The International Journal of Engineering and Science (IJES)
|| Volume || 6 || Issue || 9 || Pages || PP 33-41 || 2017 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Effects of variable viscosity and thermal conductivity on unsteady micropolar fluid about a permeable cylinder under moving boundaries

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Abstract: *Effects of temperature dependent viscosity and thermal conductivity on heat transfer flow of unsteady micropolar fluid about a permeable cylinder with moving boundaries have been considered for study in presence of magnetic field. In the process, viscosity and thermal conductivity are assumed to be linear functions of temperature. Using similarity transformation the governing partial differential equations (PDEs) of the problem are reduced to ordinary differential equations (ODEs). Adequate numerical technique is used to solve these equations for some of the parameters such as unsteady, suction, injection, radiation, viscosity and thermal conductivity. The effects of skin friction, Nusselt numbers are given in tabular form. The results are henceforth analyzed with the help of graphical presentations*

Key words: *Micropolar fluid, unsteady flow, heat transfer, moving cylinder, thermal radiation.*

Date of Submission: 13-09-2017 Date of Publication: 28-09-2017

I. INTRODUCTION

The study on micropolar fluid has been flourishing gradually and as a consequence paid due attention in different practical areas by the researchers. The effect of molecular spine was not considered earlier in the derivative of classical contour. Navier Stokes equation is considered in case of micro continuum theory which was developed by Eringen[1, 2] and extension works in this regards have been found due to Lukaszewicz et al.[10], Arimann et al.[17] In micropolar fluid theory each material point is of a finite size particle having six degrees(three translation and three rotation) of freedom. But in continuum mechanics, each point has only three degrees of freedom (DoF). Extra three DoF of rotation can be used to describe the gyration, not the vorticity. Angular momentum is given to take into account the effect of molecular spin. The phenomena can be observed in the field of ferrofluids, blood flows, bubbly liquids, liquid crystals and so on. Sakiadis[4] was one of the pioneer to study the boundary layer flow under the circumstances and then more contributions were found due to Peddision and Mc Nitt[12].

The boundary layer flow over a stretching or shrinking surface is a relevant type of flow appearing in many industrial and engineering processes, such as, the extraction of polymer and rubber sheets, melt spinning, hot rolling, paper production, wire drawing and glass fiber production etc. Wang [5] investigated the steady flow of incompressible viscous fluid outside a stretching hollow cylinder in an ambient fluid at rest. Iskak[3] extended the idea on injection and suction of the same. Bhattacharyya et al. [13] studied the sleep effects on boundary layer and heat transfer towards a shrinking sheet. Although a limited contribution has been found towards the study of unsteady viscous flow over stretching cylinder. In unsteady nature, a wide range of fluid flow has practical importance because all boundary layer which occur in practice are in unsteady sense and which are more or less familiar to fluid mechanics. Ali, Pop.et al.[6], Sajjad[16], Wang et.al.[18] also discussed various problems of unsteady flow in different environments.

In all the above problems, the thermo physical properties, especially fluid viscosity and thermal conductivity were assumed as constant. Seddeek and Salama[14] studied the flow and the effect of variable viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi infinite vertical porous plate on variable suctions. The effect of variable viscosity and thermal conductivity on non-Newtonian micropolar fluid flow with heat generation was discussed by Hazarika and Phukan[9]. Borthakur et al.[15], discussed the effect of variable viscosity and thermal conductivity of boundary fluid flow on heat transfer of micropolar fluid near an axisymmetric stagnation point on a moving cylinder. In this study, it is intended to take up the effects of variable viscosity and thermal conductivity of unsteady micropolar fluid flow about a permeable moving cylinder under consideration of heat transfer by radiation effects. Some light on the effects of Skin friction coefficient, Nusselt's number and suction / injection effects of the fluid motion due to various parameters also considered for analysis.

II. MATHEMATICAL FORMULATION

The unsteady viscous incompressible micropolar fluid flow past a permeable cylinder which shrinks in axial direction is considered. The cylinder diameter varies as a function of time with unsteady radius $a(t)=a_0\sqrt{1-\Omega t}$, for $\Omega > 0$ the radius become smaller with time and for $\Omega < 0$ the radius become larger with time. Due to axial symmetry, there are only two components in the cylindrical coordinates system i.e., (r, z). The velocity components are given by (u(r, z),0,w(r, z)) and micro rotation components are given by (0, N_2(r, z), 0). Here the viscous dissipation is neglected but the effect of radiation is considered along with the effects of variable viscosity and thermal conductivity. Under this assumption, the fluid equations of the governing problem would appear as below:

Equation of continuity:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0.............. \quad (1)$$

Equation of linear momentum:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial r}+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)+2\frac{\partial \mu}{\partial r}\frac{\partial u}{\partial r}+ \\ \frac{\partial \mu}{\partial z}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)-\kappa\frac{\partial N_2}{\partial z}-\sigma B^2 u.............. \quad (2)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+(\mu+\kappa)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+2\frac{\partial \mu}{\partial z}\frac{\partial w}{\partial z}+ \\ \frac{\partial \mu}{\partial r}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)-\kappa\left(\frac{N_2}{r}+\frac{\partial N_2}{\partial r}\right)-\sigma B^2 w............ \quad (3)$$

Equation of angular momentum:

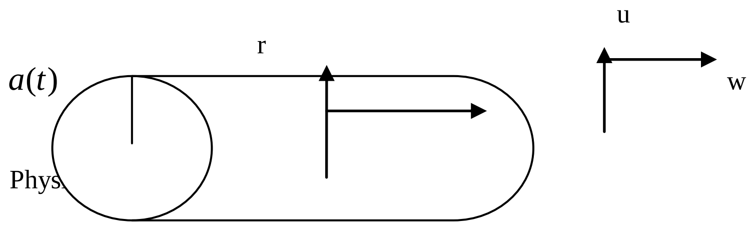
$$\rho j\left(\frac{\partial N_2}{\partial t}+u\frac{\partial N_2}{\partial t}+w\frac{\partial N_2}{\partial t}+\frac{uN_2}{r}\right)=\gamma\left(\frac{\partial^2 N_2}{\partial r^2}+\frac{1}{r}\frac{\partial N_2}{\partial r}+\frac{\partial^2 N_2}{\partial z^2}-\frac{N_2}{r^2}\right)+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\right)-2\kappa N_2 \\ .... \quad (4)$$

Energy equation:

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}+\right)=\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}-\frac{N_2}{r^2}\right)+2\frac{\partial \lambda}{\partial r}\frac{\partial T}{\partial r}+\frac{\partial \lambda}{\partial z}\frac{\partial T}{\partial z}-\frac{\partial}{\partial y}q_r \\ .... \quad (5)$$

Where μ is the co-efficient of dynamic viscosity , κ is the vortex viscosity , P is the pressure, j is the micro-inertia , B is total magnetic field , C_p is specific heat , λ is thermal conductivity and q_r radiation parameter . With the boundary conditions as:

$$\left.\begin{aligned} & u=\frac{U}{\sqrt{1-\Omega t}} \quad ,w=\frac{4\nu z}{a_0(1-\Omega t)} \quad ,N_2=0 \ ,T=T_w \ \ at \ \ r=a(t) \\ & w\to 0, \quad N_2\to 0 \quad ,T\to T_\infty \ , \ r\to\infty \end{aligned}\right\} .... \quad (6)$$



Following Rosseland approximation (1972) the heat flux q_r , is modeled as $q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}$

where σ^* and k^* are Stefan-Boltzmann constant and mean absorption coefficient. Assuming temperature difference within the flow so small, and using Taylor series and neglecting higher order terms T^4 may be expressed as linear function of temperature $T^4 \cong 4T_\infty^3 T - T_\infty^4$

The fluid viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. According to Lai and Kulacki[7]

$$\left.\begin{aligned} &\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1+\delta(T-T_\infty)\right] = b(T-T_r) \\ &where \quad b = \frac{\delta}{\mu_\infty} \quad , T_r = T_\infty - \frac{1}{\delta} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

And

$$\left.\begin{aligned} &\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left[1+\delta(T-T_\infty)\right] = c(T-T_k) \\ &where \quad c = \frac{\xi}{\lambda_\infty} \quad , T_k = T_\infty - \frac{1}{\delta} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (8)$$

Also b, c , T_r , T_k are constants and their values depend on the reference state and thermal properties of fluid i.e. ν and λ . In general b , c > 0 for liquids and b ,c < 0 for gas.
Using similarity transformation as:

$$\left.\begin{aligned} &u = \frac{-2\nu f(\eta)}{a(t)\sqrt{\eta}} \quad , w = -\frac{4\nu z}{a^2(t)} f^{/}(\eta) \quad , N_2 = \frac{8\nu z\sqrt{\eta}}{a^3(t)} g(\eta) \\ &\theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty} \quad , \quad \eta = \left(\frac{r}{a_0}\right)\frac{1}{(1-\Omega t)} \ , \ B = \frac{B_0}{\sqrt{1-\Omega t}} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots (9)$$

The equation (1) is identically satisfied by using (9); based on the equation (2) and (9), it is found that the fluid pressure is independent of z. Equations (3)-(5) give equations (10)-(12) respectively as follows :

$$\left.\begin{aligned} &[1 - C1(\frac{\theta-\theta_r}{\theta_r})](\eta f^{///} + f^{//}) + \frac{\theta^{/}}{\theta-\theta_r} f^{//} - C_1(\eta g^{/} + g) - Mf^{/} \\ &= S(\eta f^{//} + f^{/}) - ff^{//} - (f^{/})^2 \end{aligned}\right] \qquad \ldots\ldots\ldots\ldots (10)$$

$$\eta g^{//} = (C_3 S\eta - C_3 f - 2)g^{/} + (\frac{C_3}{\eta} f + C_3 f^{/} + 2C_3 S + 2C_2)g + C_2 f^{//} \qquad \ldots\ldots\ldots\ldots (11)$$

$$(1+\frac{4}{3}R)\eta\theta^{//} + \theta^{/} + \frac{4}{3}R\theta^{/} + P_r(f - S)\theta^{/} + 2\eta\frac{\theta^{/}}{\theta-\theta_r} = 0 \qquad \ldots\ldots\ldots\ldots (12)$$

$P_r = \frac{\mu c_p}{\lambda}$ is Prandtl number, $S = \frac{\beta a_0^2}{4\nu}$ unsteadiness parameter, $M = \frac{\sigma B_0^2 a_0^2}{4\mu_\infty}$ magnetic parameter, $R = \frac{4\sigma^* T_\infty^3}{kk^*}$ radiation parameter , C_1, C_2 and dimensionless micropolar parameters given by $C_1 = \frac{k}{\mu_\infty}$, $C_2 = \frac{\kappa a^2}{4\gamma}$ and $C_3 = \frac{\mu_\infty j}{\gamma}$.

Here the boundary conditions becomes:

$$\left.\begin{aligned} &f(1) = f_w \;\; where \;\; f_w = -\frac{a_0 U}{2\nu} \;, \\ &f^{/}(1) = 1, \, g(1) = 0 \;\;, \theta(1) = 1 \, when \;\; \eta = 1 \\ &f^{/}(\infty) = 0, \, g(\infty) = 0 \;\;, \theta(1) = 0 \; when \;\; \eta \to \infty \end{aligned}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots..(13)$$

Here in this study rate of shear stress given by skin friction C_f , rate of heat transfer for the surface i.e Nusselt number Nu are determined for various parameter.

$$C_f = \frac{2\tau_w}{\rho w^2} \text{ where } \tau_w = (\mu + \kappa)\frac{\partial w}{\partial r}\Big]_{r=a(t)} \text{ which give } \quad C_f z / a(t) = (C_1 - \frac{\theta_r}{\theta - \theta_r}) f^{//} \quad \ldots\ldots\ldots\ldots..(14)$$

And $Nu = \frac{z q_w}{k(T - T_\infty)}\Big]_{r=a(t)}$ where $q_w = -k(\frac{\partial T}{\partial r})$ give $Nuz / a(t) = -2(\frac{\theta_k}{\theta - \theta_k})\theta^{/}(1)$ ……………(15)

III. RESULTS

The system of equations (10)-(12) with boundary conditions (13) are solved for various combination of the parameters involved in the equations using a well controlled algorithm based on the fourth order Runge –Kutta method with shooting technique .The solutions of the problems are presented in the form of a curves as velocity distributions, temperature distributions and microrotation distributions. Numerical computations have been made by developing programs in Matlab for reasonable range of the parameters involved in the equations. The values of the Skin frictions, Nusselt's number presented in tabular form with the help of the relation (14) & (15).Initially the numerical value of different parameters are taken as M=0.0, P_r=0.72, θ_r=-10, θ_k=-10 , C1=0.5, C2=0.1, C3=2.0 ,S=-1.0, $f_w = 0.0$ etc.

The velocity distributions for the various parameter is represented in fig:1-5. From fig:1 it can be observed that velocity decreases with the increase of magnetic parameter *M*. It is because that the application of magnetic field will result a resistive force (Lorentz force) similar to drag force, which tends to resist the fluid flow and reducing its velocity. In fig:2 fluid velocity speed up for injection (f_w <0) in comparison to suction.(f_w >0). Fig.3-5 velocity again speed up for increasing values of unsteady parameter , viscosity and vortex viscosity(coupling constant) parameter *C1*. It is observed that temperature decreases with the (fig: 7) increasing value of P_r which is due to the reason that increasing values of the Prandtl number the viscosity increases and as a result temperature decrease. Fig:10 displays the change in temperature distribution for radiation parameter R, here temperature increases for radiation.

Fig:8, fig:11 represent that the temperature function θ(η) increase slowly due to the effect of magnetic parameter M ,but increase significantly with f_w from suction to injection. Fig:12 microrotation also increase significantly with f_w from suction to injection. The microrotation function g(η) in fig:12-16 decreasing for *M* and f_w but increasing for S ,viscosity parameter ,*C1* vortex viscosity parameter.

With the help of the tables :1-5 , the effect of rate of heat transfer given by Nusselt number and skin friction co efficient for various different parameters analyzed. In table 1, the increasing value of the viscosity parameter slightly increases the skin friction coefficient but decrease the Nusselt number. Table:5, display that the increasing value of the radiation parameter increase both the skin friction coefficient and Nusselt number but opposite case arises in the change of thermal conductivity. Significant changes occur when the value of parameter changes from suction to injection and the unsteady parameter. Table:6 shows the comparisons of the values of f'' with the paper Fang et.al.(2012) [18],W.M.K.A. Wan Zaimi et. al.(2013)[19] and also with the values of S.Hussain(2016)[16] . In all these cases with the decreasing values of unsteadiness parameter f'' also decreases . When we consider the micropolar fluid with the effect of variable viscosity and thermal conductivity the values have significant changes although the trend of decreasing remain same. . From above analysis it reflect that variable viscosity and thermal conductivity influence the fluid flow. Table 7: shows the comparison of various values as f'', θ', g' with respect to f_w ,from that it is easy to say that variable viscosity and thermal conductivity have significant effect on in micro rotation function.

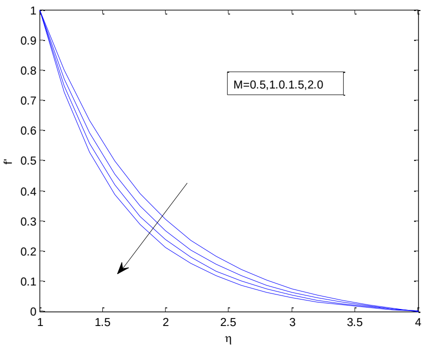


fig:1

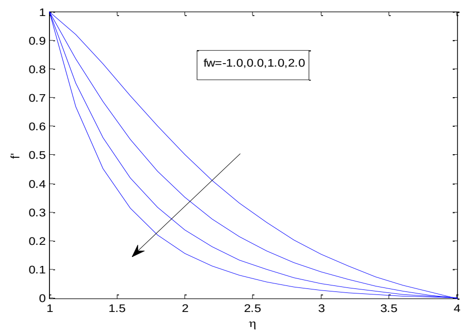


fig :2

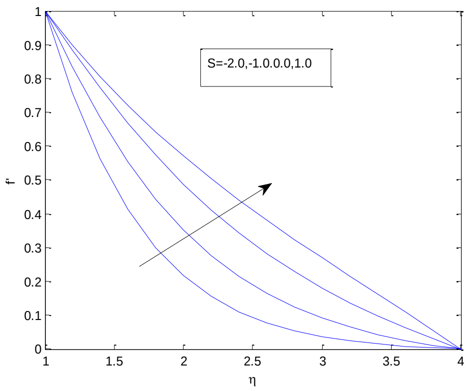


fig:3

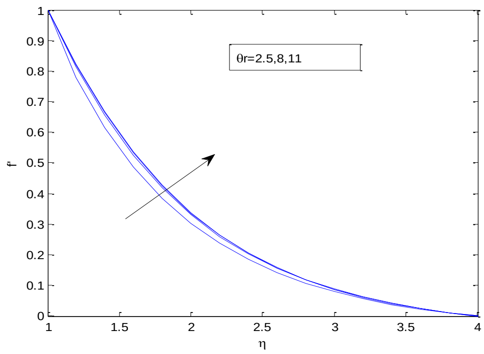


fig:4

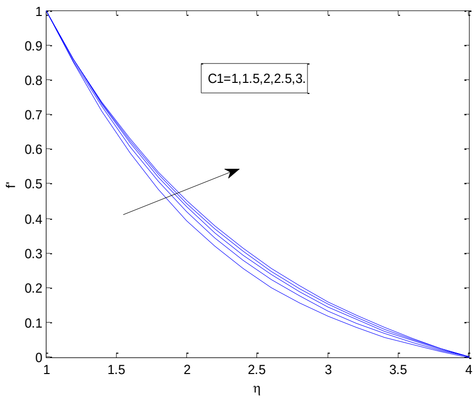


fig:5

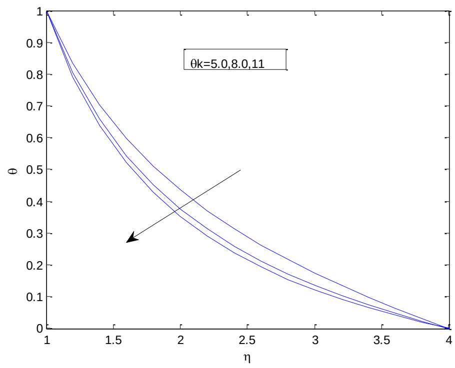


fig:6

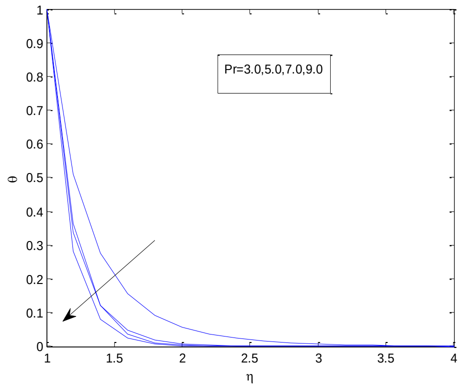


fig: 7

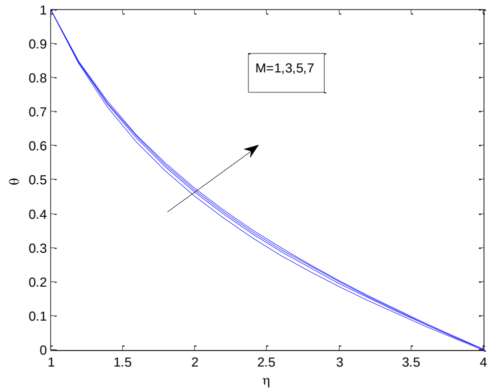


fig:8

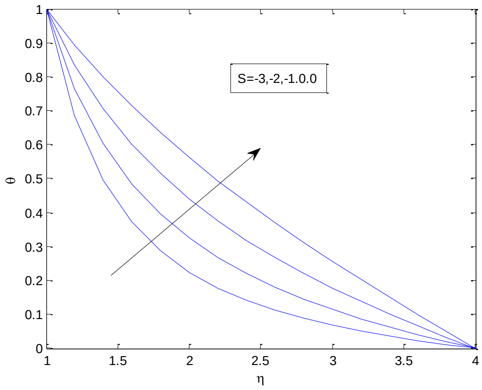


fig: 9

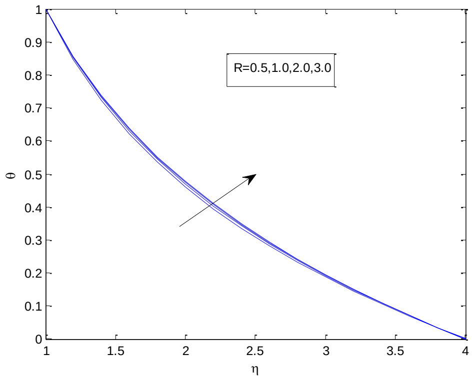


fig:10

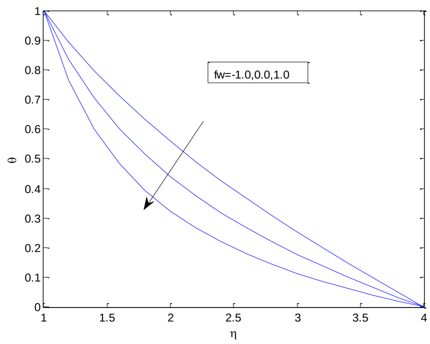


fig:11

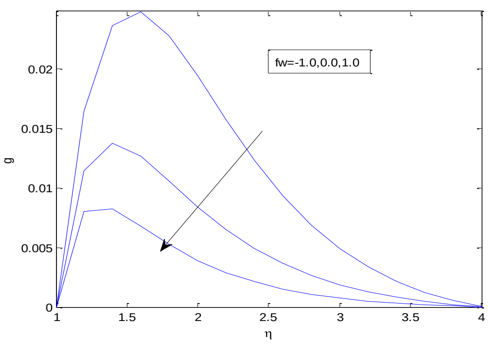


fig:12

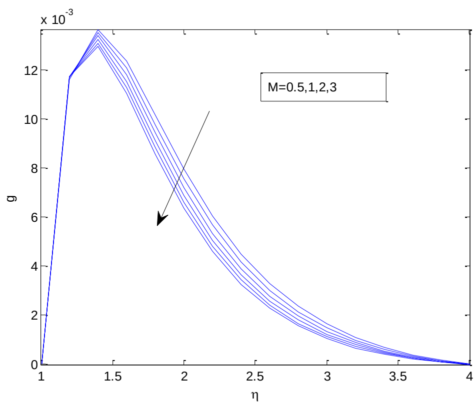


fig :13

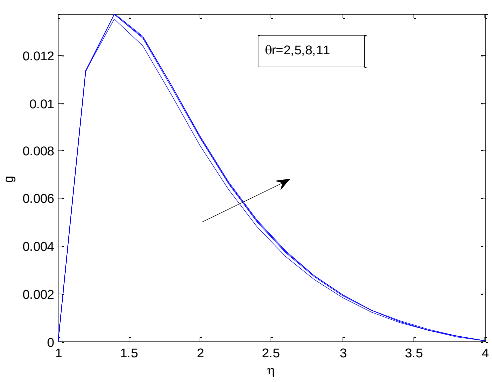


fig:14

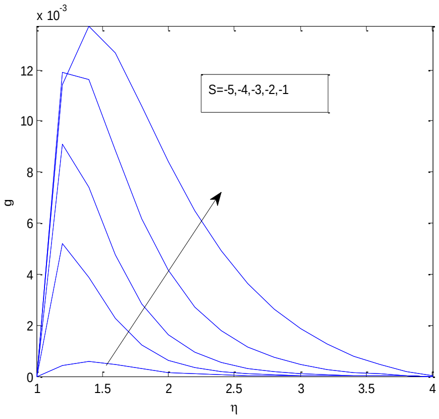


fig :15

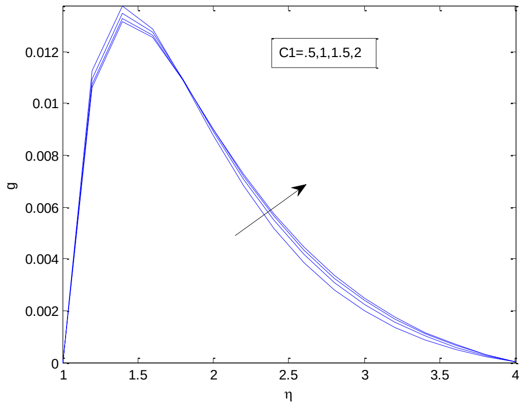


fig:16

Table:1

θ_r	C_f	Nu
-12	-1.72699	-1.88151
-11	-1.72277	-1.88173
-10	-1.71779	-1.882
-9	-1.71182	-1.88232
-8	-1.70453	-1.88271
-7	-1.69545	-1.8832
-6	-1.68381	-1.88384
-5	-1.66835	-1.8847
-4	-1.64686	-1.88594
-3	-1.61501	-1.88784
-2	-1.56329	-1.89115

Table:2

θ_k	C_f	Nu
-12	-2.18244	-3.17291
-11	-2.18387	-3.20408
-10	-2.18556	-3.24123
-9	-2.18759	-3.28628
-8	-2.19008	-3.34204
-7	-2.19319	-3.41284
-6	-2.1972	-3.50572
-5	-2.20256	-3.63287
-4	-2.21008	-3.81752
-3	-2.2214	-4.10995
-2	-2.24026	-4.643

Table:3

f_w	C_f	Nu
-5	1.560546	-0.08987
-4	1.322174	-0.17882
-3	0.988127	-0.34905
-2	0.315505	-0.64572
-1	-0.75745	-1.13125
0	-2.11176	-1.86449
1	-3.65842	-2.86519
2	-5.32475	-4.09776

Table: 4

R	C_f	Nu
-12	-2.09132	-1.4944
-11	-2.0912	-1.49236
-10	-2.09106	-1.48988
-9	-2.09088	-1.48681
-8	-2.09065	-1.48291
-7	-2.09036	-1.47776
-6	-2.08995	-1.47069
-5	-2.08934	-1.46034
-4	-2.08838	-1.44376
-3	-2.08657	-1.41291
-2	-2.08198	-1.33554

Table:5

S	C_f	Nu
-7	-11.8849	-9.98668
-6	-10.0277	-8.47739
-5	-8.27871	-6.96479
-4	-6.59002	-5.48408
-3	-4.94791	-4.09155
-2	-3.40031	-2.86252
-1	-2.11176	-1.86449
0	-1.33916	-1.12016
1	-1.14588	-0.61151
2	-1.2551	-0.3033

Table-:6

	Fang et al.(2012)	W.M.K.A.Wan Zaimi et al.(2013)	Sajjad.H.(2016)	Present result
S	$f^{//}$	$f^{//}$	$f^{//}$	$f^{//}$
0	0	0	-1.64494	0.66768
-1	-0.5791	-0.57912	-2.09355	-1.05408
-2	-1.6973	-1.69730	-2.76411	-1.69826
-3			-3.57354	-2.47201
-4			-4.45475	-3.29306
-5	-7.0031	-7.00350		-8.27871
-6		-9.27666		-10.0277

Table:7

Sajjad.H.(2016)				Present result		
f_w	f''	θ'	g'	f''	θ'	g'
-1.0	-1.06818	-1.09692	-0.00719	-0.37766	-0.56372	0.107811
0.0	-1.70090	-1.39744	-0.00911	-1.05408	-0.93034	0.096597
1.0	-2.47377	-2.02436	-0.01055	-1.82717	-1.43068	0.091679

IV. CONCLUSION

Effects of variable viscosity and thermal conductivity of unsteady micropolar fluid towards a permeable cylinder with moving boundaries was solved numerically using fourth order Runge-Kutta method with shooting technique. Some conclusion can be drawn from the above analysis.

1. Increasing value of the suction parameter skin friction co-efficient and Nusselts number increases.
2. Suction /injection parameter significantly effect the micro-rotation function g(η)
3. The flow velocity, skin friction, temperature profile clearly effected by variable viscosity and thermal conductivity.
4. Due to increasing value of the unsteady parameter, viscosity the micro rotation parameter g(η) increases but it is decreases in magnitude with the increasing value of *M*.
5. The radiation increases the fluid temperature but the temperature reduces by the increase of Prandtl number
6. Increasing value of the unsteady parameter S increase the magnitude of three quantities velocity , temperature and micro-rotation.

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