

Flow Field and Temperature Distribution in a Trapezoidal Rooftop Enclosure using Comsol Multiphysics

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-----ABSTRACT-----

In this study, laminar air flow and temperature distribution in a trapezoidal rooftop enclosure when heated isothermally from the inclined walls (summer condition) and when heated from the base wall (winter condition) are examined using COMSOL MULTIPHYSICS code. The effects of Rayleigh number, pitch angle, and the heating side on the flow structure and temperature distribution within the enclosure are investigated. Results indicate that, for winter condition, at low Raleigh number, the flow patterns within the enclosure are smooth and perfectly normal to the isotherms indicating the dominance of conduction. As Rayleigh number increases, flow slowly become convective dominant with more cells developed within the enclosure. At a low pitch angle, multiple heat flow circulations with high intensity between the cold inclined and hot base wall results in multicellular flow structure within the enclosure. But as the pitch angle increases, the number of cells reduces as small cells emerged to form bigger ones. In summer condition, two large counter-rotating cells are observed for the Ra and aspect range ratio considered. The practical significance of the results is that the flow patterns and thermal characteristics of the attic space presented will be of a great value to professionals engaged in design and analysis of building attics, to ensure proper functions of the attics and its energy efficiency.

KEYWORDS: Laminar Natural Convection, Pitch Angle, Trapezoidal Rooftop Shape

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I. INTRODUCTION

Natural convection heat transfer is usually occurred due to buoyancy forces and temperature difference in enclosure and its application is common in engineering field such as solar collector, heat exchanger, building heating and cooling etc. In most of these applications, the cavity shapes can be curvilinear such as triangular and trapezoidal. Natural convection in rooftop enclosure is gaining importance for various processing applications in engineering field as proposed by [1],[2],[3],[4]. In roof design, the attic space is often given paramount consideration because its thermal characteristics have great influence on the conditions of the space directly below it. In tropical climate, conventional types of roof construction suffer from excessive midday overheating due to the high solar radiation incident on the surface area. Solar radiation absorption will cause the roofing sheets to become very hot in the midday sun and a low-pitched roof, common in the tropics, is apt to trap heat in the attic. The significant amount of the cooling load in residential and industrial buildings for an air conditional system is as a result of the heat transfer across the ceiling from the attic to the interior part of the building as stated by [5].

One of the most important objectives for design and construction of houses is to provide thermal comfort for occupants and to minimise the energy costs associated with heating and air-conditioning. It is, therefore, desirable to have a thorough knowledge of the flow pattern and thermal characteristics of the attic space in realistic conditions.

Natural convection heat transfer and fluid flow in enclosed spaces has been studied extensively in recent years especially in response to energy-related applications [6],[7] for the enclosures heated from below (winter condition), [8] numerically analyzed laminar natural convection in a roof with an isosceles triangular cross section for wintertime conditions. Base angles varying from 15 degrees to 75 degrees, was used for Rayleigh numbers ranging from 10^3 to 10^5 . Finite-volume method was used for the discretization of the governing equations. The effects of the Rayleigh number and base angle on the flow field and heat transfer were analyzed. It was observed that roofs having low base angles were not suitable for wintertime conditions because of high heat transfer rates from the isosceles triangular attic space of the building. [9]. Employed the Galerkin weighted residual finite element method to a stream function-vorticity-energy formulation for $7 \times 10^2 \leq Ra \leq 7 \times 10^5$ and $0.2 \leq AR \leq 20$ to obtain steady-state solutions for all the seven possible non-trivial combinations of isothermal and adiabatic boundary conditions at the walls of isosceles triangular enclosures. [10] studied the heat transfer details within a system of entrapped triangular enclosures filled with fluid with Prandtl number

ranging from 0.015 to 1000 for Rayleigh number within 10^2 to 10^5 using the Galerkin finite element method. At low Ra, it was found that the heat lines were smooth and perfectly normal to the isotherms indicating the dominance of conduction in the upper and lower triangles. As Ra increases, the flow slowly became convection dominant. Multiple heat flow circulations with high intensity were formed within the lower triangular domain especially for low Prandtl numbers, whereas, less intense convective heat flow circulations were observed for the upper triangle. Multiple circulations were absent for both the triangular domains involving fluids with higher Prandtl numbers. [11], observed that, for 10^2 to 10^5 values of Ra studied, the number of cells increases with the Rayleigh number, using a finite volume method. [12], observed that, for $10^2 \leq Ra \leq 10^5$ considered, the flow bifurcation is time-dependent. In another work conducted by [13] on the effect of attaching baffles to reduce the heat loss through the attic during winter shows that the purpose could be achieved and also a desired temperature could be maintained in the attic. [14] investigated the occurrence of pitchfork bifurcation under winter conditions for isoflux cases, it was observed that multicellular flow patterns sensitive to the pitch angles were obtained.

For the cases of the enclosure heated from the inclined walls (summer condition), [15] investigated the effects of roof pitch on air flow and heating load of sealed and vented attics for Gable-Roof residential buildings. A 2D unsteady computational fluid dynamics (CFD) model was employed to investigate the effects of roof pitch on air flow and heating load of both sealed and vented attics for gable-roof residential buildings. The simulation results show that air flow in the sealed attics was steady and asymmetric, while that in the vented attics was a combination of an essentially symmetric base flow and a periodically oscillating flow. For both the sealed and vented attic cases, the heating load was found to increase with the roof pitch. [16] numerically investigated convection patterns in isosceles triangular enclosure using finite difference method for solving the Navier–Stokes and energy equations. Two cases of thermal boundary conditions were considered for Grashof numbers (Gr) in the range of $10^3 \leq Gr \leq 10^8$ and for various aspect ratios in the range $0.25 \leq H/B \leq 2.0$, where H/B is the aspect ratio. They observed that the maximum values of stream functions and Nusselt numbers perform damping oscillations around the steady state values. [17] discovered that the thermal and flow fields remain always stable and the flow remained laminar for all cases of the Rayleigh number considered. [18] employed a finite-difference representation of the steady-state stream function-vorticity-energy formulation to analyse the flow pattern and heat transfer rate of air in a right-triangular enclosure for Ra range of 5.6×10^2 to 4.5×10^4 . They observed single cell laminar flow for all Ra. An analogous problem was simulated by [19]. They used the alternating direction implicit (ADI) numerical technique and finer grid to obtain multicellular flow structure for Ra up to 10^6 and AR range of 0.125 to 1.0.

[20] attempted comparing numerically-obtained data with previous experimental measurements for summer and winter conditions. For the summer condition, they found that conductive heat transfer prevails up to Ra value of 10^6 while multicellular flow patterns were obtained for the winter condition. [21] used finite volume method to study triangular enclosures for both summer and winter conditions within the range $10^3 \leq Ra \leq 10^5$ for $15^\circ \leq \theta \leq 75^\circ$. It was observed that, for winter condition, at small pitch angle, increasing Ra resulted to multicellular flow structure while, for summer condition, the temperature profile is always stable and stratified for all Ra and pitch angles. [22] studied the effect of alternating thermal boundary conditions on the vertical and inclined walls of a right-angled triangular enclosure with the horizontal bottom adiabatic for $10^3 \leq Ra \leq 10^5$ and $0.07 \leq Pr \leq 1000$. The results show that as the Rayleigh number increases, the flow structure changes from conduction to convection dominate. [23] carried out a comprehensive review of natural convection heat transfer in triangular enclosure.

II. METHOD/APPROACH

A trapezoidal roof enclosure of air-filled ($Pr=0.71$) with a cross-section as shown in Fig. 1 was used. The enclosure extension in the direction perpendicular to the cross-section is assumed more than double its width so that the flow and the heat transfer are taken to be two-dimensional as proposed by [24]. Two sets of boundary conditions were considered: enclosure heated from the inclined walls (summer condition) and enclosure heated from the base wall (winter condition).

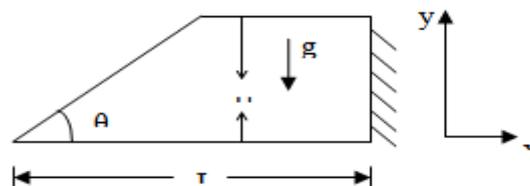


Fig. 1: Physical Mode

For summer condition, the inclined wall with upper horizontal wall represent aluminum exposed to solar radiation and heated to temperature of 30°C, 50°C and 70°C. The vertical wall is anadiabatic wall and the lower horizontal base, representing a ceiling above an air-conditioned space, was assumed cooled isothermally and maintained or kept at 23°C.

For the winter condition, the lower horizontal base depicting a ceiling was assumed heated by the warm air from the hearth within the space below to a temperature of 23°C, and the pitched roof was assumed covered with snow at 0°C. Three pitch angles 18°, 30°, 45° representing an aspect ratio range $0.3 \leq AR \leq 0.9$ were simulated. This, in combination with the thermal boundary condition, results in a range of Rayleigh number (Ra), $1.57 \times 10^5 \leq Ra \leq 9.85 \times 10^5$. The computational domain coincides with the physical domain (geometry). The computational model was developed as a whole using the COMSOL MULTIPHYSICS Version 3.5 code. The geometry for the trapezoidal roof enclosure was first developed, and then the subdomain and boundary conditions were defined for both the laminar flow, conductive and convective heat transfer using the appropriate equations and properties. Then tetrahedral meshes were developed using a higher density at the vertices for better accuracy. Hence, about 10^4 grid elements, for elemental thick, were used for the simulation.

The Thermo-physical properties of the fluid in the trapezoidal roof enclosure shown in Fig. 1 are assumed constant except the density. The Boussinesq approximation is invoked to relate the variation of density with temperature in the body force term and in this way, the temperature field and flow fields are coupled. With these assumptions, the governing equations for laminar natural convection flow in the trapezoidal cavity using conservation of mass, momentum and energy, are written[25].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(vu) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(vv) = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - g[1 - \beta(T - T_o)]$$

$$\frac{\partial}{\partial x}(uT) + \frac{\partial}{\partial y}(vT) = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

The equations can be transformed into a non- dimensional form using the relations below:

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha}, \quad V = \frac{vH}{\alpha}, \quad \Theta = \frac{T - T_a}{T_H - T_a}, \quad Nu = \frac{hH}{K}, \quad Pr = \frac{\mu}{\rho\alpha}$$

$$Ra = g\beta(T - T_a)H^3Pr/\nu^2$$

The non-dimensional form of the governing equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$\frac{\partial U^2}{\partial X} + \frac{\partial UV}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$

$$\frac{\partial UV}{\partial X} + \frac{\partial V^2}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \theta$$

$$\frac{\partial U\theta}{\partial X} + \frac{\partial V\theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)$$

Here X and Y are dimensionless coordinates varying along the horizontal and vertical directions, respectively; U and V are the dimensionless velocity components in the X and Y-directions, respectively; θ is the dimensionless temperature; P is the dimensionless pressure; Ra and Pr are Rayleigh and Prandtl numbers, θ is pitch angle, The height of the enclosure H is taken as the characteristic length in the definition of the Nusselt Number, h is the mean heat transfer Coefficient and K is the thermal conductivity of the fluid.

III. RESULTS AND DISCUSSION.

Effect of Pitch angle

It was observed that as the pitch angle increases, the flow field within the enclosure changes. The effect of the pitch angle on the velocity field is presented in Fig. 2: when the enclosure was heated from the base walls isothermally. In Fig. 2(a), the 18°-pitch makes the cold inclined and hot base walls to be closer, allowing excessive heat to be transfer between them resulting to a multicellular flow structure within the enclosure. In

Fig. 2(b), with the pitch angle increased to 30° , the four counter-rotating cells present in Fig. 2(a) have reduced to two but bigger cells. The bigger of the two cells occupies the mid-center of the enclosure. Also, velocity is high around the circumference of the cells but very low near the corners. For the 45° -pitch enclosure, Fig. 2(c), the two counter-rotating cells observed in Fig. 2(b) grew bigger but constrained to the bottom corners. The stagnant region has extended to about one-third of the height of the enclosure and the cell indicates a higher concentration of heat. The result agrees with the report of [26] on a right-angled triangular enclosure heated from the base wall that shows multicellular flow patterns that change with the aspect ratio.

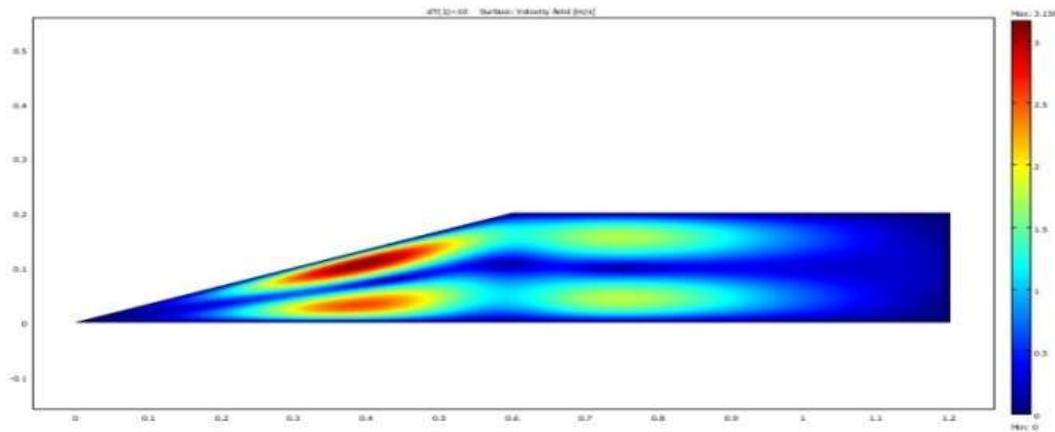


Fig. 2(a): Shows the Velocity field distribution when the enclosure was heated from the base walls at 18° -pitch,

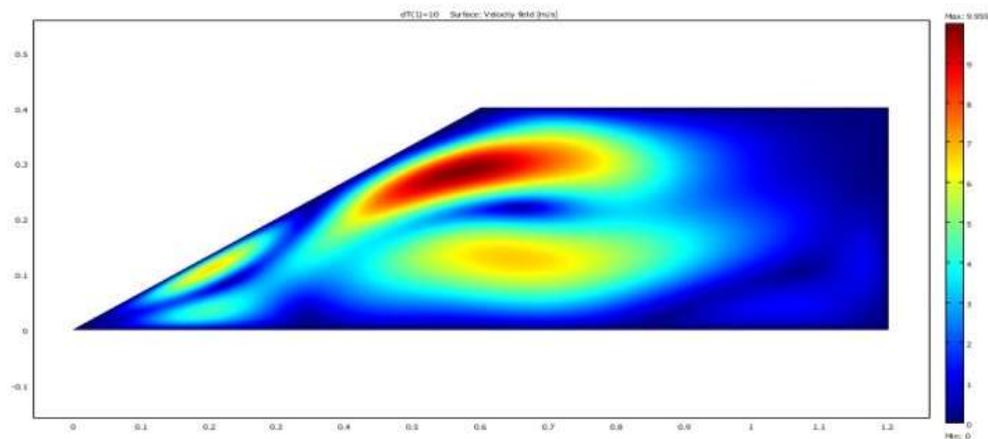


Fig. 2(b): Shows the Velocity field distribution when the enclosure was heated from the base walls at 30° -pitch,

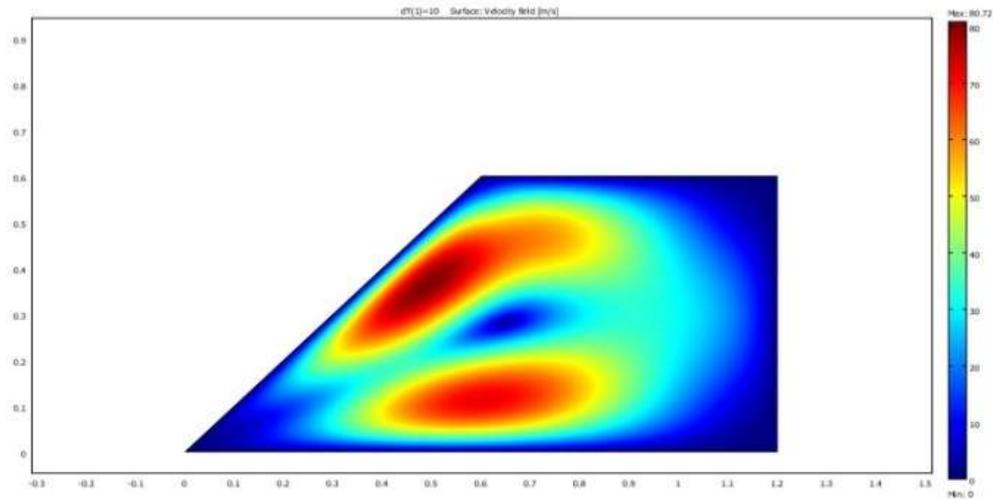


Fig. 2: Shows the Velocity field distribution when the enclosure was heated from the base walls at 45° -pitch.

Effect of Rayleigh number

The increase in Rayleigh number create a multicellular flow structure within the enclosure, as the Rayleigh number increases the number of cells increases, as shown in Fig. 3 (a-c) for 18° -pitch enclosure. At $Ra = 1.57 \times 10^5$ in fig. 3(a), a single cell fill the space. An elongated cell occupies about two-third of the space but as Ra increased to 5.41×10^5 , the elongated cell in Fig. 3(a) increase to three cells. At higher $Ra=9.85 \times 10^5$ as shown in Fig. 3(c), the cells increased to five, as the smaller cell in Fig 3(b) trifurcate. This scenario shows thorough mixing of the fluids within the enclosures.

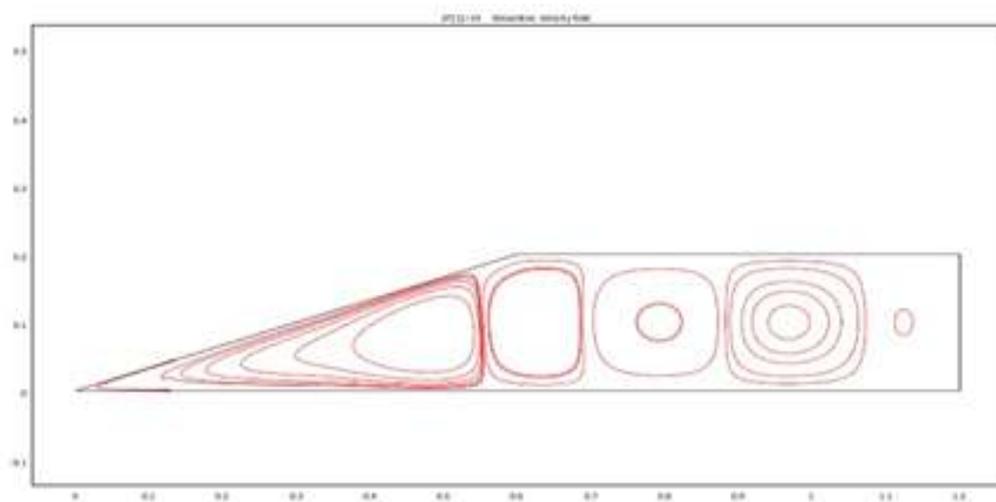


Fig. 3(a): Streamlines of the enclosure heated below at 18° - pitch, $Ra=1.57 \times 10^5$, $AR=0.3$

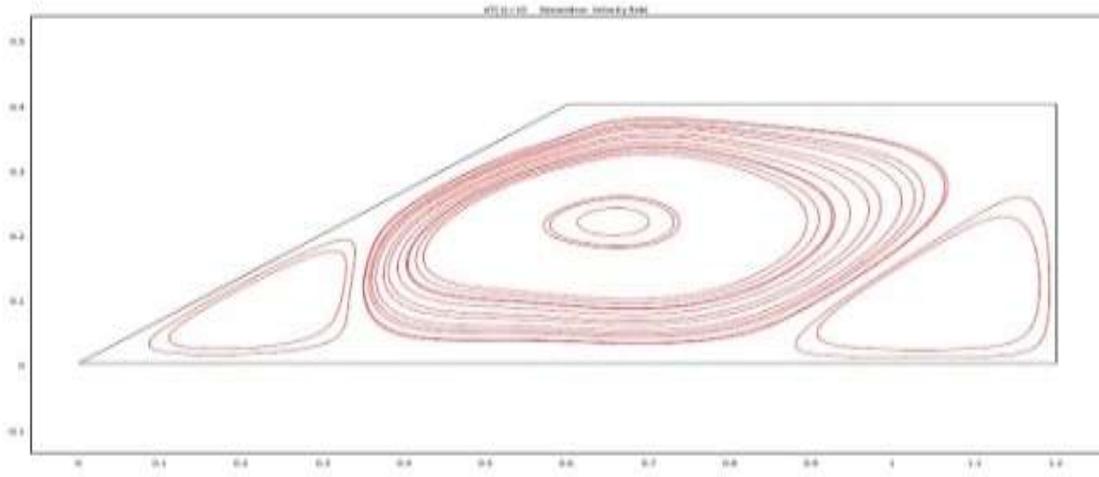


Fig. 3(b): Streamlines of the enclosure heated below at 30⁰- pitch, Ra=5.41 x 10⁵, AR=0.3

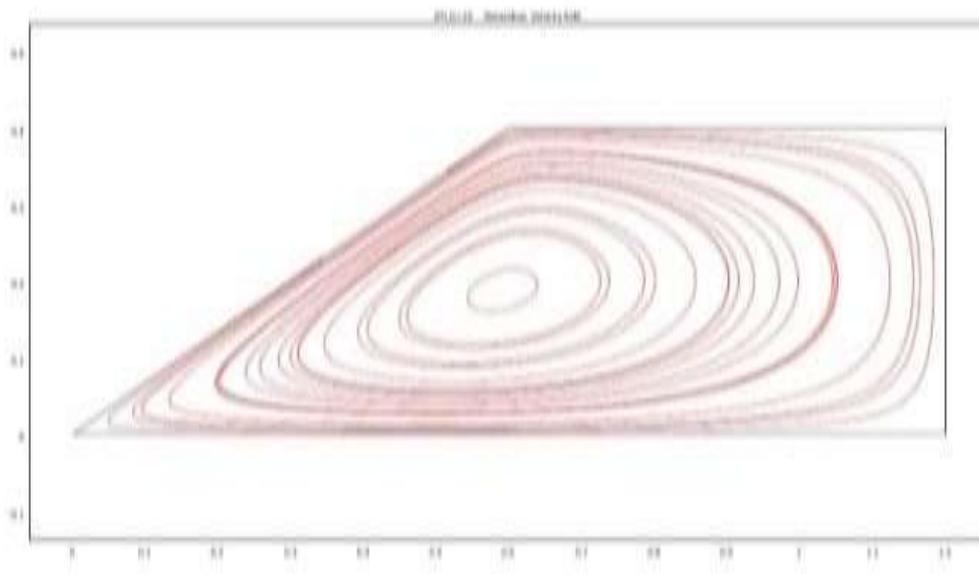


Fig. 3(c): Streamlines of the enclosure heated below at 45⁰- pitch, Ra =9.85 x 10⁵, AR = 0.3.

This result is similar to that of [27] who numerically investigated laminar natural convection in a triangular enclosure using a finite volume method and discovered that the number of counter-rotating cells increases with the Rayleigh number.

Effect of Heating Side

The heating side of the walls on the trapezoidal roof enclosure was observed to have a strong influence on the temperature and flow field pattern within the enclosures.

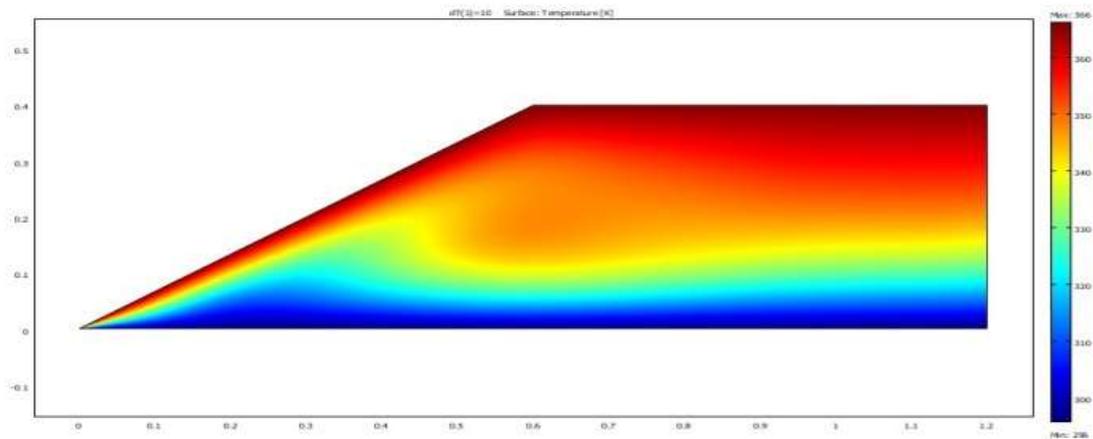


Fig. 4(a): Shows the mean temperature distribution within the enclosure at 30⁰-pitch, isothermally heated through the inclined walls $Ra=2.50 \times 10^6$

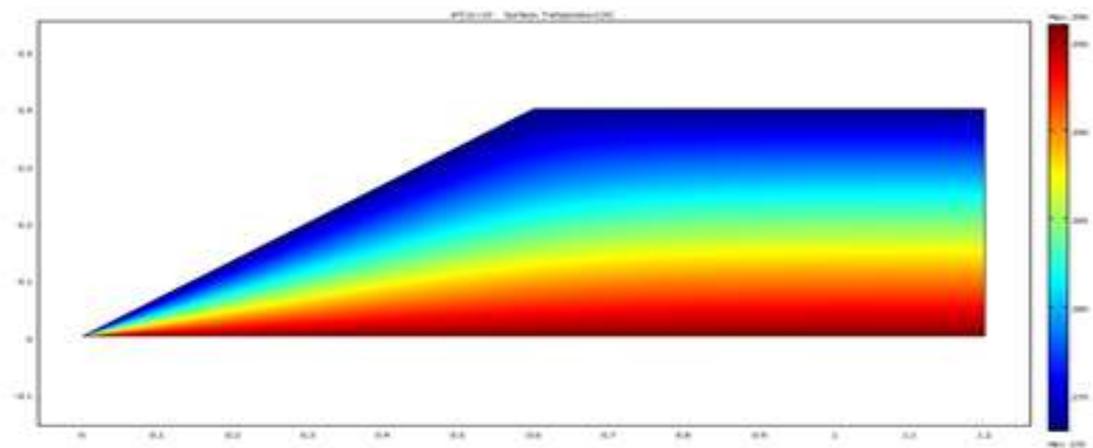


Fig. 4: shows the mean temperature distribution within the enclosure at 30⁰-pitch, isothermally heated through isothermally cold through the inclined wall, $Ra=2.31 \times 10^6$.

When the enclosure was heated isothermally through the inclined walls as shown in Fig. 4(a), the temperature is thermally stratified near the bottom corners. Hot air was found constrained to the upper part of the enclosure and movement of fluid at the mid-center is little. The results were supported by [28]. In Fig. 4(b), for the enclosure heated below, a large stratified cell having mild temperature was observed at the mid-center of the enclosure and hot air was found constrained to the base of the enclosure.

The Practical Significance of the Study

The practical significance of the study is to predict the positioning of heat extraction devices at high intensity areas of a trapezoidal roof enclosure. The unwanted heat trapped within the enclosures could be effectively extracted when heat extraction device is placed at high heat intensity areas as predicted. This would help in reducing the size of the air-conditioning system required to cool the space below, minimize the use of thermal insulation across the ceiling, and extends the period of thermal comfort within the space without reliance on mechanical air-conditioning thereby reducing the annual energy cost. The work is, also, expected to

be very useful to agriculturists engaging in rooftop drying of agricultural produce and also to professionals engage in the design and construction of buildings.

IV. CONCLUSION

In this study, flow field and temperature distribution in a trapezoidal rooftop enclosure using COMSOL Multiphysics as the design modeler, has been investigated for both summer and winter conditions. For the winter condition, results show that the high heat transfer between the hot base wall and the cold inclined walls led to multicellular flow structure with the number of cells reducing as the pitch angle increases. At low Raleigh number, the flow patterns within the enclosure are smooth and perfectly normal to the isotherms indicating the dominance of conduction. As Rayleigh number increases, flow slowly become convective dominant with more cells developed within the enclosure. For the summer condition, there were two counter-rotating cells within the enclosure for all the Rayleigh number examined.

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