The International Journal of Engineering and Science (IJES)
|| Volume || 6 || Issue || 9 || Pages || PP 16-25 || 2017 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Modeling and Numerical Simulation for the Double Corrugated Cardboard under Transverse Loading by Homogenization Method

*Duong Pham Tuong Minh
Thai Nguyen University of Technology, Thai Nguyen city, Vietnam
Corresponding Author: Duong Pham Tuong Minh

ABSTRACT

In this paper, an analytic homogenization model for the double corrugated cardboard plates under transverse loading is presented. This model is essentially based on the theory of stratification and then improved by using the theory of sandwich. The proposed analytical homogenization model allows modelling the 3D complex double corrugated cardboard by a 2D equivalent homogenized plate. This work helps to significantly reduce the computational time as well as time to build the geometry of model. To validate the model, in this work, we carry out the simulation for the corrugated cardboard plates in case of transverse loading. A very good agreement is obtained between the three-dimensional shell simulations and H-2D model demonstrating the accuracy and efficiency of our model. The homogenization model can be used not only for corrugated cardboard plates, but also for many types of sandwich panels.

Keywordsl: *Corrugated cardboard, Homogenization, Orthotropic plate, Sandwich panels, Transverse shear.*

Date of Submission: 25 AUG 2017 — Date of Accepted: 17 SEP 2017

I. INTRODUCTION

Sandwich panels have been successfully used for many years in the aviation and aerospace industries, as well as in marine, mechanical and civil engineering applications. This is due to the attendant high stiffness and high strength to weight ratios of sandwich systems [1]. The corrugated cardboards are produced by a manufacturing process, in which three or more layers are laminated together. They are the orthotropic sandwiches with the flat layers (liners) providing bending stiffness, separated by a lightweight corrugated core (flutes) that provides shear stiffness [2]. The cores and liners are glued along the edges of the facing plates to form a wide sandwich panel. The manufacturing process gives three characteristic directions (Fig. 1): The machine direction (MD) corresponds to the direction of manufacturing of the materials coinciding with the x-axis, the cross direction (CD) corresponds to the transverse direction coinciding with the y-axis, and the thickness direction (ZD).

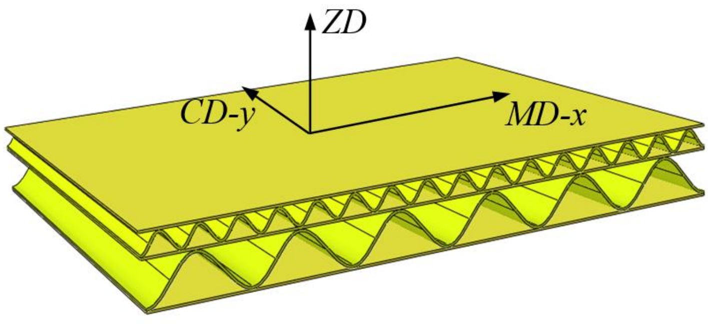

Fig. 1. The double corrugated cardboard

Corrugated core sandwich plates possess high strength and stiffness per unit weight. A common application of this material is in corrugated board boxes for shipment of goods. These boxes are often stacked on top of each other during transportation and in storage. Thus, an important design parameter for a box is the top-to-bottom compression strength. Before collapse, the vertical sides undergo large out-of-plane deformations with loads often exceeding the critical buckling load. The processes of load redistribution and final collapse are governed by the boundary conditions, the transverse shear rigidities and the flexural rigidities of the panel [3]. Several approaches to the modeling of these sandwich panels are discussed, in which finite element method is known as an effective method and used popularly [4]. When we use the finite element in the commercial software, an actual geometry of the model is represented [5]; however, this method can be easily done with a tiny, normal size and symmetrical plates. It is much more advantageous to homogenize the sandwiches in order to obtain an equivalent orthotropic plate (2D plate or shell modelling) [6].

Many homogenization models were obtained by analytical, numerical and experimental methods [5-11]. Luo et al. [7] made an analytic study on the bending stiffnesses of corrugated board. Nordstrand et al [8-9] presented some homogenized properties of the corrugated board by an analytical method. They have studied the buckling and post-buckling behaviors of an orthotropic plate including the transverse shear effect. Aboura et al. [2] also developed an analytical homogenization model based on the theory of laminated plate and compared its results with numerical and experimental results. Buannic et al. [5] proposed a homogenization theory based on the asymptotic expansion method and presented the FE computation of the effective behavior properties. Biancolini [10] used a FE numerical approach for evaluating the stiffness parameters. Among the existing analytical models, there are still some questionable problems such as the behaviors under the transversal shear efforts and the torsion moments.

In this paper, we present a homogenization model to simulate the mechanical behaviors of corrugated cardboards. The homogenization is carried out by calculating analytically the global rigidities of the double corrugated cardboards and then this 3D structure is replaced by an equivalent homogenized 2D plate. The simulations in case of transverse loading of Abaqus-3D and H-2D model for the corrugated cardboards will be studied in this article. This 2D homogenization model is very fast and has close results comparing to the 3D model using the Abaqus shell elements. The comparison shown many outstanding advantages of proposed model such as reduced time for modeling, time for calculation … We can use this model, of course, for other core structures, types of load or many other types of sandwich panels.

II. MINDLIN'S THEORY AND THEORY OF LAMINATED PLATES

For a thick or composite plate, the Mindlin theory must be used. It assumes that a straight segment perpendicular to the mean surface remains straight but not perpendicular to the mean surface after deformation. This hypothesis makes it possible to take into account the transverse shear deformations. On the mean surface of a plate, the *x* and *y*-axes are established in the surface and the *z*-axis perpendicular to the surface, the Mindlin theory takes the following field of displacements:

$$\begin{cases} u_q = u + z\beta_x \\ v_q = v + z\beta_y \\ w_q = w \end{cases} \tag{1}$$

where u_q, v_q and w_q are the displacements of a point $q(x, y, z)$, u, v and w are the displacements of the point $p(x, y, 0)$ on the mean surface, β_x is the angle of rotation of the normal from z to x or the angle of rotation around y ($\beta_x=\theta_y$), β_y is the angle of rotation of the normal from z to y or the angle of rotation around $-x$ ($\beta_y=-\theta_x$).

Thus, the following deformation field is obtained:

$$\begin{aligned} \varepsilon_x &= u_q,_x = u,_x + z\beta_x,_x \\ \varepsilon_y &= v_q,_y = v,_y + z\beta_y,_y \\ \gamma_{xy} &= 2\varepsilon_{xy} = u_q,_y + v_q,_x = u,_y + v,_x + z(\beta_x,_y + \beta_y,_x) \\ \gamma_{xz} &= 2\varepsilon_{xz} = u_q,_z + w_q,_x = w,_x + \beta_x \\ \gamma_{yz} &= 2\varepsilon_{yz} = v_q,_z + w_q,_y = w,_y + \beta_y \\ \varepsilon_z &= w_q,_z = 0 \end{aligned} \tag{2}$$

where the first three expressions are the plane deformations and the 4^{th} and 5^{th} expressions are the transverse shear deformations. The plane deformations can be decomposed into membrane parts and bending:

$$\{\varepsilon\} = \{\varepsilon_m\} + z\{\kappa\} \tag{3}$$

where $\{\kappa\}$ is the vector of curvatures.

The five constraints are defined by the following behavior laws:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = [Q] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{\nu_{yx}E_x}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{with } \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\{\sigma_\gamma\} = \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = [C] \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} G_{xz} & 0 \\ 0 & G_{yz} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{5}$$

The membrane forces, the bending and torsion moments and the transverse shear forces are obtained by integrating the stresses through the thickness:

$$\{N(x,y)\} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} dz \tag{6}$$

$$\{M(x,y)\} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} dz \tag{7}$$

$$\{T(x,y)\} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} dz \tag{8}$$

If we consider a composite panel consisting of several layers, the resulting forces defined above may be combined in layers:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}_m + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right) dz \tag{9}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} z dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}_k \left(z \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}_m + z^2 \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right) dz \tag{10}$$

$$\begin{Bmatrix} T_x \\ T_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}_k \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} dz \tag{11}$$

After the integration along the thickness, we obtain the overall stiffness matrix that links the generalized deformations with resultant forces:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ T_x \\ T_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & B_{33} & 0 & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & 0 & 0 \\ 0 & 0 & B_{33} & 0 & 0 & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xm} \\ \varepsilon_{ym} \\ \gamma_{xym} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{12}$$

in which

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n}\left[h^k - h^{k-1}\right] Q_{ij}^k = \sum_{k=1}^{n} Q_{ij}^k t^k \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n}\left[\left(h^k\right)^2 - \left(h^{k-1}\right)^2\right] Q_{ij}^k = \sum_{k=1}^{n} Q_{ij}^k t^k z^k \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n}\left[\left(h^k\right)^3 - \left(h^{k-1}\right)^3\right] Q_{ij}^k = \sum_{k=1}^{n} Q_{ij}^k \left[t^k\left(z^k\right)^2 + \frac{\left(t^k\right)^3}{12}\right] \\ F_{ij} &= \sum_{k=1}^{n}\left[h^k - h^{k-1}\right] C_{ij}^k = \sum_{k=1}^{n} C_{ij}^k t^k \end{aligned} \tag{13}$$

The law of behavior above can be written in matrix form:

$$\begin{bmatrix} \{N\} \\ \{M\} \\ \{T\} \end{bmatrix} = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [F] \end{bmatrix} \begin{bmatrix} \{\varepsilon_m\} \\ \{\kappa\} \\ \{\gamma_s\} \end{bmatrix} \tag{14}$$

where $\{N\}$, $\{T\}$ and $\{M\}$ are the internal forces and moments; $[A]$, $[D]$, $[B]$ and $[F]$ are the stiffness matrices related to the membrane forces, the bending-torsion moments, the bending-torsion-membrane coupling effects and the transverse shear forces respectively; $\{\varepsilon_m\}$ is the membrane strain vector, $\{\kappa\}$ is the curvature vector and $\{\gamma_s\}$ is the transverse shear strain vector.

III. HOMOGENIZATION MODEL FOR THE DOUBLE CORRUGATED CARDBOARD

The corrugated cardboard panels are made of multilayer including face-sheets and cores that have fluting cores and the cavities between layers. To apply the calculation method using the theory of plates, the matrix (9) obtained by the theory of laminated plates should be modified [5, 6]. Considering a double corrugated cardboard and using *a*, *b*, *c*, *d*, and *e* to represent the lower liner, the lower flute, the intermediate liner, the upper flute and the upper liner (Fig. 2). The geometry of each flute is defined by the following equations:

$$\begin{cases} H^b(x) = \dfrac{h^b - t^b}{2}\sin\left(\dfrac{2\pi}{P^b}x\right) \\ \theta^b(x) = \tan^{-1}\left(\dfrac{dH^b(x)}{dx}\right) \end{cases} ; \quad \begin{cases} H^d(x) = \dfrac{h^d - t^d}{2}\sin\left(\dfrac{2\pi}{P^d}x\right) \\ \theta^d(x) = \tan^{-1}\left(\dfrac{dH^d(x)}{dx}\right) \end{cases} \tag{15}$$

To homogenize a panel corrugated double wall, we consider a representative volume element (RVE). This volume must be sufficiently small relative to the dimensions of the entire panel. Once the overall stiffness of each slice are obtained by integrating the thickness, homogenization along *x* is performed to calculate the average stiffness of all tranches over a period:

$$[A]=\frac{1}{P}\int_0^P[A(x)]\,dx \quad ; \quad [B]=\frac{1}{P}\int_0^P[B(x)]\,dx$$
$$[D]=\frac{1}{P}\int_0^P[D(x)]\,dx \quad ; \quad [F]=\frac{1}{P}\int_0^P[F(x)]\,dx \qquad (16)$$

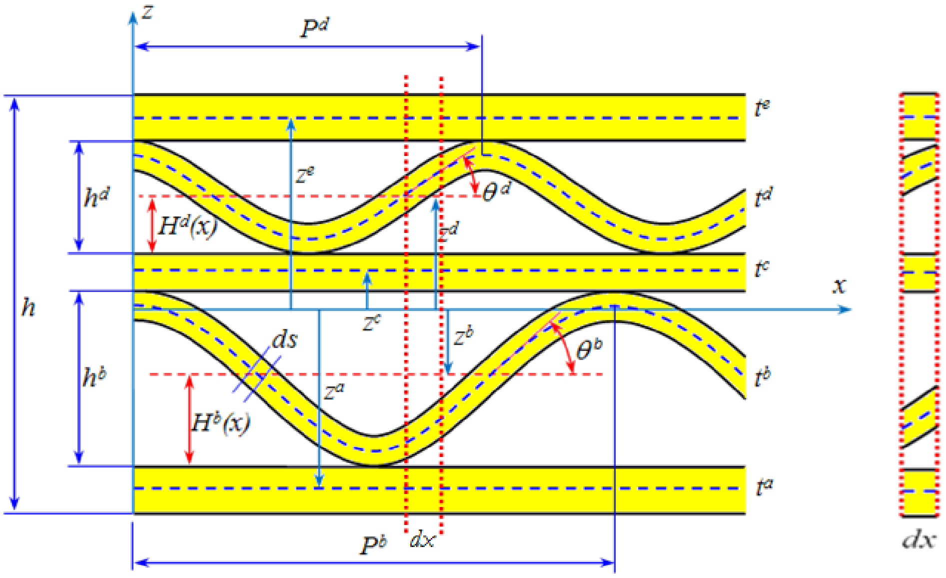

Fig. 2. Geometry of a double corrugated cardboard

Noting that the mechanical properties of the core achieved by experiments are valid only in its face, so we need to calculate the local coordinate system. Once the overall stiffness of each slice is obtained by integrating over the thickness of the plate, homogenization along x will be performed to calculate the average stiffness of all slices in one period [5].

3.1 Traction and bending stiffnesses related to N_x, M_x, N_y, M_y

The vertical position (z) of a groove portion (ds) is a function of x and a thickness over its vertical section is a function of the angle of inclination of the groove. Eq. (8) can be write:

$$A_{ij}=Q_{ij}^a t^a+Q_{ij}^b\frac{t^b}{\cos\theta^b}+Q_{ij}^c t^c+Q_{ij}^d\frac{t^d}{\cos\theta^d}+Q_{ij}^e t^e$$
$$B_{ij}=Q_{ij}^a t^a z^a+Q_{ij}^b\frac{t^b}{\cos\theta^b}z^b+Q_{ij}^c t^c z^c+Q_{ij}^d\frac{t^d}{\cos\theta^d}z^d+Q_{ij}^e t^e z^e$$
$$D_{ij}=Q_{ij}^a\left[t^a\left(z^a\right)^2+\frac{1}{12}\left(t^a\right)^3\right]+Q_{ij}^b\left[\frac{t^b}{\cos\theta^b}\left(z^b\right)^2+\frac{1}{12}\left(\frac{t^b}{\cos\theta^b}\right)^3\right]+ \qquad (17)$$
$$+Q_{ij}^c\left[t^c\left(z^c\right)^2+\frac{1}{12}\left(t^c\right)^3\right]+Q_{ij}^d\left[\frac{t^d}{\cos\theta^d}\left(z^d\right)^2+\frac{1}{12}\left(\frac{t^d}{\cos\theta^d}\right)^3\right]+Q_{ij}^e\left[t^e\left(z^e\right)^2+\frac{1}{12}\left(t^e\right)^3\right]$$

where

$$h=t^a+h^b+t^c+h^d+t^e$$
$$z^a=-\frac{h}{2}+\frac{t^a}{2} \quad ; \quad z^e=\frac{h}{2}-\frac{t^e}{2} \quad ; \quad z^c=-\frac{h}{2}+t^a+h^b+\frac{t^c}{2}$$

$$z^b(x) = -\frac{h}{2} + t^a + \frac{h^b}{2} + \frac{1}{2}\left(h^b - t^b\right)\sin\left(\frac{2\pi}{P^b}x\right); \quad \frac{dz^b}{dx} = \frac{\pi\left(h^b - t^b\right)}{P^b}\cos\left(\frac{2\pi}{P^b}x\right); \quad \theta^b(x) = \tan^{-1}\left(\frac{dz^b}{dx}\right)$$

$$z^d(x) = \frac{h}{2} - t^e - \frac{h^d}{2} + \frac{1}{2}\left(h^d - t^d\right)\sin\left(\frac{2\pi}{P^d}x\right); \quad \frac{dz^d}{dx} = \frac{\pi\left(h^d - t^d\right)}{P^d}\cos\left(\frac{2\pi}{P^d}x\right); \quad \theta^d(x) = \tan^{-1}\left(\frac{dz^d}{dx}\right)$$

For each of the two grooves, a homogenization on their period (along *x*) should be calculated numerically according to equation (15).

3.2 Transverse shear stiffnesses related to T_y

In laminate theory, the shear stiffness relative to the shear force T_y on a CD section is calculated by the sum of the layers. However, the CD section of the sandwich is not a continuous medium and the transverse shear deformation is not constant or linear on the section, so the theory of laminates is no longer valid. The shear force T_y on a CD section causes a coupling between bending and transverse shear. Thus, it is very difficult to directly determine the transverse shear stiffness on a CD section relating to T_y.

To avoid the coupling between bending and transverse shear, and to obtain "pure" shear, according to the reciprocity theorem, Nordstrand et al. [3] have proposed a horizontal shear model in which transverse shear under the force T_y (force along *z* and per unit length along *x*) is replaced by shear over the thickness under the force *T* (along *y*) (Fig. 3). The shear modulus thus obtained is equivalent to the transverse shear modulus.

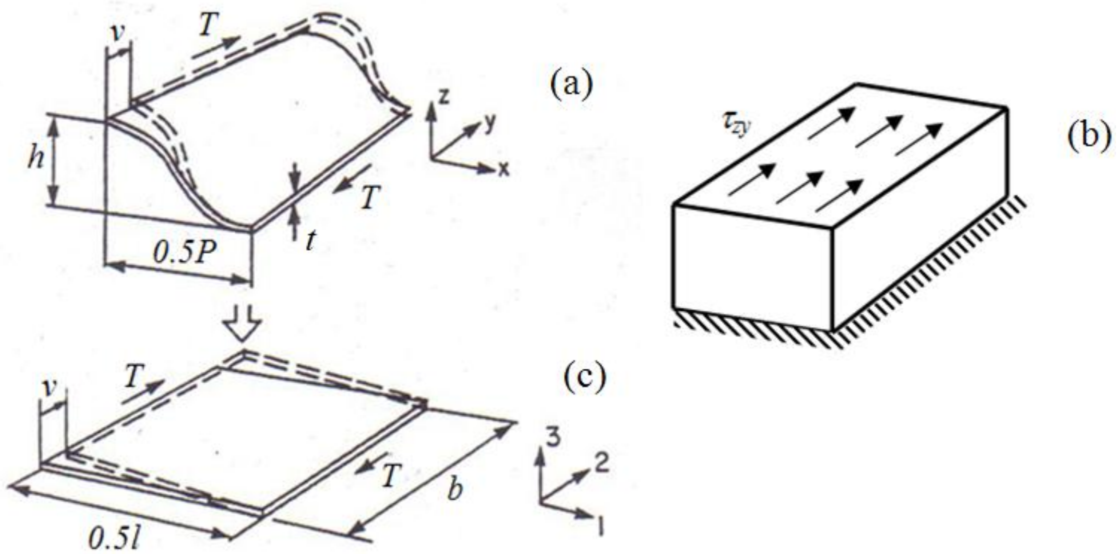

Fig. 3. The equivalent model to determine shear stiffness on CD section

The deformations due to the shearing of the flat skins are much less than those due to the shearing of the groove, and therefore negligible. We make equivalence between one-half period of the corrugated core sandwich (with both skins not shown) (Fig. 3a) and a solid with a dimension of $P/2 \times b \times h$ (Fig. 3b). A pair of shear forces *T* exerted on the groove by the upper and lower faces gives the sliding *v*. The shear of the homogeneous solid can be defined by:

$$\tau_{zy} = \frac{T}{0.5Pb} = G_{zy}\frac{v}{h} \tag{18}$$

The shear stress in the 3D corrugated core (Fig. 3a) is equivalent to the shear in the flattened groove (Fig. 3c). This gives us:

$$\tau_{12} = G_{12}\gamma_{12} \quad \Rightarrow \quad \frac{T}{bt} = G_{12}\frac{v}{0.5l} \quad \Rightarrow \quad v = \frac{0.5Tl}{G_{12}bt} \tag{19}$$

Substituting Eq. (19) in (18), we obtain the shear modulus of the solid:

$$G_{zy}^* = G_{12}\frac{4ht}{Pl} \tag{20}$$

In the case of a double corrugated core cardboard, we have:

$$G_{zy}^{b*} = G_{12}^{b} \frac{4e^b h^b}{P^b l^b} \quad ; \quad G_{zy}^{d*} = G_{12}^{d} \frac{4e^d h^d}{P^d l^d} \tag{21}$$

for the lower and upper groove respectively.

If we replace the lower groove between the faces t^a and t^c by a homogeneous solid of thickness h^b (the same for the upper groove), we obtain:

$$\tau_{zy} = G_{zy}^{b*}\gamma^b = G_{zy}^{b*}\frac{v^b}{h^b} \quad \Rightarrow \quad v^b = \frac{\tau_{zy}h^b}{G_{zy}^{b*}} \quad idem \quad v^d = \frac{\tau_{zy}h^d}{G_{zy}^{d*}} \tag{22}$$

The effect of shear on the thicknesses of three faces is very low and easy to calculate:

$$\tau_{zy} = G_{zy}^{a}.\gamma_{zy}^{a} = G_{zy}^{a}\frac{v^a}{e^a} \quad \Rightarrow v^a = \frac{\tau_{zy}e^a}{G_{zy}^{a}} \quad idem \quad v^c = \frac{\tau_{zy}e^c}{G_{zy}^{c}} \quad and \quad v^e = \frac{\tau_{zy}e^e}{G_{zy}^{e}} \tag{23}$$

However, the effect of shear in the plane of the intermediate face t^c is significant (5.4% for the considered interlayer). This effect can be modeled as shown in Fig. 4 where $F = \tau_{zy}.1.P^d$ is the force exerted over a period of the face t^c by one vertex (or two) of the upper groove. We can define the internal strain energy and calculate the following displacement along F as follows:

$$U = \frac{1}{2}\int_0^{\gamma P^d} \frac{\left[(1-\gamma)\tau_{zy}P^d\right]^2}{G_{xy}^{c}S}ds + \frac{1}{2}\int_{\gamma P^d}^{P^d} \frac{\left[\gamma\tau_{zy}P^d\right]^2}{G_{xy}^{c}S}ds \tag{24}$$

$$v^\gamma = \frac{\partial U}{\partial\left(\tau_{zy}P^d\right)} = \frac{(1-\gamma)^2\tau_{zy}P^d}{G_{xy}^{c}.e^c.1}\gamma P^d + \frac{\gamma^2\tau_{zy}P^d}{G_{xy}^{c}.e^c.1}(1-\gamma)P^d$$

$$\Rightarrow v^\gamma = \frac{\tau_{zy}\left(P^d\right)^2}{G_{xy}^{c}e^c}\left[\gamma - 2\gamma^2 + \gamma^3 + \gamma^2 - \gamma^3\right] = \frac{\tau_{zy}\left(P^d\right)^2}{G_{xy}^{c}e^c}\gamma(1-\gamma) \tag{25}$$

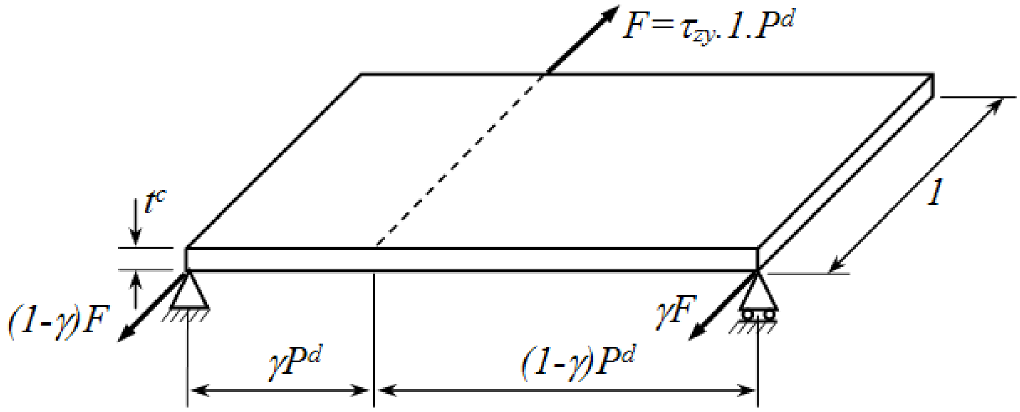

Fig. 4. Shearing in the plane of the groove t^c

It should be noted that: 1) It is necessary to take the greatest period between P^b and P^d; 2) If $P^b \neq P^d$, on max(P^b, P^d), we can have 1, 2 or more forces; 3) If $P^b = n.P^d$ (n = 2,3, ...), we have n forces on P^d in different positions, we can also calculate v^γ; 4) We will interpolate if P^b is not an integral multiple of P^d. The shear stiffness on the total thickness of the cardboard is obtained as follows:

$$\tau_{zy} = G_{zy}^{*}\gamma_{zy} = G_{zy}^{*}\frac{v^a + v^b + v^c + v^d + v^e + v^\gamma}{t^a + h^b + t^c + h^d + t^e} \tag{26}$$

By substituting the equations (22, 23, 25) in (26), the shear stiffness is obtained over the total thickness of the carton:

$$G_{zy}^{*} = \frac{h}{\frac{t^a}{G_{zy}^{a}} + \frac{h^b}{G^{b*}} + \frac{t^c}{G_{zy}^{c}} + \frac{h^d}{G^{d*}} + \frac{t^e}{G_{zy}^{e}} + \frac{\gamma(1-\gamma)\left(P^d\right)^2}{G_{xy}^{c}t^c}} \tag{27}$$

Finally, transverse shear stiffness is obtained on CD section:

$$F_{22} = G^*_{zy}.h = \frac{h^2}{\frac{t^a}{G^a_{zy}} + \frac{h^b}{G^{b*}} + \frac{t^c}{G^c_{zy}} + \frac{h^d}{G^{d*}} + \frac{t^e}{G^e_{zy}} + \frac{\gamma(1-\gamma)(P^d)^2}{G^c_{xy}t^c}} \quad (28)$$

3.3 Transverse shear stiffnesses related to T_x

In laminate theory, the transverse shear stiffness relative to the shear force T_x on MD section is also calculated by the sum of the layers. However, it is also difficult to determine this stiffness because of the coupling between the bending and transverse shear. Nordstrand et al. [3] proposed to replace the transverse shear under the force T_x (on MD section and along *z*) by shearing on the thickness under the force $T = T_x$ along *x*. In fact, this problem is not really a problem of shear of the five layers; it is dominated by the flexion of three plane faces and especially by the flexion of the two grooves. The shear stiffness can also be determined numerically, the equivalent model (for single or double groove) is shown in Fig. 5. The horizontal shear modulus (equal to that of transverse shear) for the homogenized plate is defined as follows:

$$G^*_{zx} = \frac{\sigma_{zx}}{\gamma_{zx}} = \frac{F / bL}{u / h} = \frac{F}{u}\frac{h}{bL} \quad (29)$$

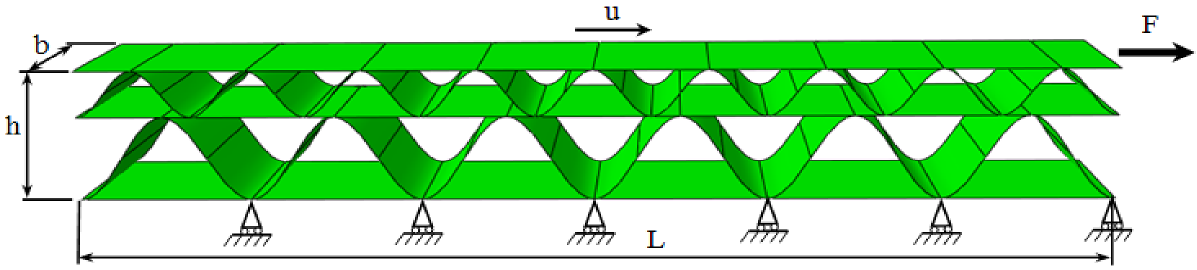

Fig. 5. The equivalent model to determine shear stiffness on MD section

The problem is to determine the *F/u* ratio analytically or numerically. In order to eliminate the edge effect, we take a period in the middle of the length of the carton to define *ΔF/Δu* instead of *F/u*. Finally, the transverse shear stiffness of the corrugated board is obtained:

$$F_{11} = G^*_{zx}h = \frac{\Delta F}{\Delta u}\frac{h^2}{bP} \quad (30)$$

If we assume that a double corrugated cardboard is the superposition of two single corrugated cardboard, then the shear moduli of these two cardboards G^{b*}_{zx} and G^{d*}_{zx} can be analytically calculated by the analytical formulas of Nordstrand et al. [3]. It should be noted that a small error is introduced by the use of the flat layer in the middle for both upper and lower parts. Then, equations (29) and (30) give us:

$$\begin{aligned} G^{b*}_{zx} &= \frac{\sigma_{zx}}{\gamma^b_{zx}} = \frac{\sigma_{zx}}{\Delta u^b / h^b} \quad ; \quad G^{d*}_{zx} = \frac{\sigma_{zx}}{\gamma^d_{zx}} = \frac{\sigma_{zx}}{\Delta u^d / h^d} \\ G^*_{zx} &= \frac{\sigma_{zx}}{\gamma_{zx}} = \frac{\sigma_{zx}}{\Delta u \ / h} \quad ; \quad \Delta u = \Delta u^b + \Delta u^d \quad ; \quad h = h^b + h^d \end{aligned} \quad (31)$$

These five equations allow us to analytically obtain the transverse shear stiffness on the MD section (relative to T_x) for a double corrugated cardboard:

$$F_{11} = G^*_{zx}h = h^2\left(\frac{h^b}{G^{b*}_{zx}} + \frac{h^d}{G^{d*}_{zx}}\right)^{-1} \quad (32)$$

IV. VALIDATION OF HOMOGENIZATION MODEL

To validate our H-model, we first discretize the five layers of corrugated cardboard by shell elements S4R of Abaqus to obtain the model Abaqus-3D; Then, we discretize the middle surface of corrugated cardboard by shell elements S4R of Abaqus combined with our H-model (using "user's subroutine UGENS") to obtain H-2D model. The confrontation of the results allow us to evaluate the efficiency and accuracy of our homogenization model.

The calculations and comparisons are made on a corrugated panel having CD section illustrated in Fig. 6. Geometric data are: period (or step) and height of the lower groove P^b = 9 mm and h^b = 5.2 mm, those of the upper groove P^d = 6 mm and h^d = 2.9 mm, thicknesses $t^a = t^c = t^e$ = 0.25 mm, $t^b = t^d$ = 0.26 mm. The properties of materials are given in Table 1 [12]. The rigidities of 2D equivalent plate are calculated as shown in Table 2.

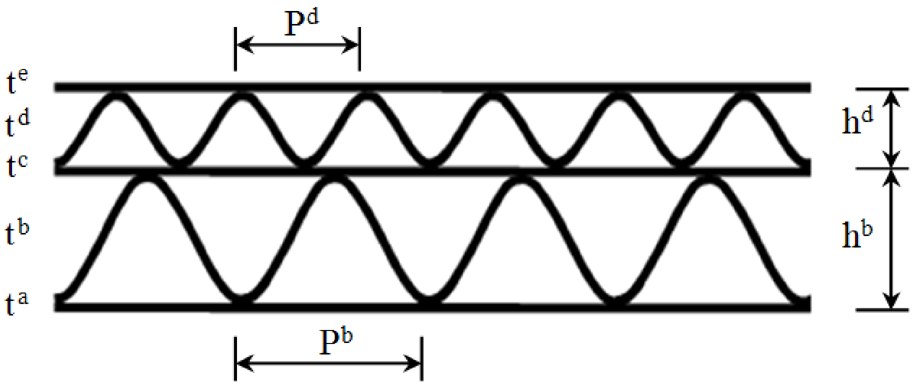

Fig. 6. Geometry of the CD section of double corrugated cardboard

Table 1. The material properties of five layers formed corrugated cardboard plate

Layers	E_1 *(MPa)*	E_2 *(MPa)*	E_3 *(MPa)*	G_{12} *(MPa)*	G_{13} *(MPa)*	G_{23} *(MPa)*	ν_{12}	ν_{13}	ν_{23}
a	8250	2900	2900	1890	7	70	0.43	0.01	0.01
b, d	4500	4500	3000	1500	3.5	35	0.40	0.01	0.01
c, e	8180	3120	3120	1950	7	70	0.43	0.01	0.01

We use a corrugated panel having length L = 162mm and width B = 162mm. For the simulation of the homogenized plate using our H-2D model, the middle surface is discretized into 2916 S4R quadrilateral elements and 3025 nodes. However, for the Abaqus-3D simulation, 52116 S4R quadrilateral elements and 48293 nodes are needed. Indeed, to fully describe the geometry of the groove, it takes at least 16 elements over a period of groove.

Table 2. Rigidities of the 2D equivalent plate

Rigidities	A_{11} (N/mm)	A_{12} (N/mm)	A_{22} (N/mm)	B_{11} (N)	B_{12} (N)	B_{22} (N)
Values	6606.2	1055.1	5989.8	2507.1	526.1	2914.5
Rigidities	D_{11} (N.mm)	D_{12} (N.mm)	D_{22} (N.mm)	F_{11} (N/mm)	F_{22} (N/mm)	
Values	75214.1	11870.5	49672.4	8.5	284.5	

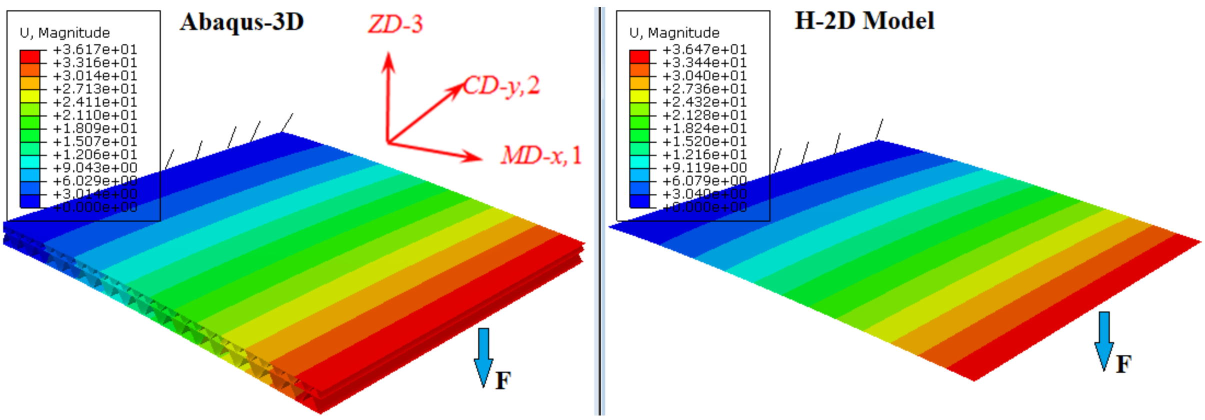

Fig. 7 Simulation of Abaqus-3D and H-2D Model under transverse loading on MD section

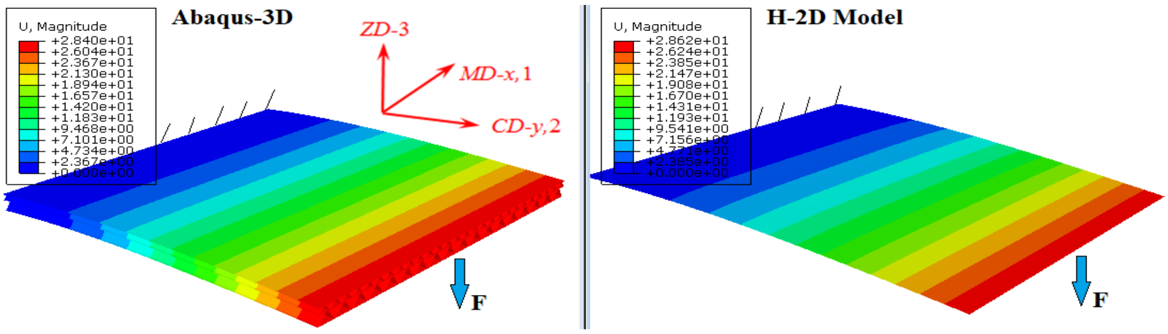

Fig. 8 Simulation of Abaqus-3D and H-2D Model under transverse loading on CD section

Table 3. Comparison between Abaqus-3D and H-2D model under transverse loading

Vertical force F_3 = 150N		Abaqus-3D	H-2D Model	Error (%)
MD	Displacement U_3 (mm)	36.17	36.47	0.83
	CPU time (s)	79.9	2.0	39.95 times
CD	Displacement U_3 (mm)	28.40	28.62	0.77
	CPU time (s)	80.2	2.2	36.45 times

We fix the plate at the left end and apply the vertical force (F = 150 N) at the right end. In both types of simulations (Abaqus-3D model and H-2D model), a rigid plate is bonded to the MD or CD section at the right end of the corrugated cardboard panel to better apply forces. The deformed shapes together with the iso-values of displacement of the panel under transverse loading obtained by 3D shell Abaqus and our H-2D model simulations are shown in Fig. 7. and Fig. 8. The comparisons of results obtained by the two models and the percentages of error in H-2D model compared to Abaqus-3D results for the case of transverse loading are presented in Table 3. We observe that the calculations by our H-2D model are very fast while calculations by Abaqus-3D are much longer (~40 times) and we note that the numerical results given by the two models are very close.

V. CONCLUSION

In this paper, we have proposed an analytic homogenization model for the bending and transverse shear problems of the double corrugated core sandwich panels. The comparison of the results obtained by the Abaqus 3D and the H-2D simulations have proved the validation of the present homogenization model for bending and transverse shear problems. The present H-2D model allows us to largely reduce not only the time for the geometry creation and FEM calculation, but also the computational hardware requirements for the large sandwich panels. This homogenization model can be used not only for corrugated cardboard plates, but also for naval and aeronautic composite structures.

REFERENCES

[1] Q.H. Cheng, H.P. Lee, C. Lu, A numerical analysis approach for evaluating elastic constants of sandwich structures with various cores, *Composite Structures*, 74, 2006, 226–236.
[2] Aboura Z, Talbi N, Allaoui S, Benzeggagh ML, Elastic behaviour of corrugated cardboard: experiments and modelling, *Composite Structure, 63,* 2004, 53–62.
[3] Nordstrand T, Carlsson LA, Allen HG, Transverse shear stiffness of structural core sandwich, *Composite Structure, 27*, 1994, 317–329.
[4] Noor AK, Burton WS, Bert CW, Computational models for sandwich panels and shells, *Applied Mechanics Reviews 155 (3)*, 1995, 155-199.
[5] Buannic N, Cartraud P, Quesnel T, Homogenization of corrugated core sandwich panels, *Composite Structure, 59*, 2003, 299-312.
[6] Talbi N, Batti A, Ayad R, Guo YQ, An analytical homogenization model for finite element modelling of corrugated cardboard, *Composite Structure,* 69, 2005, 322–328.
[7] Luo S. The bending stiffnesses of corrugated board. AMD-Vol. 145/MD-Vol. 36, *Mechanics of cellulosic materials. ASME*, 1992, 15–26.
[8] Carlsson LA, Nordstrand T, Westerlind B, On the elastic stiffness of corrugated core sandwich plate, *Journal of Sandwich Structures and Materials, 3,* 2001, 253–267.
[9] Nordstrand T, Analysis and testing of corrugated board panels into the post-buckling regime, *Composite Structure, 63*, 2004, 189–199.
[10] Biancolini, Evaluation of equivalent stiffness properties of corrugated board, *Composite Structure, 69*, 2005, 322–328.
[11] Triplett MH, Schonberg WP, Static and dynamic finite element analysis of honeycomb core sandwich structures, *Structural Engineering and Mechanics*, 6 (1), 1998, 95–113.
[12] Nordstrand T., On buckling loads for edge-loaded orthotropic plates including transverse shear, *Composite Structures*, 65, 2004, 1–6.

Duong Pham Tuong Minh. "Modeling and Numerical Simulation for the Double Corrugated Cardboard under Transverse Loading by Homogenization Method." The International Journal of Engineering and Science (IJES), vol. 6, no. 9, 2017, pp. 16–25.