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A Novel Three Phase Transfer Field Machine with Cage (Rotor) Windings

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ABSTRACT
The ultimate feature of every electrical machine, whether a generator, motor or transformer is its output characteristics. It is the yard-stick upon which the machine is evaluated. Obviously, the output characteristics of all conventional transfer field reluctance machines are much inferior to that of a conventional induction machine of comparable size and ratings. This is an attribute of their low direct axis reactance to quadrature axis reactance ratio, coupled with the excessive leakage reactance from the quadrature axis reactance. These are as a result of the salient nature of their rotor pole structures. To ameliorate these set-backs, their rotor designs have to be optimized. This is achieved by the introduction of cage (rotor) windings at the periphery of the machine shaft.

Key words*: Conventional induction machine, conventional transfer field machine, cage (rotor) winding, slip, slip frequency, leakage reactance etc.*

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I. INTRODUCTION

The inherent demerit of all cage-less three phase transfer field effect machines is their poor output characteristics such as low output power, low electromagnetic torque and low power factor, when compared with those of conventional three phase induction machine of related size and ratings. To enhance the output characteristics of the machine set, the leakage reactance has to be minimized. This is achieved by optimizing the rotor design. To this effect, additional windings known as rotor (cage) windings are wounded at the periphery of the rotor shaft connecting the two stack machines. Just as in auxiliary windings, the rotor windings, are transposed between the two machine sets, and then connected in parallel with the auxiliary windings. The idea for such connection is that when impedance are added to the rotor circuit, of the conventional T.f machine, the rotor power factor is improved. (Menta V. K. et al 2000).

Since the effect of improvement of power factor during starting predominates the increase in impedance or decrease of current, the torque of the motor is improved. To bring down the effect, such reduction in rotor-induced current or in the increase in rotor impedance, the auxiliary and cage windings of both machine sections are connected in parallel, but transposed between the two machine halves and then short circuited. Surely, the arrangement boosts the rotor induced current, owing to reduction in the overall impedance of the circuit.

The use of short-circuited rotor windings, would lead to considerable improvement in its output performance. The rotor windings do not only give rise to increase in the induced e.m.f. but also augment output power by effectively lowering the synchronous reactance of the output windings, thus leading to a higher output and greater synchronous stability.

To this end, there is a necessity to rise the output of the cage-less three phase transfer field machine by way of using circuits on the rotor structure, so as to augment the effect of saliency (E.S. Obe and A. Binder 2011).

II. THE MACHINE DESCRIPTION

The structural arrangement of the machine under study is shown in figure 1. Unlike the existing three phase transfer field cage –less machine counterpart, the three-phase transfer field machine with cage windings comprise two identical poly-phase reluctance machine with moving conductors (rotor windings), whose salient poles rotor are mechanically coupled together, such that their **d** and **q**axes are in space quadrature. As depicted in figure 1, the stator windings, are integrally wound. Each machine element has three sets of windings. Both sets of windings of the machine are identical. The stator (primary) and the auxiliary windings are housed at the stator slots. The main windings of the machine carry the excitation current, while the auxiliary and the rotor windings, carry the circulating current. The (2s-1) ω_0 low frequency current is confined in the auxiliary and the rotor windings without interfering with the supply. The main windings of the machine sets are connected in series while the auxiliary windings, though also in the stator are transposed between the two machine stacks. They are wound for the same pole number and both are star connected. The third set of windings known as the rotor (cage) windings are wounded at the periphery of the rotor shaft connecting the two machine sets. Just as in the auxiliary windings, the rotor (cage) windings are also transposed between the two machine stacks and then connected in parallel with the auxiliary windings (see figure 2).

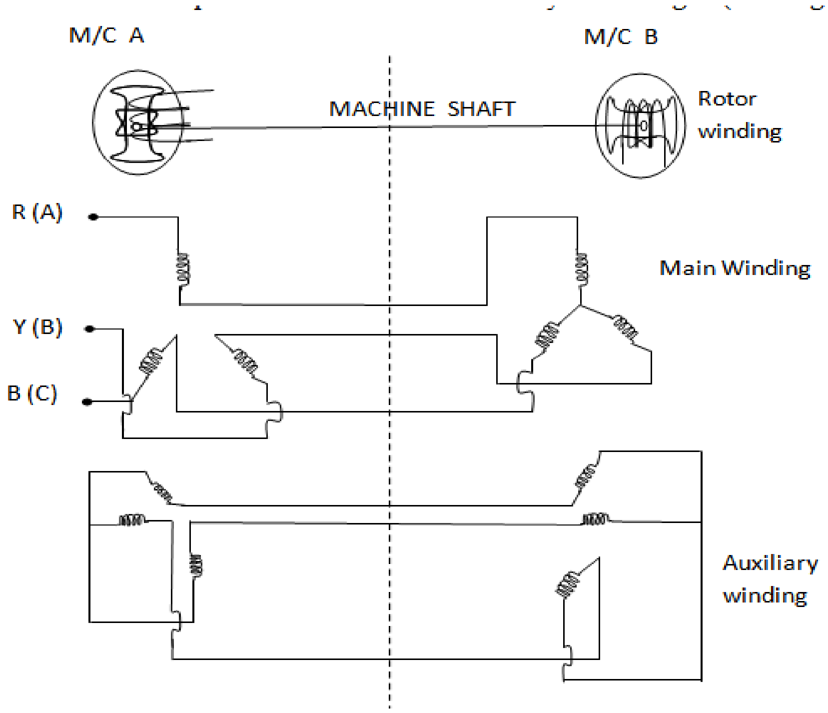

Figure 1: Connection diagram for three phase transfer field reluctance machine with rotor (cage) winding

Steady state analysis of three-phase transfer-field reluctance machine with rotor (cage) winding.
The steady state analysis of the configured machine can be done, using the schematic diagram of figure 2 below;

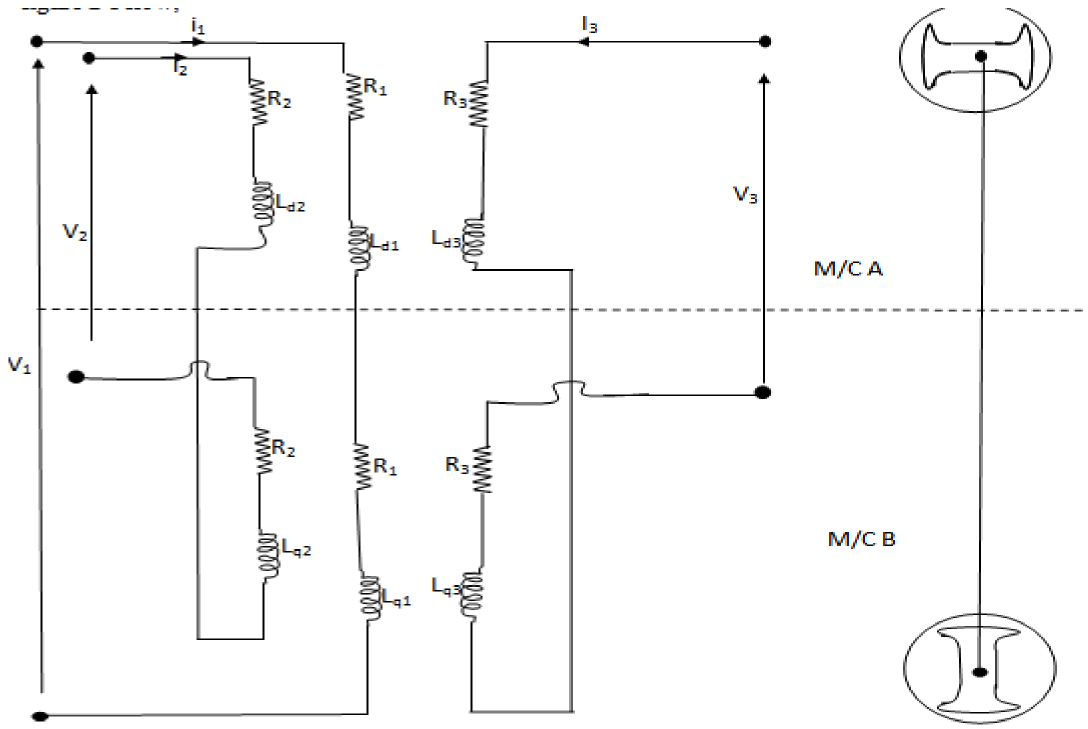

Figure 2: Per-phase schematic diagram of 3-phase transfer field machine with rotor (cage) windings.

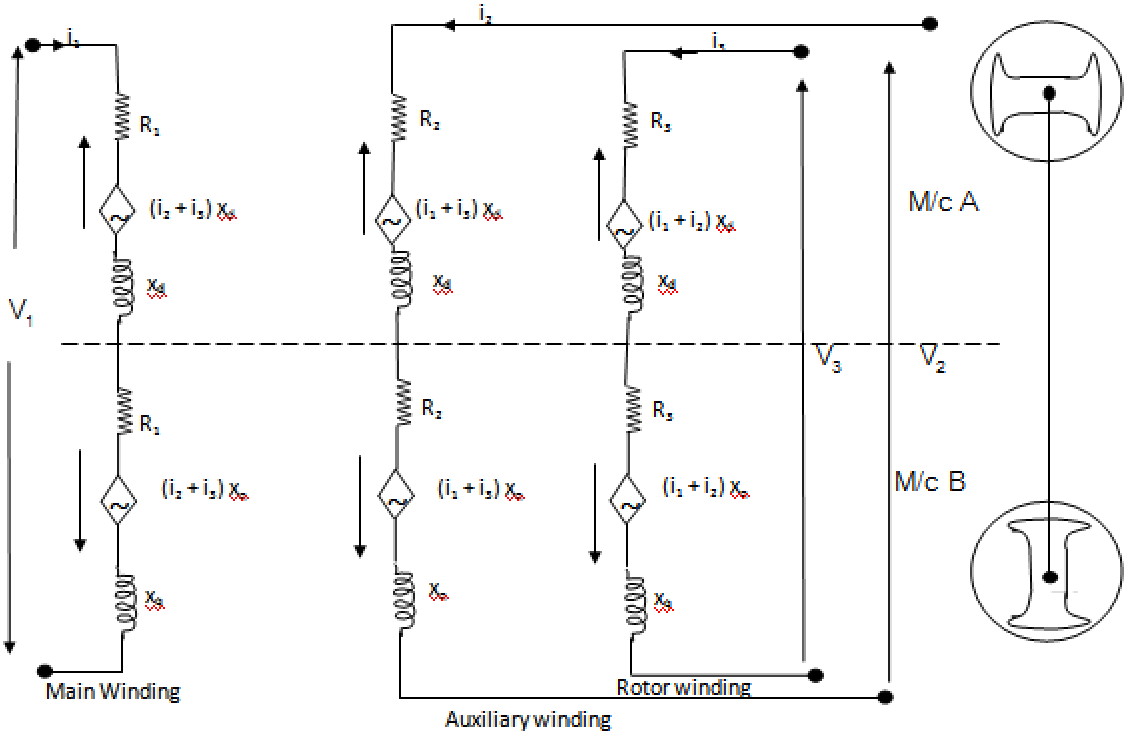

Figure 3 Modified schematic diagram of 3-phase transfer field machine with cage winding under stand-still condition, that is slip(s)=1

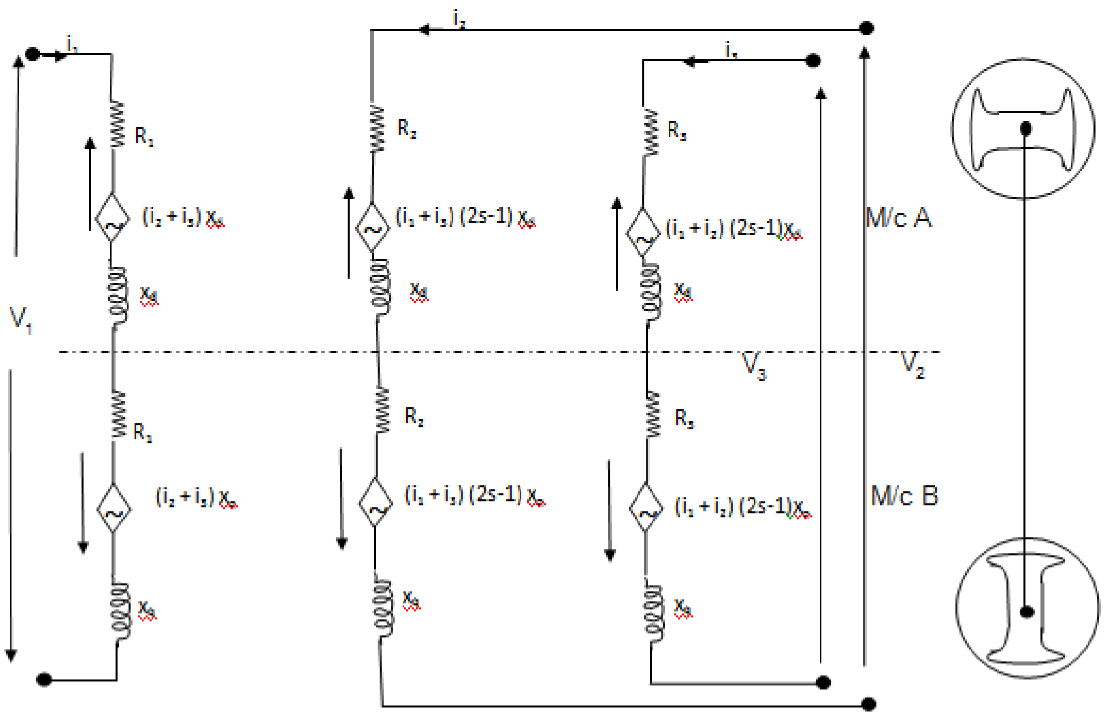

Figure 4: Per-phasemodified schematic diagram of three-phase transfer machine with cage winding under run condition, that is slip = (2s-1).

Where, V_1 = Main winding voltage
V_2 = Auxiliary winding voltage
V_3 = Cage (rotor) winding voltage

$L_{d1} = L_{d2} = L_{d3} = L_d$ = Direct axis inductances
$L_{q1} = L_{q2} = L_{q3} = L_q$ = Quadrature axis inductances
$R_1 = R_2 = R_3 = R$ = Resistance of the machine windings
i_1 = Current at the main windings
i_2 = Current at the Auxiliary windings
i_3 = Current at the rotor windings
Also , L_{12}, L_{13}. L_{21} L_{23} L_{31} and L_{32} are the mutual coupling between coil 1, 2, and 3 at the direct axis.
Similarly, $L_{1\acute{2}}$,$L_{1\acute{3}}$,L,$l_{2\acute{3}}$,$L_{3\acute{1}}$ and $L_{3\acute{2}}$ are the mutual couplings between coil 1, 2, and 3 at the quadrature axis.
Hence $L_{12} = L_{13} = L_{21} = L_{23} = L_{31} = L_{32} = k\sqrt{L_d L_d} = L_d$
Similarly, $L_{1\acute{2}}=L_{1\acute{3}}=L_{2\acute{1}}=L_{3\acute{1}} = L_{3\acute{2}} = k\sqrt{L_q L_q} = L_q$
Owing to the fact that the pole structure of the machine is salient in nature as in fig, $L_d \neq L_q$. that is;

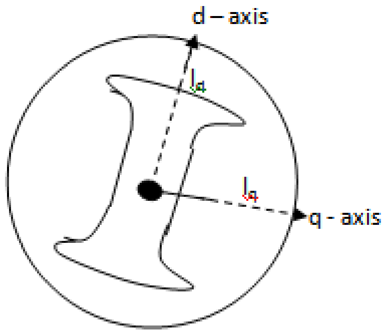

Figure 5 The Salient pole structure of the machine with d-axis and q-axis positions

From figure 5,

$$\left.\begin{aligned} L_d &= N^2P_d = \frac{N^2}{S_d}\\ L_q &= N^2P_q = \frac{N^2}{S_q}\\ S_d &= \frac{l_d}{\mu_A}\\ S_q &= \frac{l_q}{\mu_A} \end{aligned}\right\} \qquad (1)$$

Also from figure 5, $l_q > l_d$, at constant μ_A and N, $S_q > S_d$
$\Rightarrow L_d > L_q$
Where, l_d = Direct axis air-gap length,
l_q = Quadrature axis are gap length
P_d, S_d = Direct axis permeance and reluctance respectively
P_q, S_q= Quadrature axis permeance and reluctance respectively
L_d = Direct axis inductance
L_q = Quadrature axis inductance
Taking the voltage equation of the machine sections of figure 2, we obtain;

$V_1 = (R_1 + R_1)\, i_1 + L_d\frac{di_1}{d_t} + L_q\frac{di_1}{d_t} + L_{12}\frac{di_2}{d_t} - L_{1\acute{2}}\frac{di_2}{d_t} + L_{13}\frac{di_3}{d_t} - L_{1\acute{3}}\frac{d_{i3}}{d_t}$

$V_1 = (R_1 + R_1)\, i_1 + L_d\frac{di_1}{d_t} + L_q\frac{di_1}{d_t} + L_d\frac{di_2}{d_t} - L_q\frac{di_2}{d_t} + L_d\frac{di_3}{d_t} - L_q\frac{d_{i3}}{d_t}$

$V_1 = 2R_1 i_1 + j\omega L_d i_1 + j\omega L_q i_1 + j\omega L_d i_2 - j\omega L_q i_2 + j\omega L_d i_3 - j\omega L_q i_3$

$V_1 = 2R_1 i_1 + jx_d i_1 + jx_q i_1 + jx_d i_2 - jx_q i_2 + jx_d i_3 - jx_q i_3$

$V_1 = 2R_1 i_1 + j(x_d + x_q - (x_d - x_q)\, i_1 + j(x_d - x_q)i_1 + (x_d - x_q)i_2 + j(x_d - x_q)i_3$

$\therefore V_1 = 2R_1 i_1 + j\,(2x_q)i_1 + (x_d - x_q)\,(i_1 + i_2 + i_3)$ (2)

$V_2 = (R_2 + R_2)i_2 + L_d\frac{d_{i2}}{d_t} + L_q\frac{d_{i2}}{d_t} + L_{21}\frac{d_{i1}}{d_t} - L_{2\acute{i}}\frac{d_{i1}}{d_t} + L_{23}\frac{d_{i3}}{d_t} - L_{2\acute{3}}\frac{d_{i3}}{d_t}$

$V_2 = (R_2 + R_2)\, i_2 + L_d\frac{di_2}{d_t} + L_q\frac{di_2}{d_t} + L_d\frac{di_1}{d_t} - L_q\frac{di_1}{d_t} + L_d\frac{di_3}{d_t} - L_q\frac{d_{i5}}{d_t}$

$V_2 = 2R_2 i_2 + (2s - 1)\,[\, j\omega L_d i_2 + j\omega L_q i_2 + j\omega L_d i_1 - j\omega L_q i_1 + j\omega L_d i_3 - j\omega L_q i_3]$

$V_2 = 2R_2 i_2 + (2s - 1)\,[\, jx_d i_2 + jx_q i_2 + jx_d i_1 - jx_q i_1 + jx_d i_3 - jx_q i_3]$

$V_2 = 2R_2 i_2 + (2s - 1)\,[\, j(x_d + x_q - (x_d - x_q))\, i_2 + j(x_d - x_q)i_2 + j(x_d - x_q)i_1 + j(x_d - x_q)i_3]$

$V_2 = 2R_2 i_2 + (2s - 1)\,[\, j\,(2x_q)\, i_2 + j\,(x_d - x_q)\,(i_2 + i_1 + i_3)$

$\therefore \frac{V_2}{2s-1} = \frac{2R_2 i_2}{2s-1} + j(2x_q)i_2 + j(x_d - x_q)(i_1 + i_2 + i_3)$ (3)

Also, $V_3 = (R_3 + R_3)i_3 + L_d\frac{d_{i3}}{d_t} + L_q\frac{d_{i3}}{d_t} + L_{31}\frac{d_{i1}}{d_t} - L'_{31}\frac{d_{i1}}{d_t} + L_{32}\frac{d_{i2}}{d_t} - L'_{32}\frac{d_{i2}}{d_t}$

$V_3 = (R_3 + R_3)i_3 + L_d\frac{d_{i3}}{d_t} + L_q\frac{d_{i3}}{d_t} + L_d\frac{d_{i1}}{d_t} - L_q\frac{d_{i1}}{d_t} + L_d\frac{d_{i2}}{d_t} - L_q\frac{d_{i2}}{d_t}$

$V_3 = 2R_3 i_3 + (2s-1)[j\omega L_d i_3 + j\omega L_q i_3 - j\omega L_q i_1 + j\omega L_d i_1 - j\omega L_q i_2 + j\omega L_d i_2]$

$V_3 = 2R_3 i_3 + (2s-1)[jx_d i_3 + jx_q i_3 + jx_d i_1 - jx_q i_1 + jx_d i_2 - jx_q i_2]$

$= 2R_3 i_3 + (2s-1)[j(x_d + x_q - (x_d - x_q))i_3 + j(x_d - x_q)i_3 + j(x_d - x_q)i_1 + j(x_d - x_q)i_2$

$= 2R_3 i_3 + (2s-1)[j(2x_q)i_3 + j(x_d - x_q)(i_3 + i_1 + i_2)$

$\Rightarrow \frac{V_3}{2s-1} = \frac{2R_3 i_3}{2s-1} + j(2x_q)i_3 + j(x_d - x_q)(i_1 + i_2 + i_3)$ (4)

Equation 2-4 result an equivalent circuit of figure 6 below

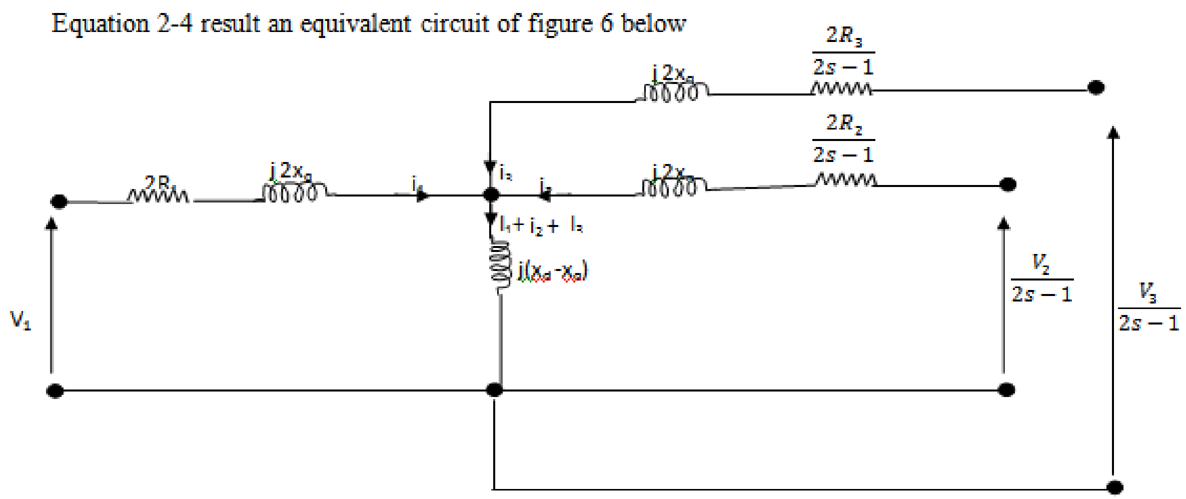

Figure 6 Per-phase steady state equivalent circuit of 3-phase transfer field machine with rotor windings

Since the rotor and auxiliary windings are short circuited, $\frac{V_3}{2s-1} = 0$, $\frac{V_2}{2s-1} = 0$. Hence, figure6 yields;

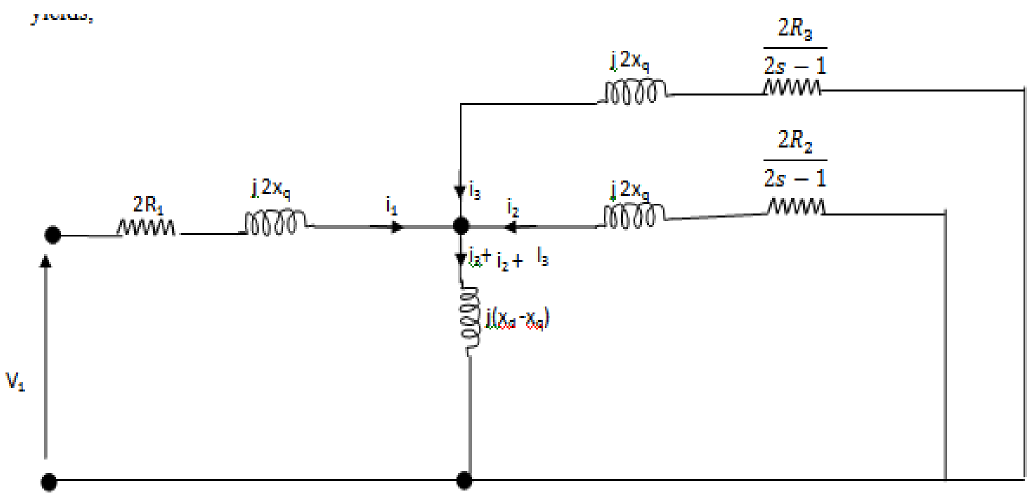

Figure 7 Per-phase steady state equivalent circuit of 3-phase transfer field machine with rotor (cage winding) when rotor and auxiliary windings are short circuited.

From figure 3, So far $R_1 = R_2 = R_3 = R$,

$$Z_2 \uparrow\uparrow Z_3 = \left[\frac{\left(\frac{2R}{2s-1}\right) + (j2x_q)\left(\frac{2R}{2s-1}\right) + (j2xq)}{\left(\frac{2R}{2s-1}\right) + j2x_q + \left(\frac{2R}{2s-1}\right) + (j2x_q)}\right]$$

$$= \frac{\left[\left(\frac{2R}{2s-1}\right) + (j2x_q)\right]^2}{2\left[\left(\frac{2R}{2s-1}\right) + (j2x_q)\right]}$$

$= \frac{\frac{2R}{2s-1} + j2\,x_q}{2}$

$= \frac{2R}{2(2s-1)} + \frac{j2\,x_q}{2}$

$\therefore\ Z_2 \uparrow\uparrow Z_3 = \frac{R}{2s-1} + jxq$ (5)

Hence, figure7 can be redrawn as below;

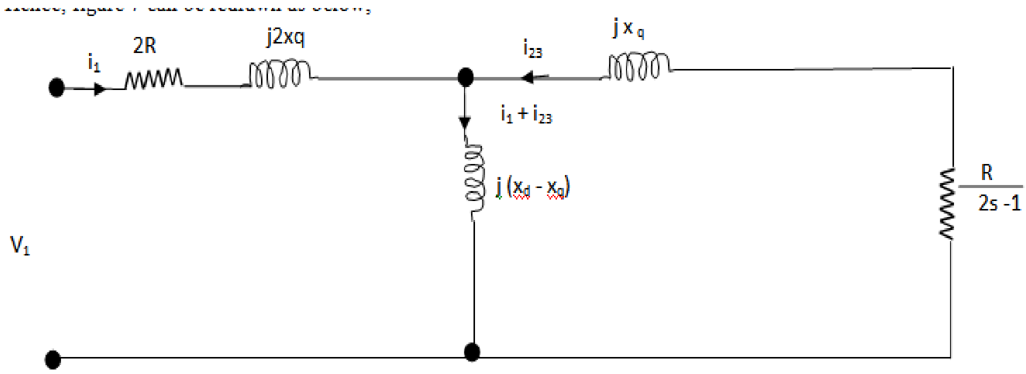

Figure 8 Modified steady state equivalent circuit of the machine under short circuit condition.

Also, $\frac{2R}{2s-1} = R + \frac{2R\,(1-s)}{2s-1}$
Hence, Figure 8 becomes;

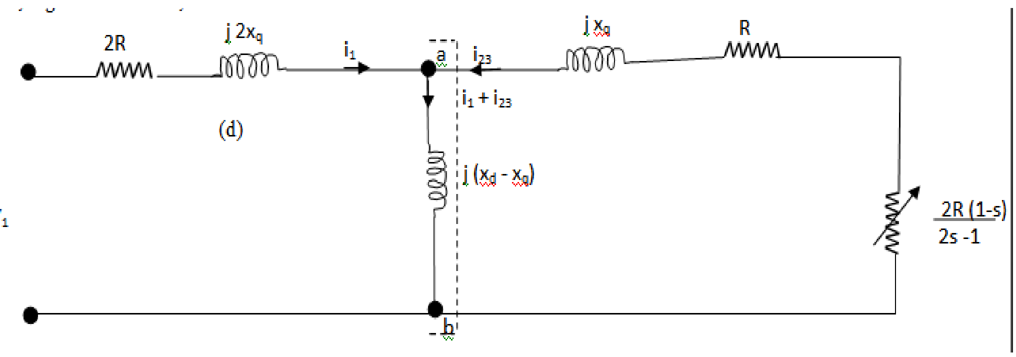

Figure 9 Per-phase steady state equivalent circuit machine with electrical (representing Cu-loss) and mechanical loads.

From figure 9,

$V_{TH} = \left[\frac{j(x_d - x_q)}{j(x_d - x_q) + (2R + j2\,x_q)}\right] V_1$

$= \left[\frac{j(x_d - x_q)}{2R + j\,(x_d - x_q + 2x_q)}\right] V_1$

If 2R <<j (xd – xq + 2 xq), we have;

$V_{TH} = \left[\frac{j(x_d - x_q)}{j(x_d - x_q + 2x_q)}\right] V_1$

$\therefore V_{TH} = \left[\frac{x_d - x_q}{x_d + x_q}\right] V_1$ volts (6)

Also

$Z_{TH} = \frac{j(x_d - x_q)(2R + j2\,x_q)}{j(x_d - x_q)(2R + j2\,x_q)}$

$= \frac{j(x_d - x_q)(2R + j2\,x_q)}{2R + j\,(x_d - x_q + 2x_q)}$

If 2R <<< j (xd – xq + 2 xq), then;

$Z_{TH} = \frac{j(x_d - x_q)(2R + j2x_q)}{j(x_d - x_q + 2x_q)}$

$= \frac{(x_d - x_q)(2R + j2x_q)}{(x_d + x_q)}$

But (xd – xq) (2R + j2 xq) = 2R (xd – xq) + j(2xq (xd – xq))

= 2R (xd – xq) + j(2xq xd – 2xq xq)) (7)

$\therefore Z_{TH} = \frac{2R(x_d - x_q)}{(x_d + x_q)} + \frac{j(2x_q x_d - 2(x_q)^2)}{(x_d + x_q)}$

But $Z_{TH} = R_{TH} + X_{TH}$

Hence $R_{TH} = \frac{2R(X_d - X_q)}{(X_d + X_q)}$ - Real value of Z_{TH} (8)

$X_{TH} = \frac{j(2xqxd - 2(x_q)^2)}{(X_d + X_q)} = \frac{j2x_q(X_d - X_q)}{(X_d + X_q)}$ - Imaginary value of Z_{TH} (9)

Hence figure 9reduces to;

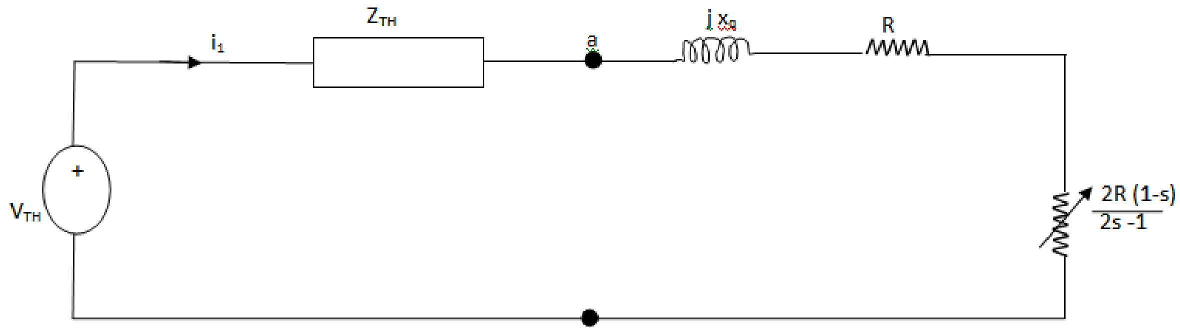

Figure 10 Thevenin equivalent circuit model of 3-phase transfer field machine with cage (rotor) windings

From figure 10,

$i_1 = i_{23} = \frac{V_{TH}}{Z_{TH}} = \frac{V_{TH}}{\left(R_{TH} + R + \frac{2R(1-s)}{2s-1}\right) + j(X_{TH} + X_q)}$

$= \frac{V_{TH}}{\left(R_{TH} + \frac{R}{2s-1}\right) + j(X_{TH} + X_q)}$ (10)

$\Rightarrow i_1^2 = \frac{(V_{TH})^2}{[\left(R_{TH} + \frac{R}{2s-1}\right) + j(X_{TH} + X_q)]^2}$

$= \frac{(V_{TH})^2}{\left(R_{TH} + \frac{R}{2s-1}\right)^2 + (X_{TH} + X_q)^2}$ (11)

Power Across Air-gap, Torque and Power Output in three-phase transfer field machine with cage (rotor) winding

With regards to the equivalent circuit of figure 9, the power crossing the terminal **ab** in the circuit is the power that is transferred from the stator windings to auxiliary and cage windings, through the machine air-gap magnetic field. This is called the power across the air gap or simply air-gap power, whose three phase value is shown below;

$P_G = 3(i_{23})^2 \frac{R}{2s-1}$ Watts (12)

Also, Auxiliary Rotor windings copper loss P_c (aux/rotor) = $3(i_{23})^2 R$ (13)

Putting equation 13 into equation 12, we have;

$P_G = \frac{P_{c(aux/rotor)}}{2s-1}$

$\Rightarrow$ Pc (aux/rotor) = (2s-1) P_G Watts (14)

But Mechanical Output (gross) Power (Pm) of the machine is given by;

$P_m = P_G$ – Pc (aux/rotor)

$\Rightarrow P_m = \left[3(i_{23})^2 \frac{R}{2s-1}\right] - \left[3(i_{23})^2 R\right] = 6(i_{23})^2 R \frac{(1-S)}{2s-1}$ Watts

$\Rightarrow P_m = 2P_G(1-s)$ Watts (15)

From equation 14 and 15, it can be inferred that high slip operation of the machine will favour auxiliary/rotor winding copper losses Pc(aux/rotor) at the detriment of the mechanical output (gross) Power (Pm), and would make the machine highly inefficient. Hence, the machine is particularly designed to operate at low slip, even at full load.

Torque/slip Characteristic of 3-phase transfer field reluctance machine with cage windings

From figure 10, the expression for the steady-state electromagnetic torque of the machine is given as below;

$$Te = \frac{P_m}{\omega_m} = \frac{P_m}{\omega(1-s)}$$

$$= [6\,(i_{23})^2 R\,\frac{(1-S)}{2s-1} x \frac{1}{\omega(1-s)}] = \frac{6(i_{23})^2 R}{\omega\,(2s-1)}$$

$$= \frac{6(i_{23})^2 R}{\omega\,(2s-1)} \text{ N-m} \tag{16}$$

Putting equation 11 into equation 16, we have;

$$Te = \frac{6}{\omega}\left(\frac{R}{2s-1}\right)\left[\frac{(V_{TH})^2}{\left(R_{TH}+\frac{R}{2s-1}\right)^2+(X_{TH}+X_q)^2}\right] \text{ N-m} \tag{17}$$

Equation 17 is the expression for torque developed as a function of voltage (V_{TH}) and slip (s).

Production of Torque/Slip Curve for 3-Phase Transfer field Machine with cage (rotor) winding

It is the interaction between the windings (main, auxiliary and the rotor) current that produce the fluxes, which is responsible for torque production.

Table 1 – Parameters for 3-phase transfer field reluctance machine with cage (rotor) windings

S/No	Parameter	Value
1	Lm_d	133.3mH
2	Lm_q	25.6mH
3	$L_{ls} = L_{la} = L_{rr}$	0.6mH
4	$r_m = r_a = r_r = 2R$	3.0Ω
5	J	$1.98 x 10^{-3} kgm^3$
6	V	220V
7	F	$50H_Z$
8	P	2

With the values of the machine parameters and using equation 6 through equation 17 a plot for the Torque/Slip curve of the machine is developed as in figure 11.

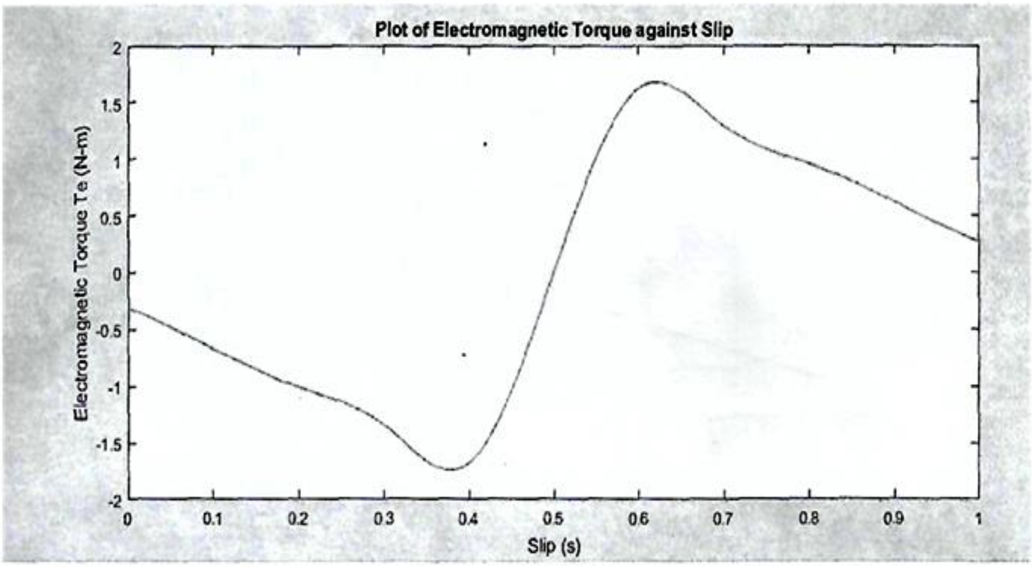

Figure 11 A plot of Torque/Slip characteristics of the 3-phase transfer field machine with cage winding.

Efficiency/Slip characteristics of the configured 3-phase transfer field machine

The efficiency-slip relationship for the configured 3-phase transfer field machine can be studied for better, using the per phase steady-state equivalent circuit of the machine as in figure 9.

The input impedance looking through the input terminals is;

$$Z = 2R + j2xq + \left[\frac{j(X_d - X_q)\,(jx_q + \frac{R}{2s-1})}{\frac{R}{2s-1} + j\,(x_q + (X_d - X_q))}\right] \quad = 2R + j2xq + \left[\frac{j(X_d - X_q)\,(jx_q + \frac{R}{2s-1})}{\frac{R}{2s-1} + jx_d}\right] \tag{18}$$

The current I_1 in the main winding is giving by;

$$\Rightarrow i_I = \frac{V_I}{Z} \tag{19}$$

Similarly, the current in the auxiliary and rotor windings (i_{23}) is given by;

$$i_{23} = \left[\frac{j(X_d - X_q)}{\frac{R}{2s-1} + j\,(x_q + (X_d - X_q))}\right] i_1 \tag{20}$$

The copper losses in the main, auxiliary and rotor winding = $3[2R(i_1)^2 + R(i_{23})^2]$

$$= 3R[2(i_1)^2 + (i_{23})^2] \tag{21}$$

But, Input Power = Output Power + Copper losses in the main, auxiliary and rotor winding, excluding windage and friction losses;

$\therefore$ Input Power = $6R\left(\frac{1-s}{2s-1}\right)(i_{23})^2 + 3R\,(2(i_1)^2 + (i_{23})^2)$

$$= 3R\left[2\left(\frac{1-s}{2s-1}\right)(i_{23})^2 + 2(i_1)^2 + (i_{23})^2\right] \tag{22}$$

$$\therefore \text{ The machine efficiency } (\varepsilon) = \frac{2\left(\frac{1-s}{2s-1}\right)(i_{23})^2}{2\left(\frac{1-s}{2s-1}\right)(i_{23})^2 + 2(i_1)^2 + (i_{23})^2} \tag{23}$$

The characteristics curve of the relationship between the machine efficiency against slip(s) is obtained, using equation 23, as in figure

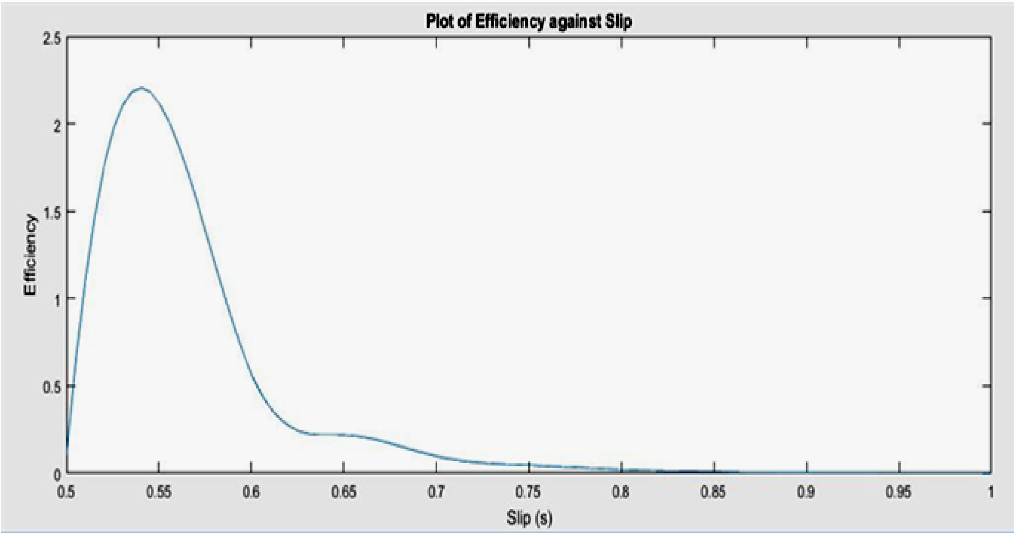

Figure 12 Efficiency/Slip characteristics of 3- phase transfer field machine with cage winding

Power factors/Slip characteristic of 3-phase transfer field machine

From the Thevenin equivalent of the configured machine of figure 9 the machine's power factor (cosθ) is given by;

$$\text{Power factor } (\cos\theta) = \frac{Real\ (Z)}{\sqrt{Real\ (Z)^2 + Imag\ (Z)^2}}$$

$$= \frac{R_{TH} + \frac{R}{2s-1}}{\sqrt{(R_{TH} + \frac{R}{2s-1})^2 + (X_{TH} + X_q)^2}} \tag{24}$$

A plot of the power factor (cosθ) against slip(s) is shown in using equation 24

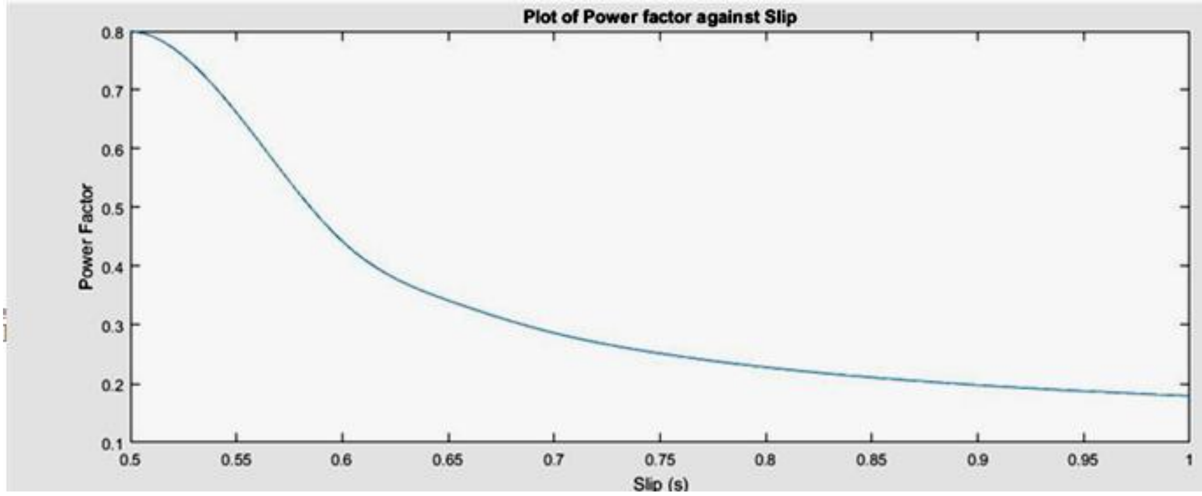

Figure 13 Power factor/Slip characteristics of 3-phase transfer field machine with cage winding

Rotor current (i_{23}) /Slip(s) characteristic of 3-phase transfer field machine with rotor windings
Using equation 10, a plot of rotor current (i_{23}) against slip(s) is obtained as in figure 14.

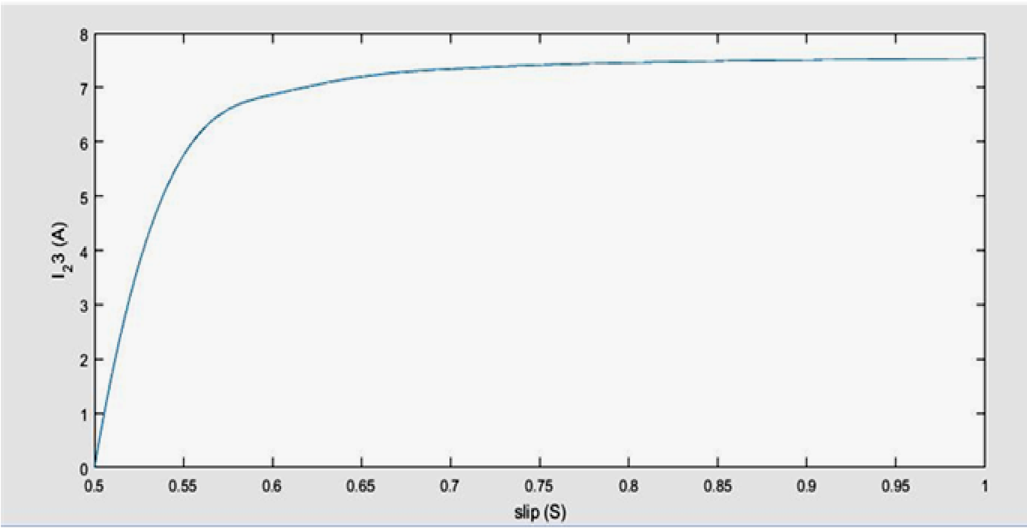

Figure 14 A plot of Rotor current/slip characteristics of 3-phase T.F machine with cage winding.

Dynamic Model of 3-Phase transfer Field Machine with cage (rotor) windings

For us to derive the dynamic equations of the circuit model of the configured transfer field reluctance machine it is paramount to take a look at the variation of inductances with rotor position since the rotor has salient poles. In general, the peameance along the **d** and **q** axes is not the same.

Since the rotor is of salient poles, its mmfs are always directed along the **d** and **q** axes. Also, the direction of the resultant mmf of the stator windings relative to these two axes will vary with the power factor. A common approach to handling the magnetic effect of the stator's resultantmmf is to resolve it along the **d** and **q** axes, where it could be dealt with systematically. Let us consider the magnetic effect of current flowing in phase **a**of the stator. The resolved components of the a-phase mmfFa, will produce the flux components:

$\phi d = P_d F_a \sin \theta r$ and $\phi q = P_q F_a \cos\theta r$ along the d and q axes respectively.

Where P = peameance
The linkage of these resolved flux components with the a-phase windings is;

$\lambda_{aa} = Ns\ (\phi d \text{ Sin } \theta r + \phi q \cos \theta r)$ Wb turn.

$= Ns\ F_a\ (P_d \sin 2\theta r + P_q \cos 2\theta r)$

$$= Ns\ F_a\ \left(\frac{p_d + p_q}{2} - \frac{p_d - p_q}{2} \cos 2\theta r\right) \tag{25}$$

Similarly, the linkage of the flux component, ϕd and ϕq by the b - phase winding that is $\frac{2\pi}{3}$ ahead may be written as:

$\lambda_{ba} = N_sF_a$ (Pd sin θr sin (θr - $\frac{2\pi}{3}$) + $p_q$cosθrcos (θr - $\frac{2\pi}{3}$))

$= N_sF_a \left(-\frac{p_d+p_q}{4} - \frac{p_d-p_q}{2}\cos 2(\theta r - \frac{\pi}{3})\right)$ (26)

Based on the functional relationship of λ_{aa} with the rotor angle, θr, we can deduce that the self inductance of the stator a-phase winding, excluding the leakage has the form;

$L_{aa} = L_o - L_{ms}\cos 2\theta r$, H

Where;

$L_o = \frac{L_{md}+L_{mq}}{2}$ and $L_{ms} = \frac{L_{md}+L_{mq}}{2}$

Those of the **b** – and **c** – phases, L_{bb}, L_{cc} are similar to that of L_{aa} but with θr replaced by (θr- $\frac{2\pi}{3}$) and (θr+ $\frac{2\pi}{3}$), respectively.

Similarly, it can be deduced that the mutual inductance between the **a** and **b** phase of the stator is of the form,

$L_{ab} = L_{ba} = \frac{L_o}{2} - L_{ms}\cos 2(\theta r - \frac{2\pi}{3})$ H (27)

Similarly, expression for L_{bc} and L_{ac} can be obtained by replacing θr with (θr- $\frac{2\pi}{3}$) and (θr+ $\frac{2\pi}{3}$) respectively.

Since a conventional T.F. effect machine is composed of two components with two windings each, if the parameter referring to the main winding is increased with the subscript A, B, C (ie phase quantities) while that referring to the auxiliary winding will have subscript a, b, c, the dynamic model can be derived as follows:

$V_{ABC} = r_{ABC}i_{ABC} + P\lambda_{ABC}$

$Vab_c = r_{abc}i_{abc} + P\lambda_{abc}$

$V_{dqrABC} = r_{dqrABC}i_{dqrABC} + P\lambda_{dqrABC}$ (28)

$V_{dqrabc} = r_{dqrabc}i_{dqrabc} + P\lambda_{dqrabc}$

Where P $\frac{d}{dt}$, λ = flux

$R_{ABC} = d_{iag}[(r_Ar_Br_C)]$ and$_{rabc}$= $d_{iag}[(r_ar_br_c)]$

The flux linkages are expressed as;

$\lambda_{ABC} = L_{GG}i_{ABC} + L_{GH}i_{abc}$

$\lambda_{abc} = L_{HG}i_{ABC} + L_{HH}i_{abc}$ (29)

where L_{GG}, L_{GH}, L_{HG} and L_{HH} are inductance matrices obtained from t**h**e inductance sub matrices of the two components machines as shown below.

Let L_{11} be the self inductance of the main winding and L_{22} be the self inductance of the auxiliary winding; then the mutual inductance between the main and the mutual inductance between the main and the auxiliary winding will be L_{12} or L_{21} as the case may be;

Now; $L_{11} = \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} \end{bmatrix}$ $L_{12} = \pm \begin{bmatrix} L_{Aa} & L_{Ab} & L_{Ac} \\ L_{Ba} & L_{Bb} & L_{Bc} \\ L_{Ca} & L_{Cb} & L_{Cc} \end{bmatrix}$

$L_{21} = \pm \begin{bmatrix} L_{aA} & L_{aB} & L_{aC} \\ L_{bA} & L_{bB} & L_{bC} \\ L_{cA} & L_{cB} & L_{cC} \end{bmatrix}$ $L_{12} = \pm \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$

So far the main and the auxiliary winding are identical,

$L_{GG} = L_{11}$ (Machine A) + L_{11} (Machine B)

$= L_{11}^A + L_{11}^B$

The individual inductance expressions are as follows;

$L_{AA} = L_{a1} + L_{a2}\cos 2\theta_r$

$L_{AB} = L_{BA} = -\frac{1}{2} L_{a1} \pm L_{a2}\cos(2\theta_r - \alpha)$

$L_{AC} = L_{CA} = -\frac{1}{2} L_{a1} \pm L_{a2}\cos(2\theta_r + \alpha)$

$L_{BC} = L_{CB} = -\frac{1}{2} L_{a1} \pm L_{a2}\cos 2\theta_r$

$L_{BB} = \frac{1}{2} L_{a1} \pm L_{a2}\cos(2\theta_r - \alpha)$

$L_{CC} = \frac{1}{2} L_{a1} \pm L_{a2}\cos(2\theta_r + \alpha)$

$L_{aa} = \frac{1}{2} L_{a1} \pm L_{a2}\cos 2\theta_r$

$L_{ab} = L_{ba} = \frac{1}{2} L_{a1} \pm L_{b2}\cos(2\theta_r - \alpha)$

$L_{bc} = L_{cb} = \frac{1}{2} L_{b1} \pm L_{b2}\cos 2\theta_r$

$L_{bb} = \frac{1}{2} L_{b1} \pm L_{b2}\cos(2\theta_r - \alpha)$

$L_{cc} = \frac{1}{2} L_{a1} \pm L_{b2}\cos(2\theta_r + \alpha)$

$L_{Aa} = L_{aA} = \frac{1}{2} L_{b12} \pm L_{b12}\cos 2\theta_r$

$L_{Ab} = L_{bA} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r - \alpha)$
$L_{Ac} = L_{cA} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r + \alpha)$
$L_{Ba} = L_{ab} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r - \alpha)$
$L_{Bb} = L_{bB} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r + \alpha)$
$L_{Bc} = L_{cB} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos 2\theta_r$
$L_{Ca} = L_{aC} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r + \alpha)$
$L_{Cb} = L_{bC} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos 2\theta_r$
$L_{Cc} = L_{cC} = \frac{1}{2}\ L_{a12}\cos\alpha \pm L_{b12}\cos(2\theta_r - \alpha)$
Where , $\alpha = \frac{2\pi}{3}$, and; $L_{a11} = L_{a22} = L_{a12} = \frac{1}{2}(L_{md} + L_{mq})$
$L_{b11} = L_{b22} = L_{b12} = \frac{1}{2}(L_{md} - L_{mq})$

However, the expressions for the individual inductances above, can further be used for the inductance matrix for the main windings for both machines A and B.

For machine A, the inductance matrix for the main winding is;

$$L_{11}^{A} = \begin{bmatrix} L_{ls} + L_a - L_{ms}\cos 2\theta_r & -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & \frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_a - L_{ms}\cos 2\left(\theta_r + \frac{2\pi}{3}\right) & -\frac{1}{2}L_o - L_{ms}\cos 2(\theta_r - \pi) \\ -\frac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2}L_o + L_{ms}\cos 2(\theta_r - \pi) & L_{ls} + L_a - L_{ms}\cos 2\left(\theta_r - \frac{2\pi}{3}\right) \end{bmatrix}$$

For machine B, the inductance matrix for the main winding is;

$$L_{11}^{B} = \begin{bmatrix} L_{ls} + L_a + L_{ms}\cos 2\theta_r & -\frac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_a + L_{ms}\cos 2\left(\theta_r + \frac{2\pi}{3}\right) & -\frac{1}{2}L_o + L_{ms}\cos 2(\theta_r - \pi) \\ -\frac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r + \frac{\pi}{3}\right) & L_o + L_{ms}\cos 2(\theta_r - \pi) & L_{ls} + L_a + L_{ms}\cos 2\left(\theta_r - \frac{2\pi}{3}\right) \end{bmatrix}$$

Hence $L_{GG} = L_{11}^{A} + L_{11}^{B} \begin{bmatrix} 2L_{LS} + 2L_O & -L_O & -L_O \\ -L_O & 2L_{LS} + 2L_O & -L_O \\ -L_O & -L_O & 2L_{LS} + 2L_O \end{bmatrix}$

Where LLs = leakage inductance, and Lo = $\frac{L_{md} + L_{mq}}{2}$

$$\therefore L_{GG} = \begin{bmatrix} 2L_{LS} + L_{md} + L_{mq} & -\frac{1}{2}(L_{md} + L_{mq}) & -\frac{1}{2}(L_{md} + L_{mq}) \\ -\frac{1}{2}(L_{md} + L_{mq}) & 2L_{LS} + L_{md} + L_{mq} & -\frac{1}{2}(L_{md} + L_{mq}) \\ -\frac{1}{2}(L_{md} + L_{mq}) & -\frac{1}{2}(L_{md} + L_{mq}) & 2L_{LS} + L_{md} + L_{mq} \end{bmatrix} \qquad (30)$$

Now for mutual inductance
For machine A, the mutual inductance matrix is given as;

$$L_{12}^{A} = \begin{bmatrix} L_{ls} + L_a - L_{ms}\cos 2\theta_r & L_o - L_{ms}\cos(2\theta_r - \alpha) & L_o\cos\alpha - L_{ms}\cos(2\theta_r + \alpha) \\ L_o\cos\alpha - L_{ms}\cos(2\theta_r - \alpha) & L_{ls} + L_a - L_{ms}\cos(2\theta_r + \alpha) & L_o\cos\alpha - L_{ms}\cos 2\theta_r \\ L_o\cos\alpha - L_{ms}\cos(2\theta_r + \alpha) & L_o\cos\alpha - L_{ms}\cos 2\theta_r & L_{ls} + L_a - L_{ms}\cos(2\theta_r - \alpha) \end{bmatrix}$$

Likewise, for machine B, the mutual inductance matrix is given as;

$$L_{12}^{B} = \begin{bmatrix} L_{ls} + L_a + L_{ms}\cos 2\theta_r & L_o + L_{ms}\cos(2\theta_r - \alpha) & L_o\cos\alpha + L_{ms}\cos(2\theta_r + \alpha) \\ L_o\cos\alpha + L_{ms}\cos(2\theta_r - \alpha) & L_{ls} + L_a + L_{ms}\cos(2\theta_r + \alpha) & L_o\cos\alpha + L_{ms}\cos 2\theta_r \\ L_o\cos\alpha + L_{ms}\cos(2\theta_r + \alpha) & L_o\cos\alpha + L_{ms}\cos 2\theta_r & L_{ls} + L_a + L_{ms}\cos(2\theta_r - \alpha) \end{bmatrix}$$

But $L_{GH} = L_{12}^{A} + L_{12}^{B}$

$$\therefore L_{GH} = \begin{bmatrix} -2L_{ms}\cos 2\theta_r & -2L_{ms}\cos(2\theta_r - \alpha) & -2L_{ms}\cos(2\theta_r + \alpha) \\ -2L_{ms}\cos(2\theta_r - \alpha) & -2L_{ms}\cos(2\theta_r + \alpha) & -2L_{ms}\cos 2\theta_r \\ -2L_{ms}\cos(2\theta_r + \alpha) & -2L_{ms}\cos 2\theta_r & -2L_{ls}\cos(2\theta_r - \alpha) \end{bmatrix}$$

$$\Rightarrow L_{GH} = -2L_{ms}\begin{bmatrix} cos\, 2\,\theta_r\; cos(2\theta_r - \alpha)\; cos(2\theta_r + \alpha) \\ \\ cos(2\theta_r - \alpha)\; cos(2\theta_r + \alpha)\; cos\, 2\theta_r \\ cos(2\theta_r + \alpha)\; cos2\theta_r \quad cos(2\theta_r - \alpha) \end{bmatrix}$$

But $L_{ms} = \frac{L_{md} - L_{mq}}{2}$

$\therefore$ - 2Lms = $\frac{2(L_{mq} - L_{md})}{2}$

$$\therefore L_{GH} = L_{mq} - L_{md}\begin{bmatrix} cos\, 2\,\theta_r\; cos(2\theta_r - \alpha)\; cos(2\theta_r + \alpha) \\ \\ cos(2\theta_r - \alpha)\; cos(2\theta_r + \alpha)\; cos\, 2\theta_r \\ cos(2\theta_r + \alpha)\; cos2\theta_r \quad cos(2\theta_r - \alpha) \end{bmatrix} \qquad (31)$$

Where, $\alpha = \frac{2\pi}{3}$

Since the main and auxiliary winding for machine A and B are identical, L_{HG} and L_{HH} will be the same as L_{GH} and L_{GG} respectively.

Rotor Winding Inductance

The stages of transformation of the voltage equations are to first transform the a.b.c. phase variables into **q-d-o** frame where the quantities are in stationary reference fame. Secondly is to convert the stationary reference **q-d-o** frame into the rotor reference frame ie**d**r and **q**r. Since the rotor of this machine is salient pole, the axis of the rotor quantities are already in the **q** and **d** axis, so that the **q-d-o** transformation need only by applied to the stator quantities.

The Machine Model in Arbitrary q-d-o Reference Frame

In order to remove the rotor position dependence on the inductances seen in equation 31, the voltage equations in equation 28 need to be transferred to q-d-o reference frame. The technique is to transform all the stator variable to an arbitrary reference frame.

Here, all the stator variable will be transform to the rotor. In the voltage equations for the main and auxiliary windings of the transfer field machine of equation 28, there is no need to include the rotor equation here since our intension is to adopt rotor reference frame.

Hence, the voltage equations of the main winding of the machine will after the transformation yield;

V_Q = ωλD + ρλQ + riQ

V_D = ωλQ + ρλD + riD (32)

V_O = ρλO + riO

Doing like – wise for the auxiliary and cage (rotor) windings, we have,

V_q = (ω-2ωr) λd + ρλq + riq

V_d = (ω-2ωr) λq+ ρλd+ rid

V_o = ρλo+ rio (33)

$V_{\acute{q}r}$ = (ω -2ωr) $\lambda_{dr} + \rho_{\lambda\acute{q}r} + r_{\acute{q}r}\, i_{\acute{q}r}$

V_{dr} = (ω -2ωr) $\lambda_{\acute{q}r} + \rho_{\lambda\acute{d}r} + r_{dr}\, i_{dr}$

Transformation of flux Linkages

The **ABC** and **abc**subscripts denote variables and parameters associated with the main and auxiliary windings respectively. Both r_{ABC} and r_{abc} are diagonal matrices each with equal non zero elements. For a magnetically linear system, the flux linkages may be expressed as;

$$\begin{bmatrix} \lambda_{ABC} \\ \lambda_{abc} \end{bmatrix} = \begin{bmatrix} L_{GG}\, L_{GH} \\ L_{HG}\, L_{HH} \end{bmatrix}\begin{bmatrix} i_{ABC} \\ i_{abc} \end{bmatrix} \text{Wb turns} \qquad (34)$$

Where G = main winding, H = Auxiliary winding.

To transform the above equation in respect to the cage winding, we have as follows,

$$\begin{bmatrix} \lambda_{ABC} \\ \lambda_{abc} \\ \lambda_{dqr\,1} \\ \lambda_{dqr\,2} \end{bmatrix} = \begin{bmatrix} L_{GG}\, L_{GH}\, L_{GRA}\, L_{GRB} \\ L_{HG}\, L_{HH}\, L_{HRA}\, L_{HRB} \\ L_{RAG}\, L_{RAH}\, L_{RARA}\, L_{RARB} \\ L_{RBG}\, L_{RBH}\, L_{RBRA}\, L_{RBRB} \end{bmatrix}\begin{bmatrix} i_{ABC} \\ i_{abc} \\ i_{dqr\,1} \\ i_{dqr\,2} \end{bmatrix} \qquad (35)$$

The inductance matrix terms L_{GG}, L_{GH}, L_{HG} and L_{HH} are obtained from inductance sub-matrices $L_{11,}$ L_{12},L_{21} and L_{22} for machine A and B.

L_{GRA} is the mutual inductance matrix between main winding of machine A and rotor winding of machine A.

L_{GRB} is the mutual inductance matrix between main winding of machine B and rotor winding of machine B.

L_{HRA} is the mutual inductance matrix between auxiliary winding of machine A and rotor winding of machine A

L_{HRB} is the mutual inductance matrix between auxiliary winding of machine B and rotor winding of machine B.

L_{RARA} is the inductance matrix of rotor winding of machine A.
L_{RARB} is the mutual inductance matrix between the rotor winding of machine A and the rotor winding of machine B.

Stator Winding inductances

To reduce the mathematical complexities of equation 34, it is rewritten in q-d-o frame as;

$$\begin{bmatrix} \lambda_Q \lambda_D \lambda_O \\ \lambda_q \lambda_d \lambda_o \end{bmatrix}^T = \begin{bmatrix} K_G L_{GG} K_G^{-1} K_G L_{GH} K_H^{-1} \\ K_H L_{HG} K_G^{-1} K_G L_{HH} K_H^{-1} \end{bmatrix} \begin{bmatrix} i_Q i_D i_O \\ i_q i_d i_o \end{bmatrix} \quad (36)$$

Where KG = $\frac{2}{3}\begin{bmatrix} cos\theta cos(\theta-\alpha) cos(\theta+\alpha) \\ sin\theta sin(\theta-\alpha) sin(\theta+\alpha) \\ \frac{1}{2}\frac{1}{2}\frac{1}{2} \end{bmatrix}$ (37)

$$K_G^{-1} = \begin{bmatrix} cos\theta sin\theta & 1 \\ cos(\theta-\alpha)\, sin(\theta-\alpha) & 1 \\ cos(\theta+\alpha)\, sin(\theta+\alpha) & 1 \end{bmatrix} \quad (38)$$

KH = $\frac{2}{3}\begin{bmatrix} cos\beta cos(\beta-\alpha) cos(\beta+\alpha) \\ sin\beta sin(\beta-\alpha) sin(\beta+\alpha) \\ \frac{1}{2}\frac{1}{2}\frac{1}{2} \end{bmatrix}$ (39)

$$K_H^{-1} = \begin{bmatrix} cos\beta sin\beta & 1 \\ cos(\beta-\alpha)\, sin(\beta-\alpha) & 1 \\ cos(\beta+\alpha)\, sin(\beta+\alpha) & 1 \end{bmatrix} \quad (40)$$

Where β = θ – = Speed of rotation of the arbitrary reference frame
θr = Angular rotor position
Therefore the flux linkage of equation 11 is now expressed as;
$\lambda_Q = (2L_L + L_{mq} + L_{md})\, i_Q - (L_{md} - L_{mq})\,(i_{q+}\, i_{q\acute{r}})$
$= 2L_L i_Q + L_{mq} i_Q + L_{md} i_Q - L_{md}\,(i_{q+}\, i_{q\acute{r}}) + L_{mq}(i_{q+}\, i_{q\acute{r}})$
$= 2L_L i_Q + L_{mq} i_Q + L_{md} i_Q + L_{md} i_Q - L_{md} i_Q - L_{md}(i_{q+}\, i_{q\acute{r}}) + L_{mq}(i_{q+}\, i_{q\acute{r}})$
$= 2L_L i_Q + 2L_{md} i_Q + L_{mq} i_Q - L_{md} i_Q - (i_{q+}\, i_{q\acute{r}}) + L_{mq}(i_{q+}\, i_{q\acute{r}})$
$= 2\,(L_L + L_{md})\, i_Q + [i_Q(L_{mq} - L_{md}) + (i_{q+}\, i_{q\acute{r}})\,(L_{mq} - L_{md})]$
$= 2\,(L_L + L_{md})\, i_Q + (i_Q + i_{q+}\, i_{q\acute{r}})\,(L_{mq} - L_{md})$
$\therefore \lambda_Q = 2\,(L_L + L_{md})\, i_Q + (L_{mq} - L_{md})\,(i_Q + i_{q+}\, i_{q\acute{r}})$ (41)
Similarly;
$\lambda_D = (2L_L + L_{mq} + L_{md})\, i_D + (L_{md} - L_{mq})\,(i_{d+}\, i_{d\acute{r}})$
$= 2\,(L_L + L_{mq})\, i_D + (L_{md} - L_{mq})\,(i_D + i_d + i_{d\acute{r}})$ (42)
$\lambda_O = 2L_L i_O$ (43)
Also, $\lambda_q = (2L_L + L_{mq} + L_{md})\, i_q - (L_{md} - L_{mq})\,(i_Q + i_{q\acute{r}})$
$= 2\,(L_L + L_{md})\, i_q + (L_{mq} - L_{md})\,(i_Q + i_q + i_{q\acute{r}})$ (44)
$\lambda_d = (2L_L + L_{mq} + L_{md})\, i_d + (L_{md} - L_{mq})\,(i_D + i_{d\acute{r}})$
$= 2\,(L_L + L_{mq})\, i_d + (L_{md} - L_{mq})\,(i_D + i_d + i_{d\acute{r}})$ (45)
$\lambda_o = 2L_L i_o$ (46)
Also; $\lambda_{q\acute{r}} = (L_{Lqr} + L_{mq} + L_{md})\, i_{q\acute{r}} - (L_{md} - L_{mq})\,(i_Q + i_q)$
$= (L_{Lqr} + 2L_{md})\, i_{q\acute{r}} + (L_{mq} - L_{md})\,(i_Q + i_{q+}\, i_{q\acute{r}})$ (47)
$\lambda_{d\acute{r}} = (L_{Ldr} + L_{mq} + L_{md})\, i_{d\acute{r}} - (L_{md} - L_{mq})\,(i_D + i_d)$
$= (L_{Ldr} + 2L_{mq})\, i_{d\acute{r}} + (L_{md} - L_{mq})\,(i_d + i_{d+}\, i_{d\acute{r}})$ (48)

NB: Upper case letters represent the main winding parameters, while the lower case letters and the primed lower case letters represent the auxiliary winding parameters and rotor winding parameters respectively
As before equations 41 - 43 represent the flux linkages of the main winding circuit while equations 44 - 46 represent the flux linkages of the auxiliary winding circuit. Also equations 47 -48 represent the flux linkages of the caged (rotor) winding circuit, and **r** in equations 32 and 33 is the sum of the resistances of the main, auxiliary and rotor windings in both machine halves. Hence equations 41 - 48 can be put into equations 32 and 33 to yield;
$V_Q = \omega\lambda_D + \rho\,[2\,(L_L + L_{md})\, i_Q + (L_{mq} - L_{md})\,(i_Q + i_{q+}\, i_{q\acute{r}})] + ri_Q$ (49)

$V_q = (\omega - 2\omega_r)\,\lambda_d + \rho\,[2\,(L_L + L_{md})\,i_q + (L_{mq} - L_{md})\,(i_Q + i_q + i_{\acute{q}r})] + r i_q$ (50)

$V_{\acute{q}r} = (\omega - 2\omega_r)\,\lambda_{\acute{d}r} + \rho[(L_{Lqr} + 2L_{md})\,i_{\acute{q}r} + (L_{mq} - L_{md})\,(i_Q + i_q + i_{\acute{q}r}) + r i_{\acute{q}r}$ (51)

$V_D = \omega\lambda_Q + \rho\,[2\,(L_L + L_{mq})\,i_D + (L_{md} - L_{mq})\,(i_D + i_d + i_{\acute{d}r})] + r i_D$ (52)

$V_d = (\omega - 2\omega_r)\,\lambda_q + \rho\,[2(L_L + L_{mq})\,i_d + (L_{md} - L_{mq})\,(i_D + i_d + i_{\acute{d}r})] + r i_d$ (53)

$V_{\acute{d}r} = (\omega - 2\omega_r)\,\lambda_{\acute{q}r} + [\rho\,(L_{Ldr} + 2L_{mq})\,i_{\acute{d}r} + (L_{md} - L_{mq})\,(i_d + i_d + i_{\acute{d}r})] + r i_{\acute{d}r}$ (54)

Also for O-variables;

$V_O = \rho\lambda_O + r i_O$

$\quad = \rho(2L_L i_O) + r i_O$ (55)

$V_O = \rho\lambda_o + r i_o$

$\quad = \rho(2L_L i_o) + r i_o$ (56)

$V_{\acute{o}r} = \rho\lambda_{\acute{o}r} + r i_{\acute{o}r}$

$\quad = \rho(L_r i_{\acute{o}r}) + i_{\acute{o}r}$ (57)

Equations 49 - 51 result the equivalent circuit shown in figure 15.

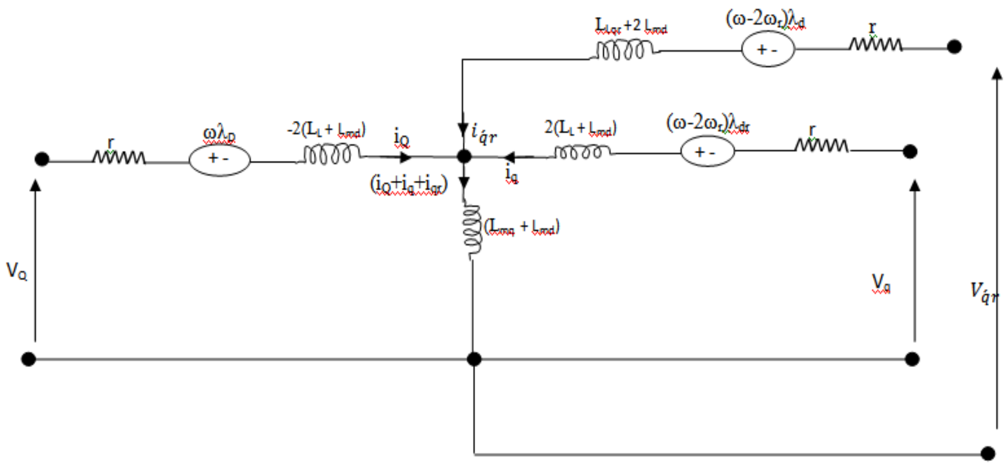

Figure 15 Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the q-variable.

Also, equations 52 - 54 result the equivalent circuit shown in figure 16

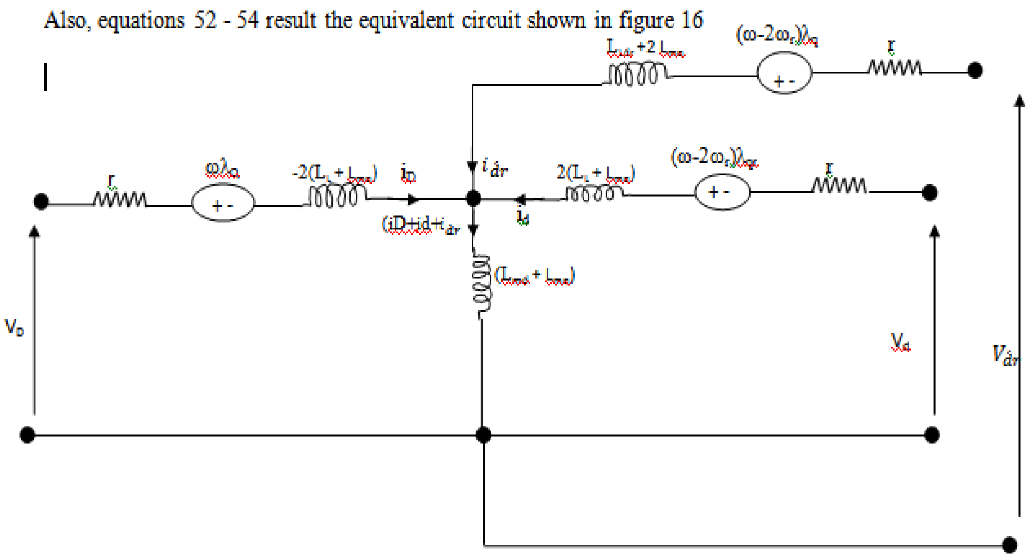

Figure 16 Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the d-variable

Similarly, equation 55 - 57 combines to yield the equivalent circuit shown in figure 17

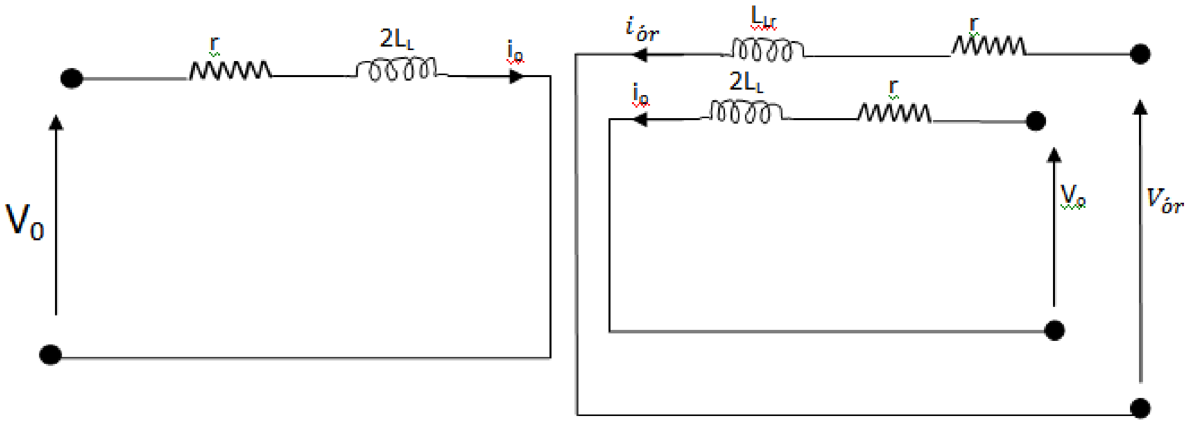

Figure 17 Arbitrary reference frame equivalent circuit for a 3-phase symmetrical transfer field machine with cage (rotor) winding in the O-variable.

15. Rotor to stator winding inductances

Obviously, both rotors of the machine halves are identical. Therefore, they possess equal and similar parameters. Let us consider the complying between the rotor winding, and the stator windings of machine A. The winding placements are depicted in figure 18 below

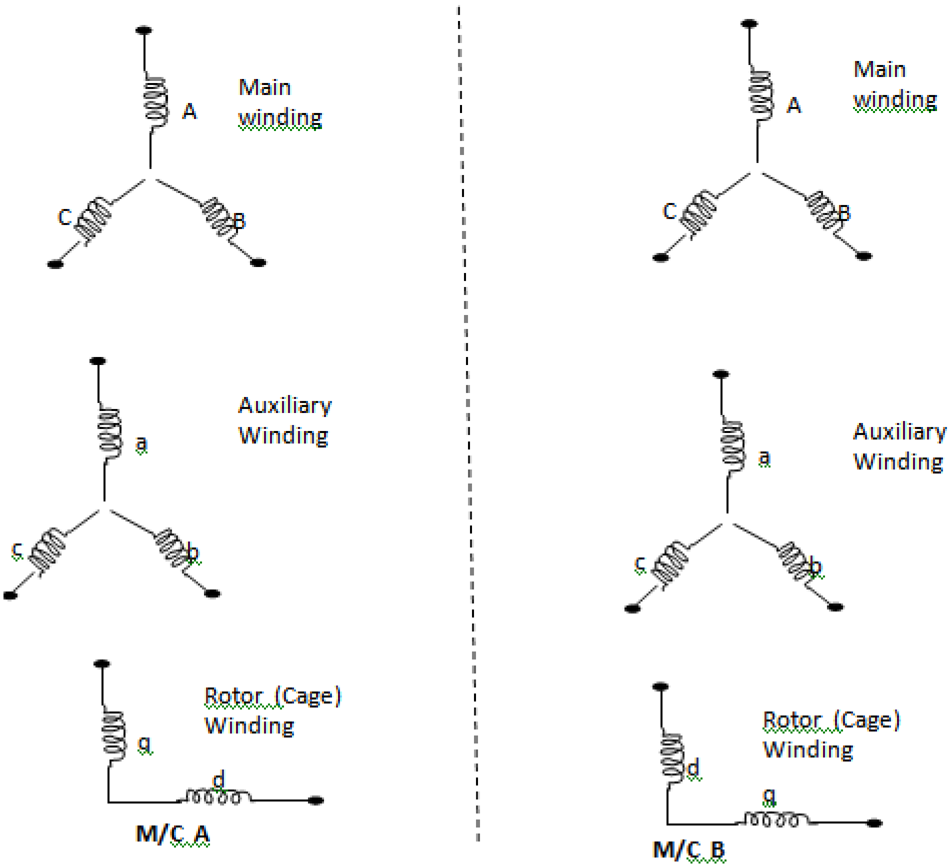

Figure 18. Rotor to Stator winding inductances

From figure 18

$$L_{GRA} = L_{RAG} = \begin{bmatrix} L_{Aq} & L_{Ad} \\ L_{Bq} & L_{Bd} \\ L_{Cq} & L_{Cd} \end{bmatrix} \tag{58}$$

$$L_{HRB} = L_{RBH} = \begin{bmatrix} L_{aq}\, L_{ad} \\ L_{bq}\, L_{bd} \\ L_{cq}\, L_{cd} \end{bmatrix}$$

NB $L_{GRA} = L_{HRB}$ on the account if the identity of the two machine halves.

Also

$L_{aq} = L_{Aq} = L_{mq}\cos\theta r$

$L_{ad} = L_{Ad} = L_{md} \sin \theta r$

$L_{bq} = L_{Bq} = L_{mq}\cos (\theta r- \frac{2\pi}{3})$

$L_{bd} = L_{Bd} = L_{md} \sin (\theta r- \frac{2\pi}{3})$ (59)

$L_{cq} = L_{Cq} = L_{mq}\cos (\theta r- \frac{4\pi}{3})$

$L_{cd} = L_{Cd} = L_{md}\sin (\theta r- \frac{4\pi}{3})$

16. Rotor to Rotor Winding inductances

On the account of identity of the two machine halves;

$$L_{RARA} = L_{RBRB} = \begin{bmatrix} L_{ldr} + L_{md}\; O \\ O L_{ldr} + L_{md} \end{bmatrix} \quad (60)$$

17. The torque equation of the 3-phase transfer field machine with cage (rotor) winding

The torque equation of the configured machine is obtained by integrating the rotor winding parameters into the derived torque equation of the conventional 3-phase transfer field machine with no rotor winding.

The expression for the torque equation of the cage winding transfer field machine is given as;

$$Te = \frac{3}{2}\left(\frac{P}{2}\right)\left[(i_{Qs} + i_{qs})X_{mq}\,(i_{Ds} + i_{ds} + i_{\acute{d}s}\right]$$
$$- \left[(i_{Ds} + i_{ds})X_{mq}\,(i_{Qs} + i_{qs} + i_{\acute{q}s})\right] \quad (61)$$

Where,

i_{Qs} is the q-axis stator current in the main winding of TF machine, i_{qs} is the q-axis stator current in the auxiliary winding of T.F machine, i_{Ds} is the d-axis stator current in the main winding of T.F machine

ids is the d-axis stator current in the auxiliary winding of TF machine $i\acute{d}r$ is the d axis rotor current in the (cage) rotor of T.F machine.

$i_{\acute{q}r}$ is the q-axis rotor current in the (cage) rotor of T.F machine.

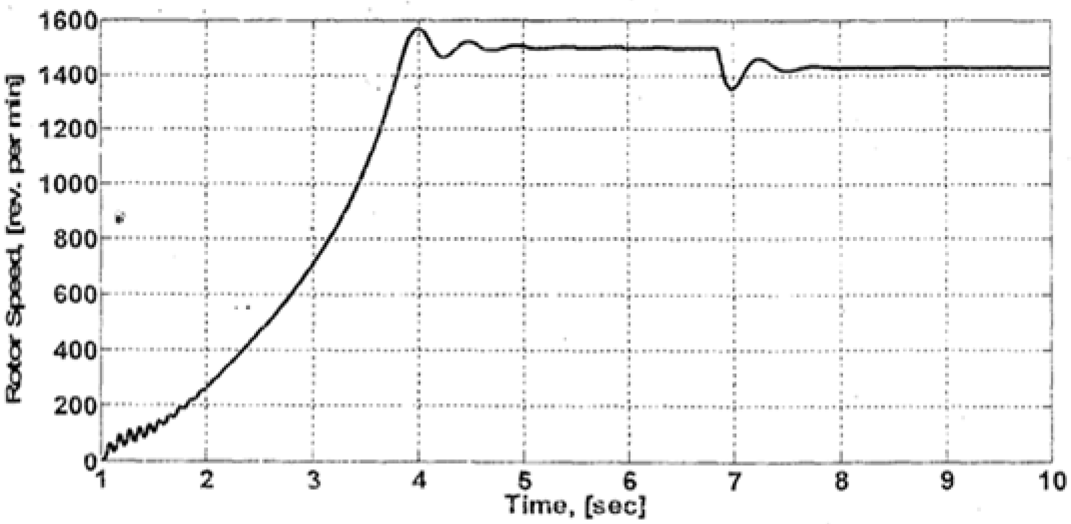

Figure 19 Rotor speed run up plot for the configured machine

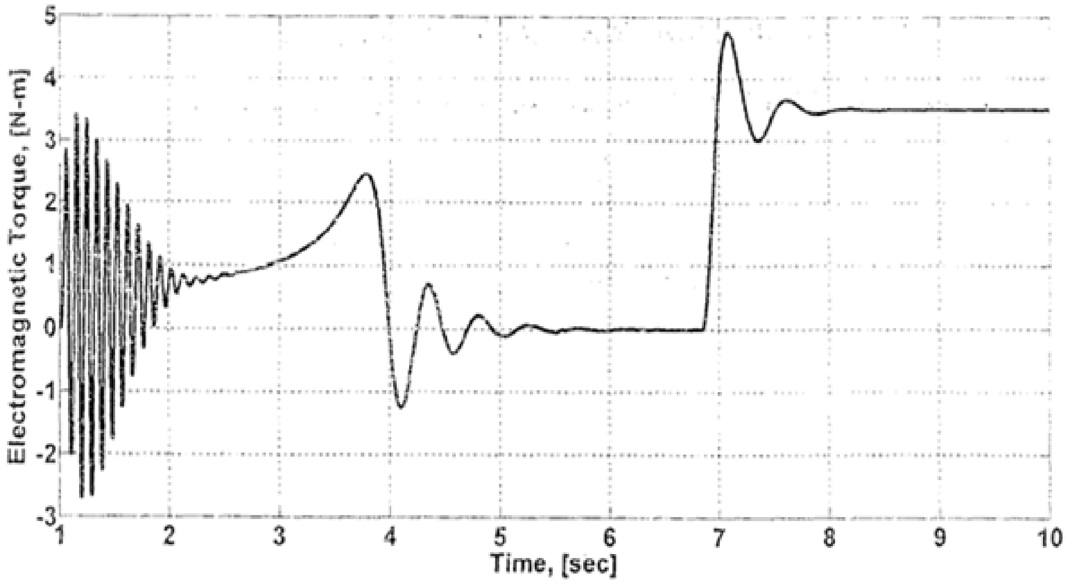

Figure 20 A Plot of Electromagnetic magnetic torque verses Time

The graphs/plots of rotor speed run up and Electromagnetic magnetic torque verses time are shown in figure 19 and 20 above respectively.

18. Result analysis of the configured three phase transfer field machine with cage (rotor) windings

For the dynamic operation of the machine, the rotor speed run-up plot against time for the configured (cage) machine is shown in figure 19. There was a little transient at different stages while rotor speed builds up before an application of load at 7 seconds. After another little transient, the rotor speed now settles to a steady-state at about 1410N-m.

Also the graph of electromagnetic torque against time for the caged machine with oscillations noticed at different stages are shown in figure 20. It is observed that on no-load, value for electromagnetic torque is zero. On application of load torque at 6.9 seconds to the machine, it oscillates and settled to a steady-state of 3.4N.

Morestill, from the steady-state electromagnetic torque versus slip characteristics curve of figure 11, the result reveals a good similarity with improved output characteristics to those of the conventional three phase transfer field machine with no cage windings. At slip (S) = 1, the injected voltage at the auxiliary and rotor windings is zero. Hence, necessitating a zero torque. However, torque may be developed at this slip, if the two windings are excited with direct current, hence, making the machine run at synchronous mode.

Due to the incorporation of rotor windings to the rotor circuit of the conventional transfer field (T.F.) machine, the machine efficiency improved tremendously due to reduction in overall impedance of the machine.

Further-still, due to additional winding (rotor winding connected in parallel with the auxiliary winding), induced rotor current at start improved, leading to concomitant boost in maximum and starting torque of the machine at better and improved power factor. For the machine, at synchronous speed, (Ns = Nr, s = 0.5) current decayed to zero, but at zero speed (Nr = 0, S = 1), starting current is maximum. This is a feature also obtainable in conventional cage-less T.F. machine.

III. CONCLUSION

From the analysis, the inclusion of rotor winding into the conventional machine provides a better output performance characteristics, necessary for its wider applications in engineering industries.

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