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Stresses Fields In Axial Compressed O-Ring Gasket

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ABSTRACT-: *When the stresses and the deformations generated during a mechanical loading exceed the allowed limits, the machine element loses these capacities to fulfill its function. So the evaluation of the stress distributions in the O-rings is primary importance to avoid their deterioration during an excessive loading. In order to ensure an analytical development of the stresses in O-ring seal, a semi-analytic approach is proposed in this work. This approach was used in the evaluation of the mechanical characteristics of a Brazilian disc during the compression test. The approach's results are perfectly measured with those of the finite element analysis using an axisymmetric model.*

KEYWORDS; *Analytical modeling, Contact pressure, FEA, O-ring; Stress fields*

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I. INTRODUCTION

The elements used to ensure the mechanical assemblies sealing are in general the secondary components with an accessible price but plays a paramount role in the mechanical function that the assembly must endure. The O-rings belong to this category of machine elements. They are exploited, considering their form and dimensions, to avoid escapes in rather restricted zones and difficult to reach.

The O-ring is compressed radially or axially between two parts. The seal reaction creates contact pressure at the interfaces joint-parts. The values of the contact pressure, as well as the form of its distribution, determine the gasket effectiveness. Theoretically, the contact pressure distribution is treated in the literature by Hertz theory. In the case of an axial compression, the nature of the load and the geometry allow representing the assembly by a 2D axisymmetric model in the form of the compressed disc between two plans, as showed in Fig. 1. The disc represents the cross-section of the O-ring. The Hertz formulas, developed for cylinder-plan contact, give the contact width, $2a$, and the maximum contact pressure, p_m, as well as the contact pressure distribution form.

According to [1], while being based on the Hertz theory for the cylinder-plan contact, and by replacing the cylinder length by the average perimeter of the O-ring, the contact width is given by Eq. (1).

$$a = 2\left[\frac{RF}{2R_m\pi^2}\left(\frac{1-\nu_g^2}{E_g}+\frac{1-\nu_p^2}{E_p}\right)\right]^{\frac{1}{2}} \tag{1}$$

The maximum value of the contact pressure, pm, is given by Eq. (2).

$$p_m = \frac{F}{aR_m\pi^2} \tag{2}$$

In addition, the contact pressure is distributed according to the Eq. (3).

$$p(x) = p_m\left[1-\left(\frac{x}{a}\right)^2\right]^{\frac{1}{2}} \tag{3}$$

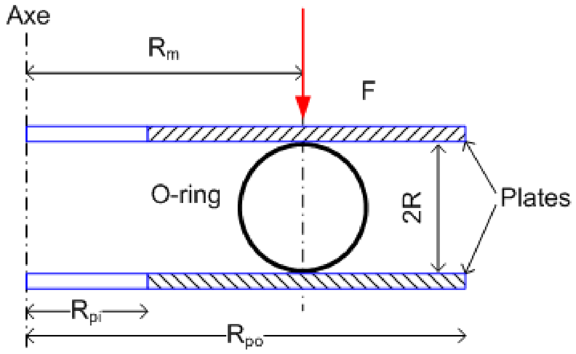

Figure 1. Unrestrained axial O-ring loading

Lindley [2] adjusted the quantities given by Eqs. (1), (2) and (3) according to experimental study results. He expressed these quantities according to the gasket compression ratio and the geometrical and mechanical characteristics of the seal.

Other studies have being carried out to clarify the O-ring behavior. Thus, George et al. [3] developed a finite elements model using the hyperplasic behavior of the seal with Neo-Hookean parameters. Dragoni et al. [4] studied the effects of grooves dimensions and friction on the mechanical behavior of the joint. Diany et al. [5] studied the O-rings relaxation by introducing the modified Maxwell model based on the temporal variation of the longitudinal modulus of elasticity. They treated the cases where the seal is solicited radially and axially with or without blocking.

The previously presented studies were more interested in contact pressure distribution without being delayed on the stress distribution inside of the gasket section and which may affect the deterioration and the degradation of this type of gasket.

This work proposes to determine analytically the stresses distributions in the O-ring cross-section using the distribution of the contact pressure confirmed by the various studies and also the equations developed in the case of the compression test of the Brazilian disc. The results of this analytical approach will be compared to those of the finite element analysis.

II. ANALYTICAL APPROACH

Figure 2 presents the distribution of contact pressure, given in Eq. (3), in the case of axial loading without lateral restraint. This pressure is in the form of bell such as the base width represents the contact width, $2a$. The maximum value, p_m, is reached in the center of the contact zone limited by the arc of 2α angle. This contact pressure does not envisage the zones where the stresses are critical and do not give any idea on the manner with which the stresses are distributed in the seal. To remedy this weakness a similarity with the compression test of the Brazilian disc will be proposed in this article to evaluate the stresses in the O-ring in axial compression.

The compression test of the Brazilian disc is an alternative test, suggested by the International Society of Rock Mechanics (ISRM) to determine the elastic mechanical properties of brittle materials, as reported in [6-7-8]. In this test, the test piece is a disc loaded by a pair of pressure, q, diametrically opposed and uniformly distributed over an arc of angle 2α, as presented in Fig. 3. Hondros [9] obtained the radial, tangential and shear stresses for each point of the disc. Eqs. (4) gives these formulas for the polar coordinates and angle α.

$$\sigma_r(r,\theta) = -\frac{2q}{\pi}\left\{\alpha + \sum_{k=1}^{\infty} f_k(r)\sin(2k\alpha)\cos(2k\theta)\right\} \quad (4a)$$

$$\sigma_\theta(r,\theta) = -\frac{2q}{\pi}\left\{\alpha - \sum_{k=1}^{\infty} f_k(r)\sin(2k\alpha)\cos(2k\theta)\right\} \quad (4b)$$

$$\tau_{r\theta}(r,\theta) = -\frac{2q}{\pi}\left\{\sum_{k=1}^{\infty} g_k(r)\sin(2k\alpha)\sin(2k\theta)\right\} \quad (4c)$$

With :

$$f_k(r) = \left[1 - \left(1+\frac{1}{k}\right)\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)^{2k-2}$$

And

$$g_k(r) = \left[1 - \left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)^{2k-2}$$

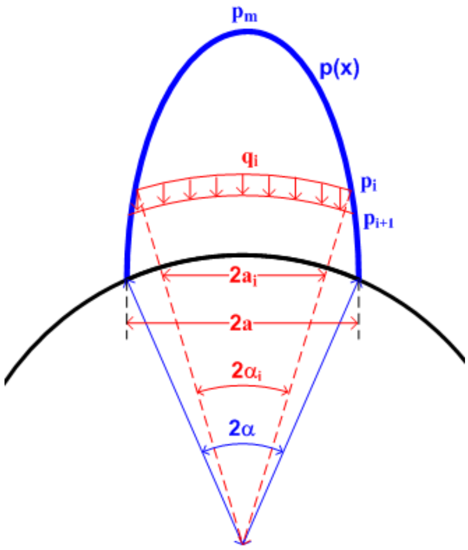

Figure 2. Contact pressure distribution

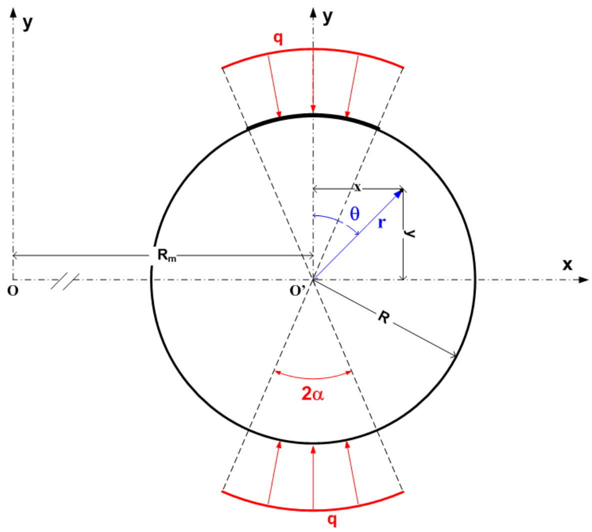

Figure 3. Compressed disc by uniform pressure

For the O-ring, the contact pressure is not uniformly distributed over the contact area. However, it is possible to subdivide the contact region in a finite number of areas centered on the vertical axis of the gasket cross-section where the pressure is considered constant. Indeed, the half width of contact, a_i, is divided into finite number of divisions defined by the abscissa, a_i, and have the corresponding value of the contact pressure. These quantities are defined by the formulas of Eqs. (5) and in Figure 2.

On each zone of elementary contact, $2a_i$, the contact angle is $2\alpha_i$ and the presumably constant contact pressure is q_i. These quantities are defined by the formulas of the Eq. (5).

$$\begin{aligned}
&For \quad i = 0,1,2,\ldots,N \\
&\qquad a_i = i\frac{a}{N} \quad , \quad p_i = p(a_i) \quad and \quad \alpha_i = \sin^{-1}\left(\frac{a_i}{R}\right) \\
&For \quad i = 1,2,\ldots,N-1 \\
&\qquad q_i = p_i - p_{i+1} \\
&then \quad q_0 = q_N = 0
\end{aligned} \tag{5}$$

For each elementary pressure, q_i, the formulas of the Eq. (1) give elementary stresses and the sum of these elementary stresses gives the general ones according to formulas of Eqs. (6).

$$\sigma_r(r,\theta) = \sum_{i=1}^{N}\left[-\frac{2q_i}{\pi}\left\{\alpha_i + \sum_{k=1}^{\infty} f_k(r)\sin(2k\alpha_i)\cos(2k\theta)\right\}\right] \quad (6a)$$

$$\sigma_\theta(r,\theta) = \sum_{i=1}^{N}\left[-\frac{2q}{\pi}\left\{\alpha_i - \sum_{k=1}^{\infty} f_k(r)\sin(2k\alpha_i)\cos(2k\theta)\right\}\right] \quad (6b)$$

$$\tau_{r\theta}(r,\theta) = \sum_{i=1}^{N}\left[-\frac{2q_i}{\pi}\left\{\sum_{k=1}^{\infty} g_k(r)\sin(2k\alpha_i)\sin(2k\theta)\right\}\right] \quad (6c)$$

The value of N is optimized to reproduce the totality of the contact pressure and to have thus a better accuracy and more precision on the computed stresses.
In the Cartesian coordinates, in accordance with Fig. 3, the stress state at each point inside the gasket is determined by the change of coordinates according to the relationships in Eqs. (7).

$$\sigma_x(x,y) = \frac{1}{2}(\sigma_\theta + \sigma_r) + \frac{1}{2}(\sigma_\theta - \sigma_r)\cos(2\theta) - \tau_{r\theta}\sin(2\theta) \quad (6a)$$

$$\sigma_y(x,y) = \frac{1}{2}(\sigma_\theta + \sigma_r) - \frac{1}{2}(\sigma_\theta - \sigma_r)\cos(2\theta) + \tau_{r\theta}\sin(2\theta) \quad (6b)$$

$$\tau_{xy}(x,y) = \frac{1}{2}(\sigma_\theta - \sigma_r)\cos(2\theta) + \tau_{r\theta}\sin(2\theta) \quad (6c)$$

III. FINITE ELEMENTS MODEL

To validate the results of the analytical approach described in the preceding paragraph concerning the case of the O-ring in axial compression, an axisymmetric finite element model is developed using Ansys software [10]. The compression of the seal is carried out using two ordinary steel plates. The upper plate is perforated and the bottom is full circular. The plates dimensions are chosen so that their flexibility does not affect the values of the joint stresses.

The assembly is modeled using 2D axisymmetric elements with eight nodes. The Moony-Rivelin model with two parameters, C_1 and C_2, is adopted to model the hyperelastic mechanical behavior of the gasket material. A 2D plane element, with elastic behavior, is used to model the plates. The geometric and mechanical characteristics for the assembly are summarized in Table 1.

O-ring	Plates
R= 3.49 mm	Rpo= 75.56 mm
Rm= 61.6 mm	Rpi= 45.64 mm
νg=0.42	νp= 0.3
Eg=4(1+ νg)(C_1+C_2)= 128 MPa	Eg= 210000 MPa

Table 1 (Assembly characteristics)

A uniformly distributed load is applied to the upper plate. This force is transmitted to the O-ring through the contact elements defined in the interfaces plates-gasket. The mesh is optimized to ensure the convergence and to have a dense distribution of stresses and displacements in the elastomeric seal.

IV. RESULTS AND DISCUSSION

A mechanical assembly is a collection of pieces gathered using various simple or compound mechanical linkages to provide a well-defined function. Each part plays a role, either primary or secondary, but important and significant to guarantee the realization and the attainment of the desired assembly task. The O-ring is a part which has a significant and principal role when the risk of leakage or lack of safety is important. This element is usually at a ridiculous price and its chemical composition is independent of the application field. However, in certain cases, negligence of the seal importance may cause unexpected human and material damage, as happened in the space shuttle Challenger disaster occurred on 1986 where the O-ring was not designed to fly under unusually cold conditions [11].

In the unrestrained axial loading, the distribution of the contact pressure can be calculated analytically using the Eq. (3) or determined by finite elements analysis [5]. Eq. (3) gives the contact pressure according to the contact width and the maximum pressure. The last two quantities are calculated either analytically using Eq. (1) and Eq. (2), or determined by the finite element method. Consequently, three curves are possible for the contact pressure, p(x), as presented in Fig. 4. The tree pressure contact distributions are determined for the same total axial force F equal to 10.8 kN. The difference between analytical and EF values of the maximum pressure contact is about 12%. This difference can be explained by the assumptions adopted in the analytical development. When the results of the EF are used to calculate the contact pressure, using Eq. (3), the obtained curve approaches closely the EF one.

As a result of the last remark, in the following paragraphs, the contact pressure distribution determined by the FEA will be used to evaluate analytically the stresses distributions inside the elastomeric gasket; it is a semi-analytical approach.

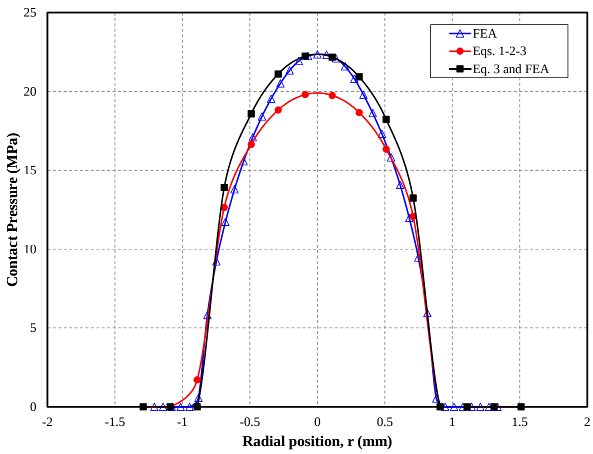

Figure 4. FE and analytical O-ring contact pressure

The formulas of the Eq. (7) give the O-ring stresses in the absolute reference (x, y). Figures 5 and 6 show the distribution of the axial stress, σ_y, determined, respectively, by the semi-analytical approach and the finite element method. Both figures have the same overall look, but it is difficult to compare the values of the stress. However, by fixing the angle θ, the inclination compared to the vertical axis, Fig. 7 compares the two approaches results. It is clear that the two methods give the same values and the same tendency of the variation according to the radial position, for the three values of θ (0, π/ 4 and π/ 2). The figure curves confirm the obvious results. Indeed, in the O-ring center, for all orientations, the stress σy has the same value of -7.5 MPa. On the other hand, at the external surface of the seal, the stress depends on the orientation: it is null for θ= π/ 4 and π/ 2 and -22.5 MPa in the middle of the contact area (θ= 0).

Figure 8 shows the three stresses together (x, σ_y and τ_{xy}) for the ray defined by θ= π/ 4. The two calculation approaches curves overlap perfectly and the three stresses are null at the outer radius.

The angle θ effect, defining the tangential position, can be appreciated in Fig. 9 which shows the variation of the radial, hoop and shear stresses, for the median radius r=R/2. The curves of this figure show clearly that the stresses vary in a periodic way and confirm the symmetry of the O-ring reaction. In fact, the stresses in the interval [0, π] are identical to those in the interval [π, 2π]. Each of these two intervals represents half of the gasket. Moreover, the extreme of the radial and tangential stresses are for θ=0 and θ=π.

In the O-ring, the state of stress is supposed plane considering the loads and the geometry. In the polar coordinates, in addition to the normal stresses, the shear stress has a considerable effect. Figure 10 presents the variation of the shear stress according to the radial position for several lines defined by the angle θ. In the center of the gasket, the shear stress takes various values dependently of the orientation θ. Indeed, it is the same state of stresses but with different orientations. On the other hand, on the external surface, the shear stress is null (free surface).

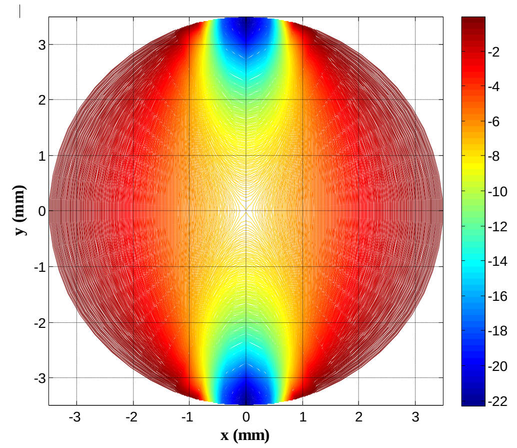

Figure 5. Analytical σ_y stress distribution

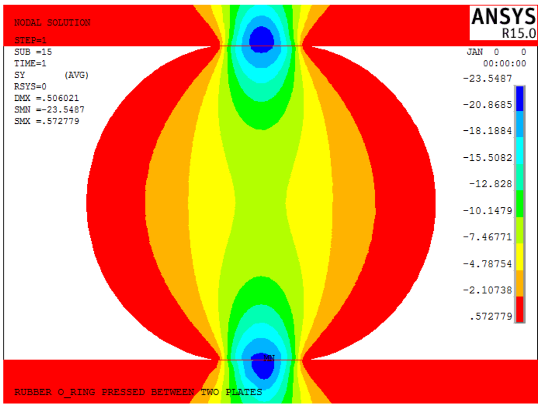

Figure 6. FE σ_y stress distribution

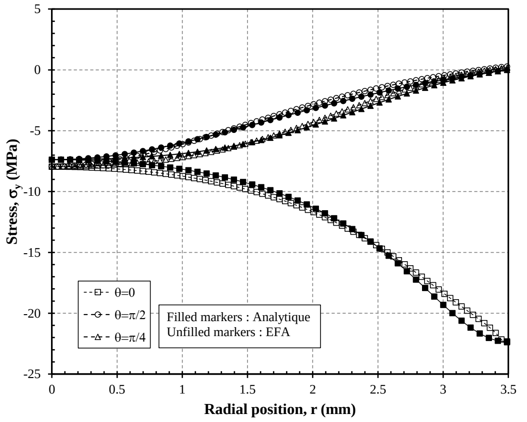

Figure 7. Comparison of FE and analytical radial variation of σ_y

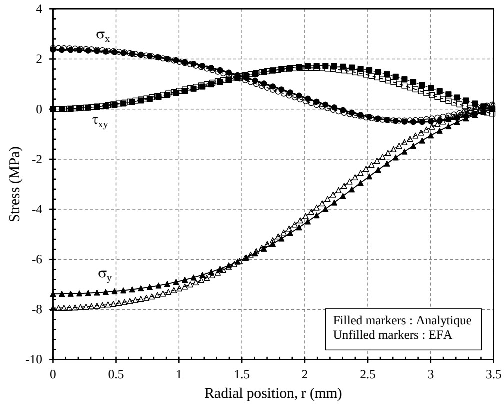

Figure 8. Stresses variation for $\theta = \pi/4$

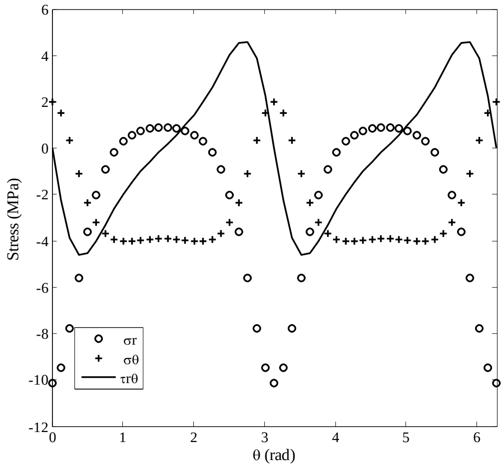

Figure 9. Analytical stresses variation for r=R/2

V. CONCLUSION

The resistance of the O-ring is affected by many parameters connected to the material type and assembly conditions. The installation conditions and the evolution of these conditions during operation involve the creation of stresses which should not reach critical limits causing the deterioration of the joint. Knowledge and determination of these stresses across the gasket prevent and help to avoid the early deterioration of the O-ring and improve the assembly's life.

The semi-analytical approach adopted in this work allowed to know the stresses distribution in the O-ring seal and to envisage the critical zones locations of the early deterioration of the O-ring.

Nomenclature

2a	: contact width (mm)
F	: the total contact force (N)
P_m	: the maximum contact pressure (MPa)
p(x)	: contact pressure distribution (MPa)
q, q_i	: contact pressures (MPa)
R	: the O-ring cross-section radius (mm)
R_m	: the O-ring mean radius (mm)
R_{po}, R_{pi}	: the outer and inner plates radius (mm)
E_g, E_p	: elastic modulus of the O-ring and the plate (MPa)
ν_g et ν_p	: Poisson's ratio of the gasket (O-ring) and the plate
α, α_i	: half angles of contact (rad)
σ_x, σ_y, τ_{xy}	: stresses in Cartesian co-ordinates (MPa)
σ_r, σ_θ, $\tau_{r\theta}$	: stresses in polar co-ordinates (MPa)
(x, y)	: Cartesian coordinats
(r, θ)	: polar coordinats

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