Lyapunov function $V(t) = \sum_{i=1}^{3}\frac{1}{2}[x_i^2(t) + y_i^2(t)]$. Calculating the derivative of $V(t)$ through system (4) we get

$$V(t)|_{(4)} = \sum_{i=1}^{3}[x_i(t)x_i^{'}(t) + y_i(t)y_i^{'}(t)]$$
$$\leq \sum_{i=1}^{3}(|a_{ii} + k_{ii}|x_i^2(t) + |\omega_i||x_i(t)y_i(t)|) + \sum_{i=1}^{3}|k_{i2}||x_i(t)x_2(t)|$$
$$+\sum_{i=1}^{3}|k_{i3}||x_i(t)x_3(t)| - \sum_{i=1}^{3}\left(x_i^4(t) + 2x_i^2(t)y_i^2(t) + y_i^4(t)\right). \quad (9)$$

Obviously, when $x_i(t)$, $y_i(t)$ ($1\leq i \leq 3$) tend to infinity, $x_i^4(t)$, $x_i^2(t)y_i^2(t)$, $y_i^4(t)$ are higher order infinity than $x_i^2(t)$, $x_i(t)y_i(t)$, respectively. Therefore, there exists suitably large L>0 such that $V(t)|_{(4)} < 0$ as $|x_i(t)| > L$, $|yit|>L$. This means that all solutions of system (4) are bounded.

III. STABILITY AND OSCILLATION OF THESOLUTIONS

Theorem 1 Assume that zero is the unique equilibrium point of system (4) (or(5)) for selecting parameter values. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ be characteristic values of matrix$A$ and B, respectively. If each$\alpha_i < 0$, or $\mathrm{Re}(\alpha_i) < 0$, and $|\mathrm{Re}(\alpha_i)| > Re(\beta_i)(1 \leq i \leq 6)$. Then the trivial solution of system (4) is stable.
Proof According to the basic theory of functional differential equations [21], the unique trivial solution of system (6) is stable under the restrictive conditions. Noting that $f(u(t))$is a higher order infinitesimal as$x_i(t) \to 0$, and $y_i(t) \to 0$. Thus, the stability of the trivial solution of system (6) implies the stability of the trivial solution of system (4). The proof is completed.

Theorem 2 Assume that zero is the unique equilibrium point of system (4) (or (5)) for selecting parameter values. Let $m = \max\{a_{11} + |-\omega_1|,\ a_{33} + |-\omega_2|,\ a_{55} + |-\omega_3|\}$, $n = \max\{|-k_{11}| + k_{21} + k_{31},\ |-k_{22}| +$ $k12+k32,\ -k33+k13+k23$, if the following inequality holds:

$$m + n > 0. \quad (10)$$

Then system (5) has an oscillatory solution
Proof Obviously, if the trivial solution of system (6) is unstable, then the trivial solution of system (5) is unstable. Therefore, we only need to prove the instability of the trivial solution of system (6). Let $v(t) = \sum_{i=1}^{3}(|x_i(t)| + |y_i(t)|)$, from (6) we have

$$v^{'}(t) \leq mv(t) + nv(t-\tau). \quad (11)$$

Consider the scalar differential equation

$$z^{'}(t) = mz(t) + nz(t-\tau). \quad (12)$$

According to the comparison theorem of differential equation, we have $v(t) \leq z(t)$.For equation (12), the characteristic equation associated with (12) is given by

$$\lambda = m + ne^{-\lambda\tau}. (13)$$

We claim that there exists a positive characteristic root of equation (13). Indeed, let

$$g(\lambda) = \lambda - m - ne^{-\lambda\tau}. \quad (14)$$

Then $g(\lambda)$ is a continuous function of λ. From condition (10), we have $g(0) = -m - n < 0$. On the other hand, $\lim_{\lambda\to\infty} e^{-\lambda\tau} = 0$. Thus, there exists a suitably large positive numberλ, say $\lambda_1$such that $g(\lambda_1) = \lambda_1 - m - ne^{-\lambda_1\tau} > 0$. It means that there exists a λ^* where $\lambda^* \in (0, \lambda_1)$ such that $g(\lambda^*) = 0$ from the Intermediate Value Theorem. In other words, λ^* is a positive characteristic root of equation (14), implying that the trivial solution of equation (12) is unstable. Since $v(t) \leq z(t)$, this means that the trivial solution of equation (11) is unstable. It suggested that the trivial solution of system (6) is unstable. Since system (4) has a unique equilibrium point, and all solutions are bounded, this instability of unique equilibrium point will force system (4) (or (5)) to generate an oscillatory solution.

IV. SIMULATION RESULTS

The simulation is based on the system (4), first the parameters are selected as follows: $a = 0.12$, $b = 0.15, c = 0.18, \omega_1 = 1.15, \omega_2 = 1.35, \omega_3 = 1.25, k_{11} = 0.65, k_{12} = 1.15, k_{13} = 0.96, k_{21} = 1.15, k_{22} = 0.55, k_{23} = 0.98, k_{31} = 0.95, k_{32} = 0.68, k_{33} = 0.58$. The time delays $\tau_1 = 0.85, \tau_2 = 0.95, \tau_3 = 0.75$. Then the characteristic values of A are $-1.3400 \pm 1.1500i, -1.4300 \pm 1.3500i, -0.8700 \pm 1.2500i$, and the characteristic values of B are $1.3624, 1.3624, -1.7597, -1.7597, -1.3845, -1.3845$. One can see that each $\mathrm{Re}(\alpha_k) < 0$, and the solutions of system (4) are stable (see Fig. 1). Then we change the parameters as: $a = 0.15, b = 0.25, c = 0.28, \omega_1 = 1.15, \omega_2 = 1.35, \omega_3 = 1.25, k_{11} = 0.45, k_{12} = 1.15, k_{13} = 0.96, k_{21} = 1.25, k_{22} = 0.55, k_{23} = 0.96, k_{31} = 1.15, k_{32} = 0.68, k_{33} = 0.58$. The time delays $\tau_1 = 0.85, \tau_2 = 1.05, \tau_3 = 1.15$. The characteristic values of A are $-1.5100 \pm 1.1500i, -1.4100 \pm 1.3500i, -0.9700 \pm 1.2500i$, and the characteristic values of B are $1.5608, 1.5608, -1.3408, -1.3408, -1.7000, -1.7000$. One can see that each $\mathrm{Re}(\alpha_k) < 0$, the solutions of system (4) still are stable (see Fig. 2). However, the condition $|\mathrm{Re}(\alpha_k)| > Re(\beta_k)(1 \leq k \leq 6)$both in figure 1 and figure 2 are not satisfied. This means that Theorem 1 only is a stronger sufficient condition. We then select the parameters as: $a = 0.5, b = 0.6, c = 0.8, \omega_1 = 0.75, \omega_2 = 0.85, \omega_3 = 0.65, k_{11} = 0.35, k_{12} = 0.45, k_{13} = 0.55, k_{21} = 0.15, k_{22} = 0.25, k_{23} = 0.32, k_{31} = 0.15, k_{32} = 0.24, k_{33} = 0.18$. The time delays $\tau_1 = 1.5, \tau_2 = 1.2, \tau_3 = 1.6$. We see that $m = 1.24, n = 1.05$, so $m + n > 0$. The conditions of Theorem 2 are satisfied. System (4) generates an oscillatory solution (see Fig. 3). When the parameters are selected as: $a = 0.8, b = 0.6, c = 0.5, \omega_1 = 0.25, \omega_2 = 0.55, \omega_3 = 0.35, k_{11} = 1.35, k_{12} = 0.75, k_{13} = 0.5, k_{21} = 0.85, k_{22} = 1.25, k_{23} = 0.62, k_{31} = 0.55, k_{32} = 0.64, k_{33} = 1.18$. The time delays $\tau_1 = 0.5, \tau_2 = 0.8, \tau_3 = 0.6$.One can see that $m = 1, n = 2.75$, so $m + n > 0$ still holds. System (4) generates an oscillatory solution (see Fig. 4). In order to see the effect of time delays, we increase delays as $\tau_1 = 3.5, \tau_2 = 3.8, \tau_3 = 3.6$. The other parameters are the same as in Fig.4, we see that the oscillatory behavior is still maintained. However, the oscillatory amplitude and frequency all are changed (see Fig. 5).

V. CONCLUSION

In this paper, we have discussed the oscillatory behavior of the solutions for a model of coupled limit cycle oscillators with delays. Based on mathematical analysis method, we provided some sufficient conditions to guarantee the stability and oscillation of the solutions. Some simulations are provided to indicate the results of the criteria.

Fig. 1 Convergence of the solutions, delays: 0.85, 0.95, 0.75.

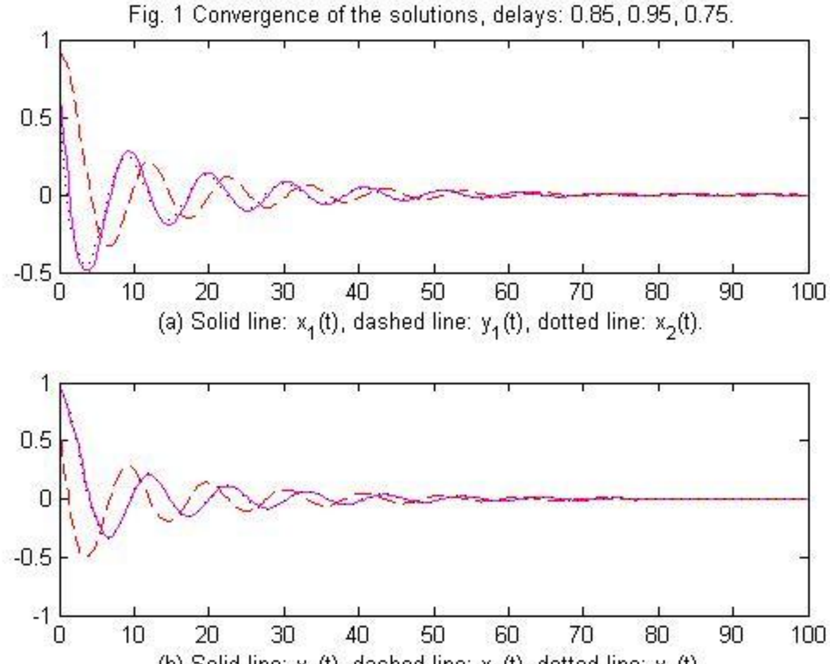


Fig. 2 Convergence of the solutions, delays: 0.85, 1.05, 1.15.

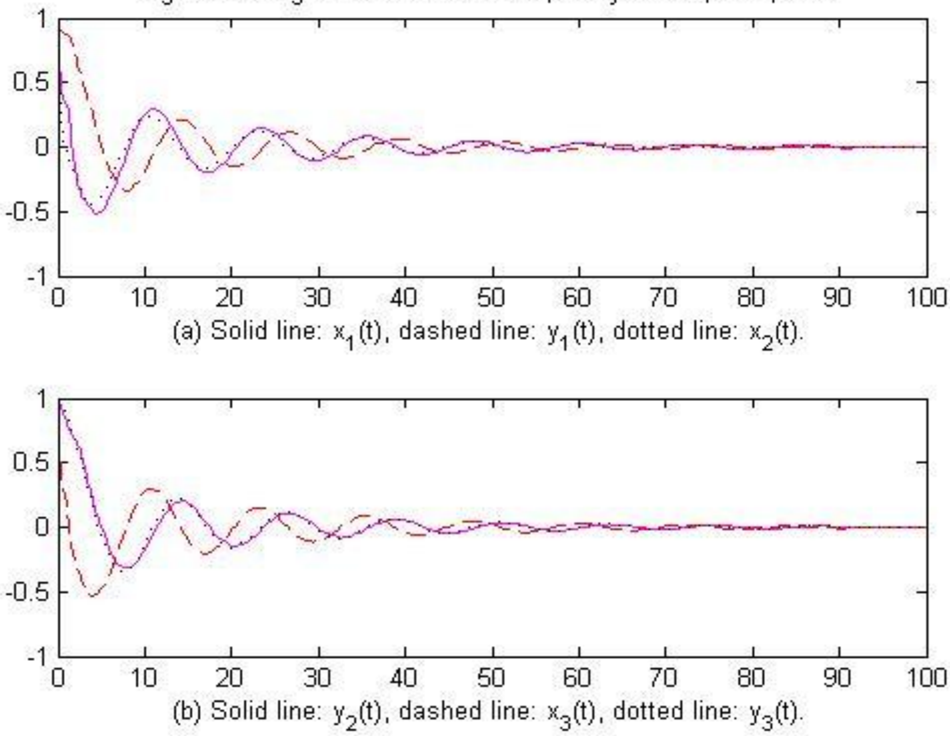


Fig. 3 Oscillation of the solutions, delays: 1.5, 1.2, 1.6.

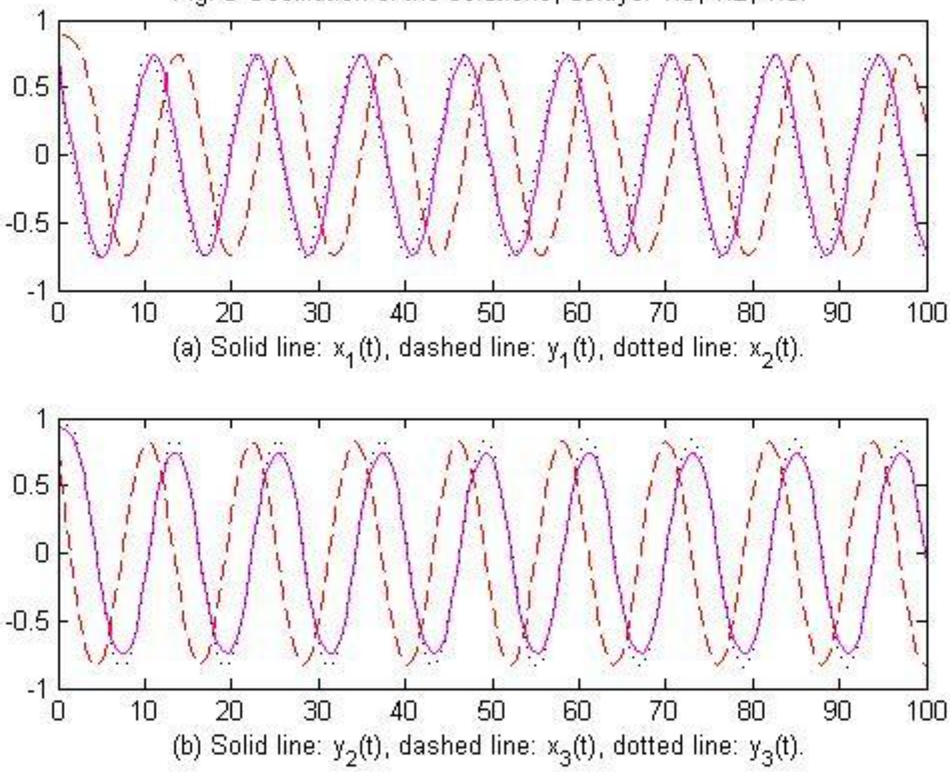
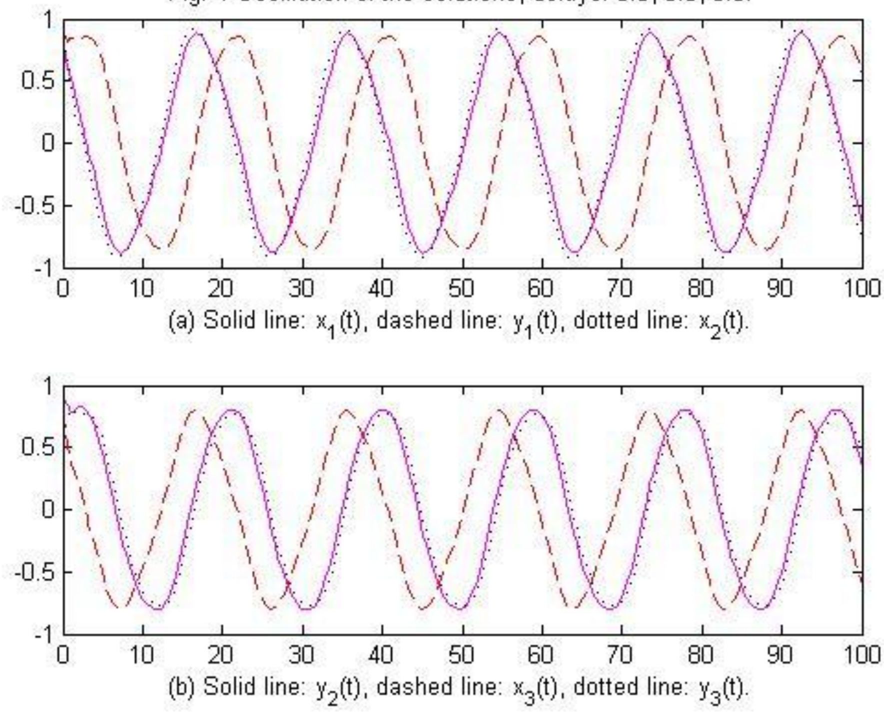


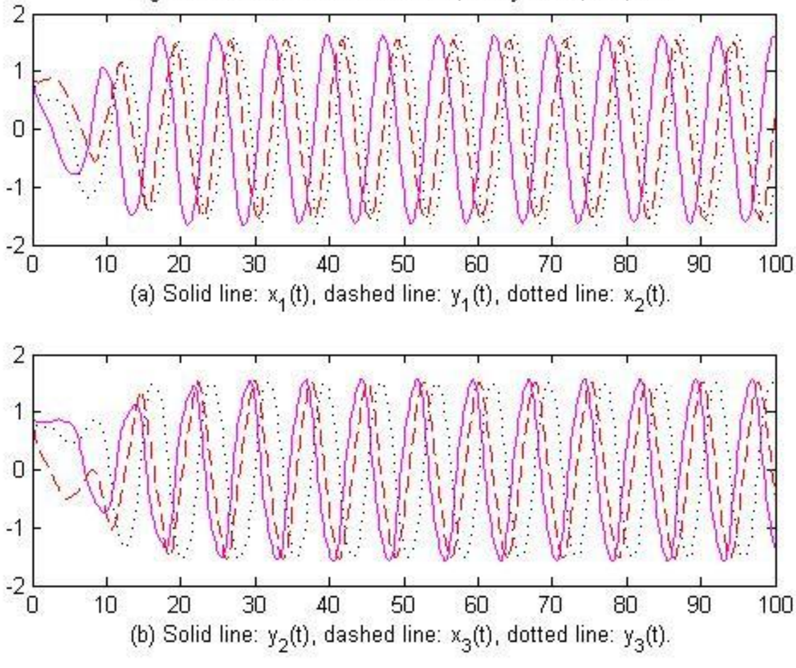
Fig. 4 Oscillation of the solutions, delays: 0.5, 0.8, 0.6.



(a) Solid line: $x_1(t)$, dashed line: $y_1(t)$, dotted line: $x_2(t)$.

(b) Solid line: $y_2(t)$, dashed line: $x_3(t)$, dotted line: $y_3(t)$.

Fig. 5 Oscillation of the solutions, delays: 3.5, 3.2, 3.6.



(a) Solid line: $x_1(t)$, dashed line: $y_1(t)$, dotted line: $x_2(t)$.

(b) Solid line: $y_2(t)$, dashed line: $x_3(t)$, dotted line: $y_3(t)$.

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