

Timoshenko's Stress-Strain Application In Determining The Variation Of Tensile Elastic Modulus In Flexible Pavements Modified With Candle Wax And Subjected To Diametral Split

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ABSTRACT

Laboratory investigation of flexible pavement was carried out using asphalt concrete samples to simulate the behaviour of actual field conditions of a flexible pavement under light, medium and heavy traffic conditions. The study was carried out in order to ascertain the variation that will result in the elastic modulus of flexible pavements modified with candle wax using Timoshenko's stress-strain analysis from Hooke's Law of Linearity. Furthermore, the concretes were subjected to tensile splitting by the application of static loads across the diametral axis of the concretes to obtain stresses and strains in both horizontal and vertical directions within the elastic region. The results obtained were imputed into Timoshenko's model for obtaining elastic modulus of materials. The results obtained revealed that elastic modulus of the modified asphalt concretes increased linearly with increasing addition of candle wax from 0-15% for all categories of traffic considered; however further additions resulted in a decrease in elastic modulus. The results of elastic modulus at the threshold candle wax content of 15% were 10,701.59Mpa, 16,937.05Mpa and 23,512.35Mpa for light, medium and heavy traffic considerations respectively.

KEY WORDS: FLEXIBLE PAVEMENT, ELASTIC MODULUS, TIMOSHENKO, STRESS-STRAIN, SPLIT TEST AND CANDLE WAX

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I. INTRODUCTION

Flexible pavements are amongst the world's largest infrastructural components especially in developed nations (Kim, 2008). Their purpose is to help make the conveyance of goods and services easier, faster and safer. With the increasing demand of both human and other forms of movement the need for well constructed road pavements cannot be overemphasized.

In pavement engineering it is acceptable to say that analysis of the elastic properties of flexible pavement will many times involve tests that will require failure of the pavement. For example before any test on any section of the pavement can be carried out, there must first be coring of the sample from the road pavement which leads to pavement failure. Therefore, in many circumstances flexible pavements are usually analyzed using properties of Hot Mix Asphalt Concretes (HMA concretes) from the laboratory which nearly simulate the pavement behaviour in real life. Thus, in pavement engineering it is commonly agreed that the properties of flexible pavement in the field can be simulated by the properties of HMA concretes from the laboratory (Igwe et al., 2016).

Although Emesiobi (2000) argued that pavement materials are isotropic; in reality the nature of flexible pavements is one that is complex because of its make-up involving different materials that are non-homogeneous and anisotropic. In addition, flexible pavements are usually subjected to different forms of environmental (temperature, moisture and oxidation) and traffic loading conditions which results in the difficulty of precise prediction of the actual pavement life (Robbins, 2009 & Igwe et al., 2009).

The design of flexible pavement will normally involve two major parts namely – a) material mix design of the asphalt wearing course largely involving the selection, proportioning and blending of the different materials that make-up the upper layer; b) the structural design of the pavement layers that make-up the pavement structure. The latter involves design of the thickness of the sub-base, base and asphalt wearing course using design methods such as the empirical or rationale methods (Oguara, 1985 and Emesiobi, 2004). However, the rationale method which encompasses the use of mechanistic design approach involving the elastic properties of the pavement materials give more stable result (Oguara and Emesiobi, 2001; Oguara, 2005).

In an earlier study; Brown and Foo (1989) stated that in flexible pavement design methods particularly mechanistic design approach involving the application of elastic theories; there is need for the elastic properties of the pavement materials to be determined. However, the study by Michael and Ramsis (1988) has previously observed that some of the common methods of measurement of elastic properties of asphalt mixes include Young's or Elastic Modulus, Shear Modulus, Bulk Modulus and Dynamic Modulus.

Emesiobi (2000) stated that knowledge of elastic properties of pavement materials is essential in elastic theory for the mechanistic design of both flexible and rigid pavements, including overlays. He further posited that in the mechanistic design method, the pavement structure is regarded as a linear multi-layered structure in which the stress-strain solutions of the materials are characterized by the Young's modulus of elasticity herein referred to as elastic modulus and Poisson's ratio.

In later separate studies; Garcia and Thompson (2007) and Robbins (2009) revealed that the most important hot-mix asphalt (HMA) property influencing the structural response of a flexible pavement is the stiffness modulus of the concrete. On this basis the assessment of the Elastic Modulus of flexible pavement became pertinent thus justifying the purpose of the present research.

The modulus of elasticity of any material is defined as the ratio of the stress to strain within the region of elasticity of the material (Emesiobi, 2000). The modulus of elasticity can be further designated in tension, compression and shear per unit area. For purpose of the present study modulus of elasticity was limited to tension alone resulting from splitting test.

1.1 Application of Timoshenko's Stress-Strain Theory

Timoshenko's stress-strain application was based on Hooke's Law of linearity which assumes that within the elastic region of a given material the ratio of stress to strain is linear and that the slope of the change in stress to strain within the linear region is constant. Furthermore, the asphalt concrete mixture when loaded diametrically exhibits a bi-axial stress system having stresses in both the horizontal and vertical axis with corresponding horizontal and vertical strains. The Timoshenko's stress-strain model for Elastic Modulus in two dimensional stress system is as presented in Equations 1- 3 below;

a) Elastic Modulus in Tension (Timoshenko and Goodier, 1951 and ASTM, 1973)

$$E = \frac{1}{\epsilon_x} (\sigma_x - \mu\sigma_y) \quad 1$$

Where

E = Elastic or Young's Modulus (MPa)

ϵ_x = Horizontal Tensile Strain

σ_x = Horizontal Tensile Stress (N/mm²)

σ_y = Vertical Compressive Stress (N/mm²)

μ = Poisson's Ratio

b) Stresses in Tension and Compression (Timoshenko, 1934 and Chong et al, 1979)

$$\sigma_x = \frac{2P}{\pi TD} \quad 2$$

$$\sigma_y = \frac{-6P}{\pi TD} \quad 3$$

Where

P = Load at failure

T = Thickness of Cylinder = 64mm

D = Diameter of Cylinder = 102mm

1.2 Objective

The objective of the research was to ascertain the changes that would occur in the elastic modulus of flexible pavements that has been modified with recycled candle wax wastes from domestic use subjected to diametral splitting by the application of Timoshenko's stress-strain relationship based on Hooke's Law of Linearity for materials within the elastic region. However, owing to the fact that a fully constructed flexible pavement cannot be subjected to splitting test which will result in pavement destruction and thus failure; thus

laboratory simulation using asphalt concrete mixtures which nearly replicates same behaviour as flexible pavement were used instead.

II. MATERIALS AND METHOD

2.1 Materials

Materials used for the research included asphalt cement which was the bituminous binder, gravel which served as the coarse aggregates and sand. In addition, recycled candle wax was used as a modifier in the asphaltic concrete mixtures.

2.2 Preliminary Tests of Materials used

In line with recommended standards preliminary tests were carried out for the materials used and the results recorded as presented in section 3. Some of the tests included penetration, softening point, viscosity and specific gravity tests for the asphalt cement. Similarly, sieve analysis and specific gravity tests were carried out for both coarse and fine aggregates. Lastly, specific gravity test was carried out for the candle wax used as modifier.

2.3 Sample Preparation

Asphalt concrete samples that simulated actual flexible pavements in the field were prepared in accordance with test method as proposed by Bruce Marshall (Asphalt Institute Manual Series, 1984; National Asphalt Pavement Association, 1982 and Roberts et al, 1996).

2.4 Indirect Tensile Test from Split Cylinder Test

The static indirect tensile strength of each specimen was determined using the procedure outlined in ASTM D 6931 where a loading rate of 51mm/minute was adopted causing Tensile failure to occur in the sample rather than the compressive failure (See Equation 3 above).

The test involved loading 102 mm diameter and 64mm thick specimen using a 13 mm wide strip to provide a uniform loading in order to produce uniform stress distribution through the diametral axis. The applied load through the diametral axis indirectly created tensile stress in the horizontal direction and corresponding compressive stress in the vertical direction of the sample and the peak load at failure of specimen was recorded. Furthermore, strains in both the horizontal and vertical directions were also measured using strain gauges.

III. RESULTS FROM LABORATORY ANALYSIS

Results from laboratory investigation plus results from analysis of laboratory tests are presented below. Furthermore, calculation sheet from excel upon which elastic modulus results were obtained using Timoshenko's stress-strain model for linear elastic materials is presented also.

Table 1: Classification Test Results of Materials

Material	Asphalt	Sand candle Wax	Gravel
Specific gravity	1.06	2.80 0.8	2.56
Grade of binder material	-	40/50	-
Mix proportion (%)	-	42	58
Viscosity of binder (poise)	-	5.7*(10 ⁻⁴)	-
Softening point	-	-	-
Penetration value	-	41.5°C 51.3mm	-

Table 2: Mix Proportion of Aggregates Blended (ASTM: 1951)

Sieve size (mm)	Specification limit	Aggregate A (Gravel)	Aggregate B (Sand)	Mix proportion (0.59A+0.41B)
19.0	100	99.1	100	99.45
12.5	86-100	86.1	100	91.80
9.5	70-90	57.5	100	74.93
6.3	45-70	21.8	100	53.86
4.75	40-60	7.5	99.5	45.22
2.36	30-52	3.5	97.3	41.96
1.18	22-40	2.3	92.3	39.20
0.6	16-30	1.8	69	29.30
0.3	9-19	1.4	28.2	12.39
0.15	3-7	1	8.4	4.0
0.075	0	0.6	0.8	0.68

Table 3: Results of Tensile Elastic Modulus of Asphalt Concrete for Light Traffic

Candle wax	ϵ_y	ϵ_x	P (N)	$\sigma_x=2p/\pi t d$	$\sigma_y=-6p/\pi t d$	$1/\epsilon_x$	$\nu=\epsilon_x/\epsilon_y$	E (MPa)
0	0.00026125	0.0001254	3,228	0.314758	-0.94427	7974.482	0.48	6124.484
5	0.0002425	0.0001164	3,455	0.336893	-1.01068	8591.065	0.48	7062.014
10	0.00021542	0.0001034	4,126	0.402321	-1.20696	9671.18	0.48	9493.846
15	0.00020333	0.0000976	4,390	0.428063	-1.28419	10245.9	0.48	10701.59
20	0.00023583	0.0001132	3,980	0.388085	-1.16425	8833.922	0.48	8365.081
25	0.00026333	0.0001264	3,410	0.332505	-0.99751	7911.392	0.48	6418.607

Table 4: Results of Tensile Elastic Modulus of Asphalt Concrete for Medium Traffic

Candle wax	ϵ_y	ϵ_x	P (N)	$\sigma_x=2p/\pi t d$	$\sigma_y=-6p/\pi t d$	$1/\epsilon_x$	$\nu=\epsilon_x/\epsilon_y$	E (MPa)
0	0.00027867	0.0001254	4,996	0.487154	-1.461461	7974.482	0.45	9129.278
5	0.00025867	0.0001164	5,312	0.517967	-1.5539	8591.065	0.45	10457.23
10	0.00022978	0.0001034	6,357	0.619863	-1.85959	9671.18	0.45	14087.8
15	0.00021689	0.0000976	7,214	0.703428	-2.110285	10245.9	0.45	16937.05
20	0.00025156	0.0001132	5,950	0.580177	-1.740532	8833.922	0.45	12044.31
25	0.00028089	0.0001264	4,855	0.473405	-1.420215	7911.392	0.45	8801.439

Table 5: Results of Tensile Elastic Modulus of Asphalt Concrete for Heavy Traffic

Candle wax	ϵ_y	ϵ_x	P (N)	$\sigma_x=2p/\pi t d$	$\sigma_y=-6p/\pi t d$	$1/\epsilon_x$	$\nu=\epsilon_x/\epsilon_y$	E (MPa)
0	0.000264	0.0001056	8,512	0.829995	-2.489984	9469.697	0.4	17291.55
5	0.00025	0.0001	8,935	0.871241	-2.613723	10000	0.4	19167.3
10	0.00024075	0.0000963	9,570	0.933159	-2.799477	10384.22	0.4	21318.27
15	0.00022775	0.0000911	9,985	0.973625	-2.920875	10976.95	0.4	23512.35
20	0.000244	0.0000976	9,460	0.922433	-2.767299	10245.9	0.4	20792.55
25	0.000265	0.000106	8,345	0.813711	-2.441132	9433.962	0.4	16888.33

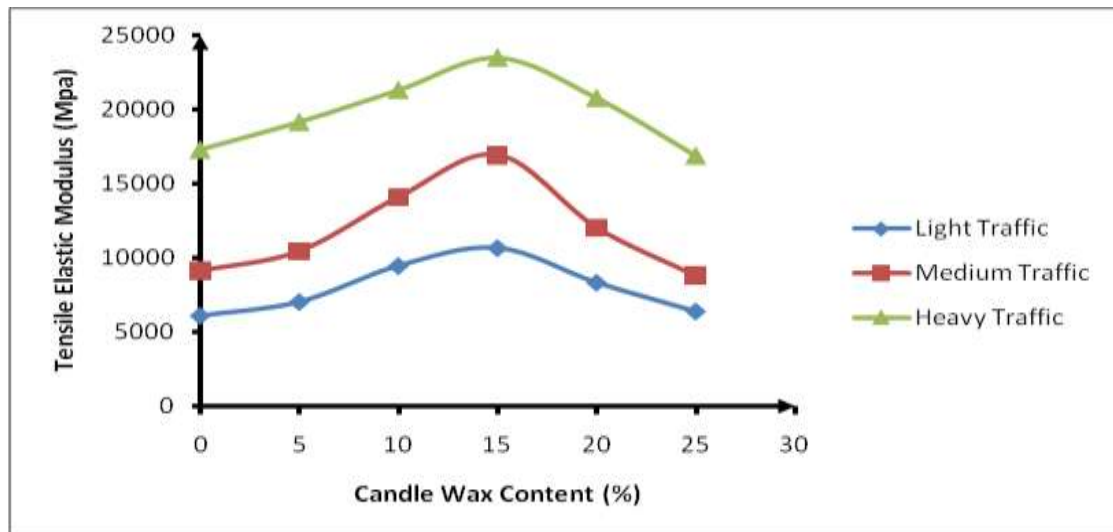


Figure 1: Variation of Tensile Elastic Modulus with Candle Wax Content

IV. RESULT DISCUSSIONS

From Tables 3-5 and Figure 1; it was observed that increasing candle wax content between 0-15 percent resulted in a simultaneous linear increase in tensile elastic modulus of the asphalt concretes for all the categories of traffic considered. Furthermore, additions beyond 15 percent of the candle wax content resulted in a decrease in elastic modulus of the asphalt concretes for all traffic categories herein considered.

For light traffic tensile elastic modulus increased from 6124.484MPa at 0 percent candle wax content to 10,701.59MPa at 15 percent candle wax content implying a 42.8 percent increase in elastic modulus of the asphalt concrete.

Similarly, for medium traffic tensile elastic modulus increased from 9129.278MPa at 0 percent candle wax content to 16,937.05MPa at 15 percent candle wax content implying a 46.1 percent increase in elastic modulus of the asphalt concrete.

Finally, for heavy traffic tensile elastic modulus increased from 17,291.55MPa at 0 percent candle wax content to 23,512.35MPa at 15 percent candle wax content implying a 26.5 percent increase in elastic modulus of the asphalt concrete.

V. CONCLUSIONS

From the laboratory tests and investigations carried out plus analysis of the laboratory results the following conclusions were made;

1. That the addition of candle wax to asphalt concrete mixtures can improve the tensile elastic modulus of the asphalt concretes whether for light, medium or heavy traffic conditions.
2. However, additions should not be more than 15 percent of the candle wax for optimal performance of the concretes.
3. That since asphalt concrete are used to simulate performance of flexible pavement; the results concludes that candle wax addition can enhance the elastic modulus of flexible pavements and therefore enhance their performance.

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