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On The Propagation of Free Waves in Viscoelastic Layered Bodies

Ro'ziyev T.R., Nuriddinov B.Z., Khojiyev A.
Corresponding Author: Engr. Ro'ziyev T.R.,

---*ABSTRACT*---
Annotation. At this work the propagation of natural waves in dissipative inhomogeneous planar bodies is discussed. Wave motions are described by linear integral-differential equations. Solving this problem, we obtain a relationship between the wave velocity and its length. The task of this kind is of great interest for geophysicists, in the field of engineering and construction.

Keywords: viscoelastic half-space, layer, equation of motion, phase velocity.

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I. INTRODUCTION

Many construction and engineering structures, working in dynamic conditions, consist of deformable bodies with different viscoelastic properties [1,2,3,4,6,7]. In addition, wave processes in elastic bodies play an important role in connection with the processing of signals, in particular in connection with the creation of mechanical resonators and filters [8, 9, 10]. The mechanisms by which the energy of elastic waves is converted into heat are not entirely clear. Various loss mechanisms are proposed [11 - 16], but not one of them does not fully meet all the requirements. Probably the most important mechanisms are internal friction in the form of sliding friction (or sticking, and then slipping) and viscous losses in pore fluids; the latter mechanism is most significant in strongly permeable rocks. Other effects that are probably generally less significant are the loss of some of the heat generated in the phase of compression of wave motion by thermal conductivity, piezoelectric and thermoelectric effects and the energy going to the formation of new surfaces (which plays an important role only near the source). Therefore, the development of a unified methodology and calculation algorithm, wave fields of dissipative inhomogeneous layered bodies, is a topical problem of the mechanics of a deformable solid [17, 18, 19].

II. FORMULATION OF THE PROBLEM

Let at the Cartesian (x, y, z) coordinate system, with the origin and the OZ axis, be given a sequence of parallel planes (Figure 1)

$$Z = 0, Z = h_1, \quad Z = h_1 + h_2 + \ldots\ldots \qquad Z = h_1 + h_2 + \ldots + h_n$$

Plane $Z = h_1 + h_2 + \ldots + h_n$, (at $n = 2$) will be called $n-m$ horizon Suppose that the spaces between the planes mentioned are filled with isotropic elastic media forming parallel layers. Layers $0 < Z < h$, characterized by permanent $\lambda_0, \mu_0, \rho_{0,}$, will be called zero. Environment, however, $h_1 + h_2 + \ldots + h_n < z < h_1 + h_2 + \ldots + h_n + h_{n+1}$ filling the space between the *n*-th and *n +1* -th horizons, characterized by constants, will be called the *n*-th layer. It will always be assumed that adjacent layers differ from each other in at least one of the constants λ, μ and ρ. In the theoretical study of the described processes, we shall assume that within each layer the wave propagation is described by the usual equations of the theory of elasticity. As for the conditions on the interfaces of adjacent layers, we assume that the components of the vector of elastic displacements and the stress tensor remain continuous when passing through them. This contact is called hard [20]. The dynamics of dissipative inhomogeneous two-layer flat structures is investigated in the article.

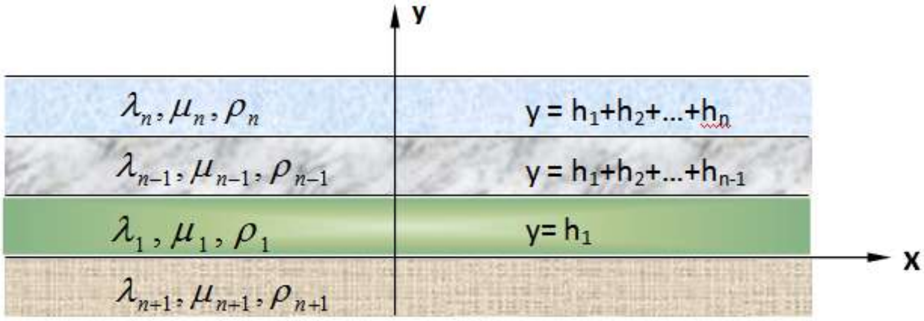


Figure 1. The design scheme: the body on the half-space.

Accounting for internal friction, caused by the dissipation of energy in the material of structures, is a more difficult task. Soft layers of multilayer structures (aggregates), as a rule, are made of materials with developed rheological properties. Therefore, the scattering of energy must first of all be taken into account for soft layers, since it mainly occurs precisely when deforming these layers. Mechanical systems, for which the viscoelastic properties of their elements are identical, will be called **dissipative homogeneous**, systems with different rheological characteristics – **dissipative heterogeneous** [21,22,23].

Equations of motion of the deformed layer in the absence of mass forces have the form [9]:

$$\tilde{\mu}_j \nabla^2 \vec{u} + (\tilde{\lambda}_j + \tilde{\mu}_j\ )grad\ di\vartheta\ \vec{u} = \rho_j \frac{\partial^2 \vec{u}}{\partial t^2}\text{ , (j =1, 2, 3..)} \quad (1)$$

Here $\vec{u}(u_x, u_y, u_z)$ - vector of displacement of points of the medium; ρ_j - the density of the material; u_i – displacement components; ν_j - Poisson's ratio;

$$\tilde{\lambda}_j = \frac{\nu_j \tilde{E}_j}{(1+\nu_j)(1-2\nu_j)}; \qquad \tilde{\mu}_j = \frac{\nu_j \tilde{E}_j}{2(1+\nu_j)}\text{ , где}$$

$\tilde{E}$ – Operational modulus of elasticity, which have the for [9,13]:

$$\tilde{E}_j\varphi(t) = E_{0j}\left[\varphi(t) - \int_0^t R_{Ej}(t-\tau)\varphi(t)d\tau\right] \quad (2)$$

$\varphi(t)$– произвольная функция времени; $R_{Ej}(t-\tau)$– the core of relaxation; E_{01} – instantaneous modulus of elasticity; We assume the integral terms in (2) to be small, then the functions $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- a slowly varying function of time, ω_R - real constant. Further, applying the freezing procedure [9], we note relations (2) with approximations of the form

$$\bar{E}_j\varphi = E_{0j}\left[1 - \Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)\right]\varphi,$$

where $\Gamma_j^C(\omega_R) = \int_0^\infty R_j(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma_j^S(\omega_R) = \int_0^\infty R_j(\tau)\sin\omega_R\tau\, d\tau$, respectively, the cosine and sine Fourier images of the relaxation core of the material. As an example of a viscoelastic material, we take three parametric relaxation nuclei $R_j(t) = A_j e^{-\beta_j t}/t^{1-\alpha_j}$. On the influence function $R_j(t-\tau)$ The usual requirements of inerrability, continuity (besides $t = \tau$), signs - certainty and monotony:

$$R > 0, \qquad \frac{dR(t)}{dt} \le 0, \;\; 0 < \int_0^\infty R(t)dt < 1.$$

$\vec{u}$ - vector of displacements of the medium j- the layer.

Two types of conditions can be defined on the boundary of two bodies:

1. In the case of a rigid contact at the interface, the condition of continuity of the corresponding components of the stress tensor and displacement vector is set, i.e.

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}; \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)};$$
$$u_x^{(1)} = u_x^{(2)}; \quad u_y^{(1)} = u_y^{(2)}. \qquad (3a)$$

If there is no friction at the interface,

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}; \ \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)} = 0; \quad u_y^{(1)} = u_y^{(2)}; \qquad (2,б)$$

2. On the free surface, the condition of freedom from stress is set, т.е.

$$\sigma_{yy}^{(1)} = 0; \quad \sigma_{yx}^{(1)} = 0, \qquad (2,c)$$

where

$$\sigma_{xx}^{(j)} = \lambda_j \theta_j + 2\mu_j \frac{\partial u_j}{\partial x}; \ \sigma_{xy}^{(j)} = \mu_j \left(\frac{\partial u_j}{\partial y} + \frac{\partial \vartheta_j}{\partial x} \right).$$

$$\sigma_{yy}^{(j)} = \lambda_j \theta_j + 2\mu_j \frac{\partial \vartheta_j}{\partial y} \qquad \theta_j = \frac{\partial u_j}{\partial x} + \frac{\partial \vartheta_j}{\partial y}.$$

III. METHODS OF SOLUTION

Now consider the solution of the differential equation (1) - (2) for one layer. The equation of motion in displacements reduces to the following form:

$$\begin{aligned}
&\bar{\mu}_n \left(\frac{\partial^2 u_n}{\partial x^2} + \frac{\partial u^2{}_n}{\partial y^2} \right) + (\bar{\lambda}_n + \bar{\mu}_n) \frac{\partial}{\partial x} \left(\frac{\partial\ u_n}{\partial x} + \frac{\partial \vartheta_n}{\partial y} \right) - \rho_n \frac{\partial^2 u_n}{\partial t^2} = 0; \\
&\bar{\mu}_n \left(\frac{\partial^2 \vartheta_n}{\partial x^2} + \frac{\partial \vartheta^2{}_n}{\partial y^2} \right) + (\bar{\lambda}_n + \bar{\mu}_n) \frac{\partial}{\partial y} \left(\frac{\partial\ u_n}{\partial x} + \frac{\partial \vartheta_n}{\partial y} \right) - \rho_n \frac{\partial^2 \vartheta_n}{\partial t^2} = 0;
\end{aligned} \qquad (3)$$

where ρ_n - density of the material. We find the solution of the problem in the form:

$$u_n = U_n(y) e^{k(x-ct)}; \quad \vartheta_n = V_n(y) e^{k(x-ct)}; \qquad n = 1,2,.......N \qquad (4)$$

where $U_n(y)$) and $V_n(y)$ – amplitude complex vector - function; k is the are number; $C = C_R + iC_i$ – complex phase velocity; a ω – complex frequency.

To clarify their physical meaning, consider two cases:

1) $k = k_R$; $C = C_R + iC_i$, then the solution (4) has the form of a sinusoid with respect to x, the amplitude of which decays in time;

2) $k = k_R + ik_I$; $C = C_R$, Then at each point x the oscillations are steady, but with respect to x they decay.

In both cases, the imaginary parts k_I or C_I characterize the intensity of dissipative processes. Substituting (4) no (3), we obtain:

$$\begin{aligned}
&\bar{\mu}_n \left(U_n'' - k^2 U_n \right) + \left(\bar{\lambda}_n + \bar{\mu}_n \right) ik \left(ikU_n + V_n' \right) + \rho_n k^2 C^2 U_n = 0; \quad (5) \\
&\bar{\mu}_n \left(U_n'' - k^2 U_n \right) + \left(\bar{\lambda}_n + \bar{\mu}_n \right) ik \left(ikU_n + V_n' \right) + \rho_n k^2 C^2 U_n = 0.
\end{aligned}$$

Thus, we have equations (5) of the second order for two domains each. We solve the problem directly, without reducing the equation to a fourth-order equation. All the arguments are given for the layer.

We find the particular solution of the system (5) in the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \begin{pmatrix} A_n \\ B_n \end{pmatrix} e^{r_n y},$$

where r_n – contact. A homogeneous algebraic system with respect to A_n and B_n has non-trivial solutions if its determinant is zero

$$\begin{vmatrix} \overline{C}_{Tn}^2 (r^2{}_n - k^2) - (\overline{C}_{Ln}^2 - \overline{C}_{Tn}^2) k^2 + \overline{C}^2 k^2 & i\left(\overline{C}_{Ln}^2 - \overline{C}_{Tn}^2\right) k r_n \\ i(\overline{C}_{Ln}^2 - \overline{C}_{TLn}^2) k r_n & \overline{C}_{TLn}^2 (r^2{}_n - k^2) + \left(\overline{C}_{Ln}^2 - \overline{C}_{Tn}^2\right) r^2{}_n + C^2 k^2 \end{vmatrix} = 0, (6)$$

$\overline{C}_{Ln}^2 = (\overline{\lambda}_n + 2\overline{\mu}_n)/\rho_n$. $\overline{C}_{Tn}^2 = \overline{\mu}_n/\rho_n$. at $\eta = 0$ magnitudes are respectively the velocity of the waves of compression and shear in an elastic medium [4]. Equation (6) can have four roots

$$(r_n)_{1,3} = \pm k\sqrt{1 - C^2/\overline{C}_{Ln}^2}; \ (r_n)_{3,4} = \pm k\sqrt{1 - C^2/\overline{C}_{Tn}^2}; n = 0,1; \quad i = 1......,4.$$

As a result, we find four particular solutions of the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \sum_{i=1}^{4} C_{ni} \begin{pmatrix} A_{ni} \\ B_{ni} \end{pmatrix} e^{(r_n)_i y}, \ \text{n} = 0,1. \qquad (7)$$

Substituting the values $(r_n)_i$ в (7) we find A_{ni}, B_{ni} at $r_n = (r_n)_i$.
Expressions for the offset are as follows:

$$\begin{pmatrix} U_1 \\ V_1 \end{pmatrix} = C_{11} \begin{pmatrix} ik \\ -k\overline{q}_1 \end{pmatrix} e^{-k\overline{q}_1 y} + C_{12} \begin{pmatrix} ik \\ k\overline{q}_1 \end{pmatrix} e^{k\overline{q}_1 y} +$$

$$+ C_{13} \begin{pmatrix} -k\overline{S}_1 \\ -ik \end{pmatrix} e^{-ks_{11} y} + C_{14} \begin{pmatrix} -k\overline{S}_1 \\ -ik \end{pmatrix} e^{+ks_{11} y};$$

$$\begin{pmatrix} U \\ V \end{pmatrix} = C_{22} \begin{pmatrix} ik \\ -k\overline{q}_1 \end{pmatrix} e^{k\overline{q}_1 y} + C_{24} \begin{pmatrix} k\overline{s} \\ -ik \end{pmatrix} e^{\ k\overline{s} y}.$$

Consequently, for both the hard and sliding contacts, we obtain a set of six boundary conditions that lead to six homogeneous equations with six unknowns C_{11}, C_{12}, C_{13}, C_{14}, C_{22}, C_{24}. For such a system of equations to have non-trivial solutions, the determinant of the coefficients must be zero. The last equation gives the dispersion equation for dissipative systems,
where

$$\overline{S}_n = (1 - C^2/\overline{C}^2{}_n)^{1/2}; \quad \overline{q}_n = (1 - C^2/\overline{C}^2{}_{Ln}), \quad n = 0,1$$

As an example, let us consider the problem of propagation of natural waves in a viscoelastic layer on a half-space.

Hard contact. The dispersion equation has the following form

$$\begin{vmatrix} (1+\bar S_1^2)\,e^{-\bar\xi q_1} & (1+\bar S_1^2)\,e^{\bar\xi q_1} & -2e^{\bar\xi q_1} & 2e^{\bar\xi q_1} & 0 & 0 \\ -2\bar q_1^{\xi q_{11}} & 2\bar q_1^{\xi q_{11}} & \left(\bar s_1+\frac{1}{\bar s_1}\right)e^{-\bar\xi q_1} & \left(\bar s_1+\frac{1}{\bar s_1}\right)e^{\bar\xi q_1} & 0 & 0 \\ (1+s_1^2)\,e^{\bar\xi q_1} & (1+\bar S_1^2)\,e^{-\bar\xi q_1} & -2e^{\xi\bar S_1} & 2e^{-\bar\xi q_1} & (1+s^2)\gamma_1 & -2/\gamma_1 \\ -2\bar q_1^{\xi q_{11}} & 2\bar q_1^{\xi \dot q_{11}} & \left(\bar s_1+\frac{1}{\bar s_1}\right)e^{\bar\xi q_1} & \left(\bar s_1+\frac{1}{\bar s_1}\right)e^{-\bar\xi q_1} & \frac{2\bar q_1}{\gamma_{1\,1}} & \left(\bar s+\frac{1}{\bar s}\right)/\gamma \\ e^{\xi\bar q_1} & e^{\xi\bar q_1} & e^{\xi\bar q_1} & e^{\xi\bar q_1} & -1 & -1 \\ \bar q\,e^{\xi\bar q_1} & \bar q_{1\,1}e^{-\xi\bar q_1} & -\frac{1}{\bar s}e^{\xi\,\bar s} & -\frac{1}{\bar s}e^{\xi\,\bar s} & -\bar q & \frac{1}{s} \end{vmatrix} = 0 \qquad (8)$$

where ζ – dimensionless wave number $\zeta = kh, \gamma_1 = \bar\mu_l / \bar\mu$ or $\lambda+2\mu=$ $= 2\,(1-\nu)/(1-2\nu)$. . As the relaxation nucleus of a viscoelastic material, we take a three-parameter core $R(t)=\frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rizhanitena-Koltunova [13], which has a weak singularity, where A, α, β - parameters materials [13]. We take the following parameters: $A = 0{,}048;$ $\beta = 0{,}05;$ $\alpha = 0{,}1.$ Using the complex representation for the elastic modulus, described earlier.

The roots of the frequency equation are solved by the Mueller method, at each iteration of the Muller method is applied by the Gauss method with the separation of the main element. Thus, the solution of equation (8) does not require the disclosure of the determinant. As the initial approximation, we choose the phase velocities of the waves of the elastic system. For free waves with R_j =0 the phase velocities and the wave number are real quantities. No the calculations, we acce the following parameters values:

$\theta = \rho_1/\rho_2 = 0,\ 75;\ \beta = 10^{-4};n = 1.$

Let us consider two variants of the dissipative system. In the first variant, the dissipative system is structurally homogeneous.

Wave number ξ varies from 0 to 3. The results of the calculations are shown in Figure 2a. Dependence of frequencies and damping on the dimensionless wave number ξ turned out to be monotonous, and the character of the dependence is the same for the frequencies and damping coefficients. In the second variant, the dissipative system is structurally inhomogeneous: the half-space under consideration, equation (8), and elastic parameters coincide with those adopted above. The results of the calculations are shown in Fig. Dependence of frequencies on ξ The same as for a homogeneous system: the corresponding curves coincide with an accuracy of up to 5%. Dependence of damping factors on ξ no monotonic.

Of particular interest is the minimum value ξ with a fixed damping factor:

δ=min ($-\omega_{I\,k}$), k= *1,2,…..K*, (9)

here δ – coefficient that determines the damping properties of the system (we call it the global damping factor).

$\omega_R, -\omega_I$

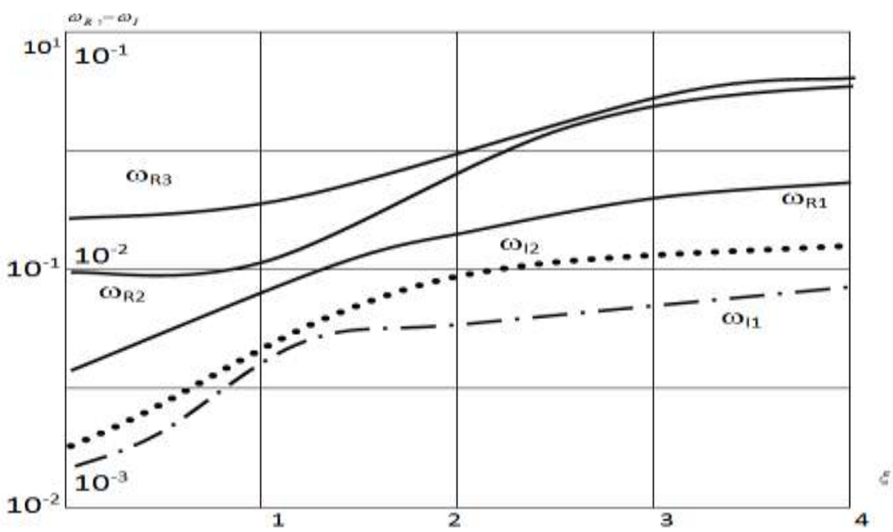


Figure 2.a. Variation of complex Eigen frequencies from the wave number.

A) Dissipatively homogeneous mechanical system.

For a homogeneous system, the coefficient δ is entirely determined by the imaginary part of the first complex-frequency modulus. For an inhomogeneous system in the role of the coefficient δ The imaginary parts of both the first and second frequencies may act as a function of their values. "Change of roles" occurs with a characteristic value ξat this value, the real parts of the first and second frequencies are the closest. Coefficient δ at the indicated characteristic value has a pronounced maximum.

Sliding contact. The dispersion equation is similar in form to equation (8). All parameter values are the same as those used above. Figures 3a and 6b show the dependences of the frequencies and damping coefficients on the wave number ξ, respectively for structurally homogeneous and inhomogeneous systems. The obtained results confirm the earlier conclusions. Change the parameter, from

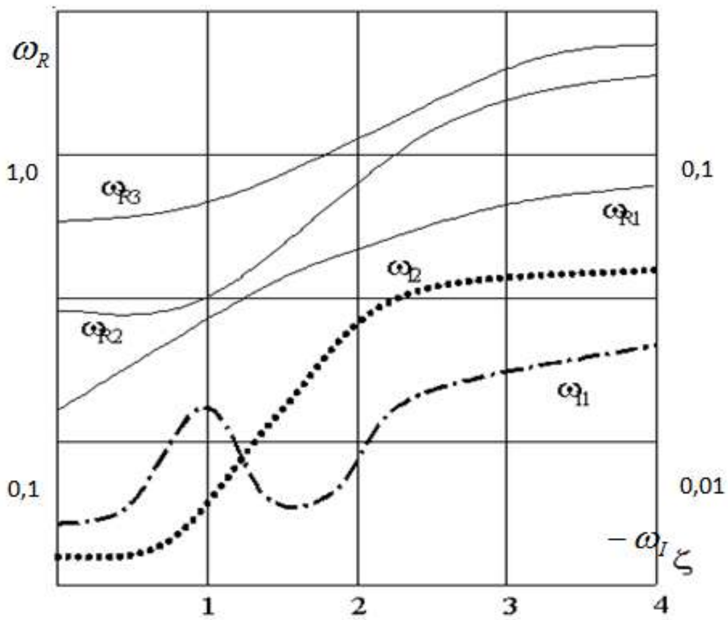
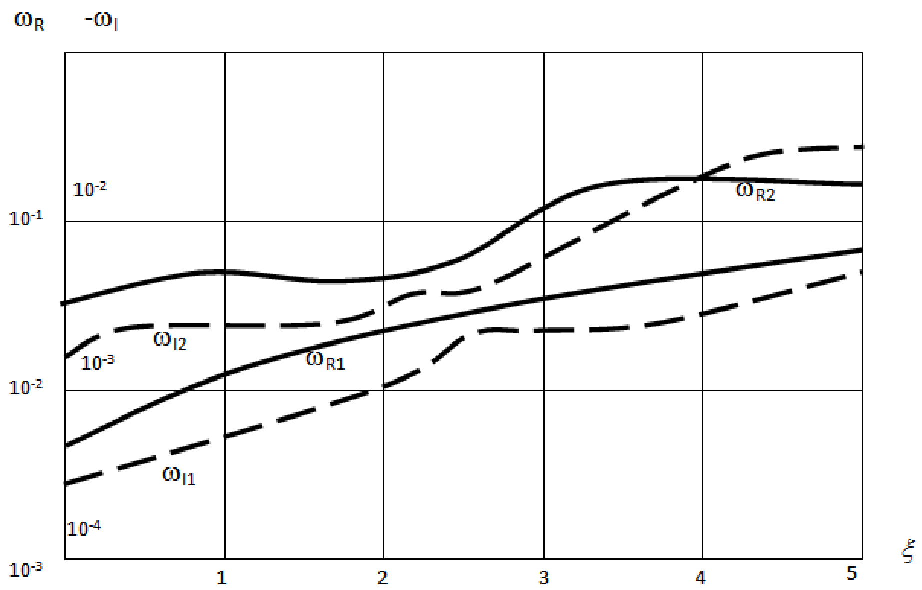
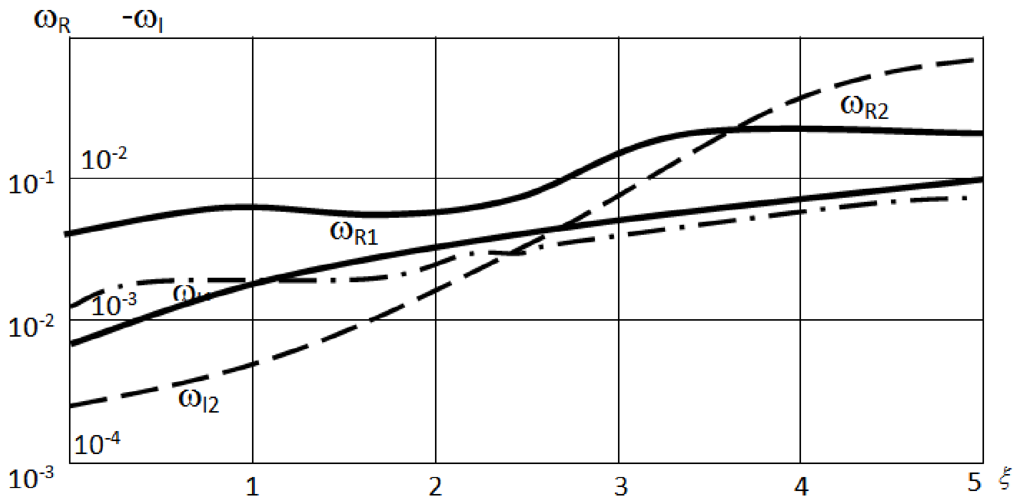


Figure 2.b. Change in complex eigen frequencies from the wave number. b) dissipative inhomogeneous mechanical system;

which depends so significantly on the coefficient δ, can be achieved by varying the geometric dimensions of the elements without changing their mechanical properties.



a) Dissipative homogeneous mechanical system.



b) Dissipative inhomogeneous system.

Figure 3. The change in complex eigenfrequencies from the wave number.

Thus, a promising opportunity for effective control of the damping characteristics of structurally inhomogeneous viscoelastic systems opens up by changing their inhomogeneous systems with close frequencies.

As a second example, let us consider the propagation of natural waves in a plane layer located in a deformable (viscoelastic) medium (Figure 3).

The results of the calculation are shown in Figure 4. Dependence of frequencies on ξ turned out to be the same as for a dissipative homogeneous system: the corresponding curves coincide with an accuracy of up to 5%. As for the coefficients of damping, their behavior has changed radically: the dependence $\omega_I \sim \xi$ became no monotonic. The global damping factor at the specified characteristic value ξ has a pronounced maximum.

IV. CONCLUSIONS

– in the course of solving the problem, the propagation of waves in dissipative - inhomogeneous media revealed no monotonic dependences of the damping rate on the physic - mechanical and geometric parameters of the system. In Dissipatively inhomogeneous media, the dependence of the phase velocity and the damping rate on the geometric and physic-mechanical parameters of the system turned out to be no monotonic;

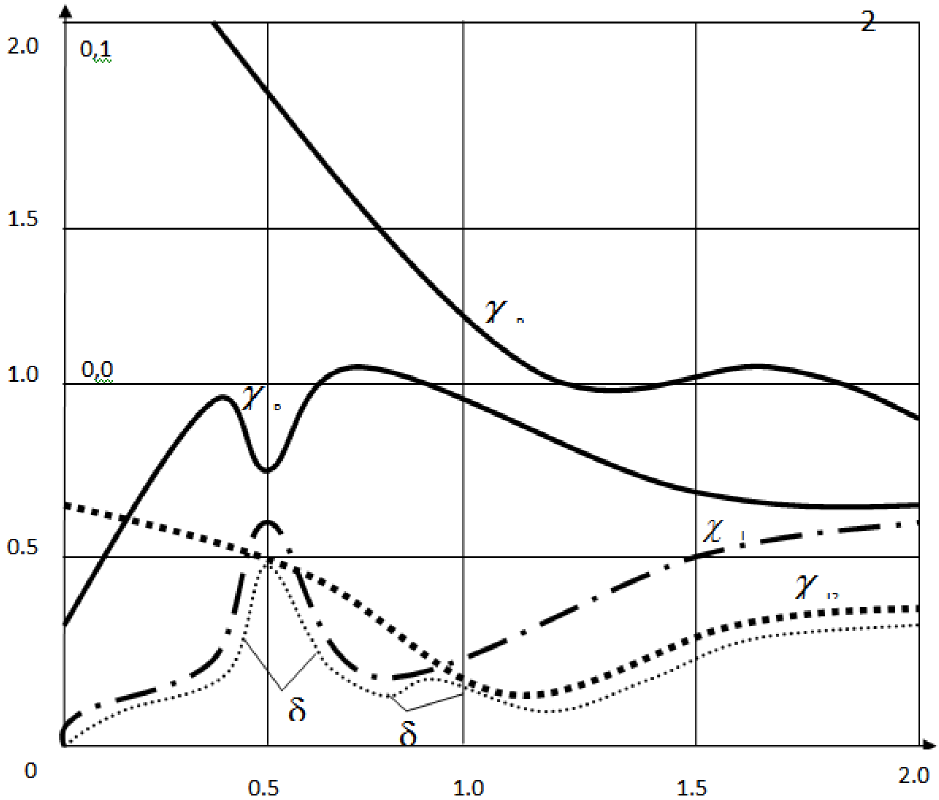


Figure 4. Dispersion curves of phase velocities

χ_{R1}, χ_{R2} and attenuation coefficient χ_{I1}, χ_{I2} for the duralumin strip in contact with the acrylic medium

C_{p1}=5400 м/с; ν_1=0.35; C_{p2}=2300 м/с; ν_1=0.35; C_{S1}=1311м/с; ρ_1 =0.138 кг/м^3; C_{s2}=1311 м/с; ρ_2 =0.126 кг/м^3.

Based on the obtained numerical results, it is revealed that the possibility of detachment of thin-walled structures from a soft layer and the effect of magnitude up to the resonance speed on the dimensions of the contact area. Also taking into account the viscous properties of the material, 15 - 10% increases the values of the phase velocities;

revealed that the phase higher forms of the expansion and torsion waves exceed the highest possible speed (C) of waves in an infinite medium, the group velocity never exceeds C. Also found that the group velocity of 10-15% exceeds the non-dispersive medium, comparison by a dispersive medium. In other words, the forms of the pulses do not remain unchanged, as in homogeneous elastic bodies.

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