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Home Energy Scheduling System: Load Planning with Production Constraints

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ABSTRACT

This paper addresses the home energy management issue. Energy consumption in buildings is one of the more challenging problems in the current energy context. Energy savings are possible by improving energy efficiency of devices and adapting end-users' demands. However, for given end-user requirements with given appliances, the schedule of the activities greatly impacts management of energy production means. In this paper the addressed home energy management issue consists in matching, as accurately as possible, the end-user demand with power grid capacity. A mixed integer linear programming formulation of the home energy scheduling problem is described after a complexity analysis of the optimization problem. Finally, some results point out the solving performances of the proposed formulation.

Keywords: *Dwelling; Home Energy Management System; Optimization; Scheduling; Mixed Integer Linear Programming.*

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I. INTRODUCTION

The energy issue is one of the major challenges of the 21st century. According to a median prediction (IEA2007), the increasing world population together with the economic growth of the emerging countries is likely to induce an increase in energy consumption of around 1.5 in 2030 compared to its current level. This demand is clearly incompatible with our present way of life with regard both to the limited worldwide fossil energy reserves and the environmental footprint. Consequently, development of alternative energy sources, increase in process efficiency and better management of energy systems through intensive use of new available technologies (networks, sensors and information systems) are the pillars for migration of energy systems to more sustainable systems. This paper proposes a contribution to the third pillar based on the spread of smart energy tools used in smart grids and smart homes.

Building related energy consumption accounts for a large part of the total energy bill. The percentage of residential electricity used by appliances and electronics in U.S. homes attains 31%. About 50% of load in houses is dedicated to refrigerators, freezers, heaters, washing machines and dryers [1]. Moreover, buildings are increasingly becoming very active nodes in the energy system due to the spread of local renewable energy production means such as solar systems and heat pumps. Buildings thus represent a major potential for optimizing energy use provided all the new degrees of freedom can be properly managed. Cooperation between technologies and infrastructure (smart homes and smart grid) may lead to additional benefits [2].

A smart grid can be defined as a power grid with communication means between devices, users and providers in order to provide services. One of the main attributes of a smart grid defined in [3] is optimal use of bulk power generation combined with distributed resources and controllable/dispatch-able loads to assure lowest cost. Intelligent energy meters are one of the first tools available to exchange information between energy providers (or service providers known as aggregators) and customers, and are the first step towards developing an energy management system. They provide signals designed to exchange information between providers and customers. Stop information is proposed in a direct control approach. The customer may refuse to stop an appliance, probably with some penalties. In the control by cost approach, electricity rates vary according to the time of the day. The customer decides whether or not he will modify his behavior taking energy prices into account.

However it is no means easy to decide on the best time to stop an appliance. Optimization systems need either to control appliances according to the customers' preferences or to provide a decision support system.

The work presented in this paper conducted in a cost context based on information exchanged between smart grid and smart home. In this paper the addressed Home Energy Scheduling Problem (HESP) aims at adjusting power consumption both according to user requests and to energy cost. Energy costs may consist of both the price of the consumed energy and its CO2 equivalent ejection. During peak consumption periods, power plants emitting high quantities of CO2 are used to satisfy consumption needs. In such periods, energy is very expensive.

Let us now discuss the energy system described in this paper and the related available results for optimal control. A building energy management system consists of two aspects: load management and local energy production management. [4,5,6,7] propose optimal control strategies for HVAC (home ventilation and air conditioning) systems, taking into account the natural thermal storage capacity of buildings. HVAC consumption has shifted from peak-period to off-peak period (slack period). [7] shows that this control strategy can save up to 10% in the building electricity bill. The combined cooling, heating, and power (CCHP) system is studied to optimize energy consumption in [8,9]. However, these approaches do not take into account energy resource constraints, which generally depend on the autonomy needs of off-grid systems [10] or on the power subscription of grid connected systems. In this case, power subscription defines the energy cost as well as the penalties should maximum power as defined in the contract be exceeded.

Generally speaking, studies in the literature focus on a particular aspect of the home energy management problem: the displays design, load control or local energy production. The joint load and production management problem is usually addressed in the literature using a global load curve to be supplied. This problem is denoted by demand side management (DSM). Few works address the issue of communication between homes and grid [2] or the derived management of activities inside the house from an energy point of view.

The goal of the study addressed in this paper is to set up a general mathematical formulation making it possible to design optimized building electrical energy management systems able to determine the best possible energy assignment plan according to given criteria. This paper focuses on the planning layer of the home energy management system. In practice this anticipative layer provides set points for a reactive layer that adjusts set points to actual productions and consumptions. The home energy management problem depicted in this paper is one of the layers of a global control architecture defined in [11].

The HESP is an application field of the generic Energy Scheduling Problem (EnSP) defined in [12]. The available electrical power at each time is the resource shared by the appliances. The tasks are the activities requested by the user, and consume the supplied power in given time windows. The HESP is demonstrated as a NP-Hard problem. A mixed integer linear program is proposed and compared with the RCPSP and the cumulative scheduling problem. The solving properties of the MILP formulation are discussed.

The basic HESP is defined in section 2 and the HESP intractability is shown. Its mathematical formulation as an MILP is depicted in section 3. An extension of the basic problem concerning the compromise between energy cost and the user's comfort is introduced in section 4 and an aggregation approach to the multi-criteria problem is given. Section 5 is devoted to experimental results..

II. THE HOME ENERGY SCHEDULING PROBLEM (HESP)

The HESP takes as its input a set of activities known as services. A distinction can be made between the power supply service providing the energy and the end-user services to be processed without interruption using the energy. The end-user services directly satisfy a request made by the users by consuming energy. Power supply services can derive power by means of various primary energies such as fuel cells based generators, photovoltaic power suppliers and grid power suppliers. End-user services include such well-known services as clothes washing, water heating, room heating, cooking and lighting. Storage of electrical power is not addressed in this paper. Energy is a resource whose availability and price are not constant. The optimization problem aims at scheduling activities by minimizing the cost of energy consumption over a given planning horizon. The availability of the resource, i.e. the available power, is used in the optimization problem to smooth the proposed planning. This limited total amount of power can correspond to the power subscription, in which case the associated constraint will not present many restrictions in the optimization problem. Otherwise it can correspond to a virtual value. Activities that can be predicted or required by users are the only one that can provide an anticipative plan. In the basic statement of the problem, energy cost is the only assumed optimization criteria. In section 3 users' satisfactions is addressed in a multi-criteria version of the basic problem. A service is identified by its index i.

2.1 Characterization of the power supply service
Two parameters characterize a power supply service *i*, they are input data of the optimization problem:
- $P(i,t)$ available power at time t
- $C(i,t)$ price of the electrical resource at time t

A power supply service *i* stands for the available power over the planning horizon and the associated cost at each time for a given production means. Several production means can be involved at home. Allocation of available energy to end-user services is the only problem addressed. Management of the total amount of energy is not addressed.

2.2 Characterization of end-user services
Three types of end-user services can be identified: *shift-able services*, *modulable services* and *unsupervised services*. Another segmentation is proposed in [13] based on the level of automation and the number of start-ups in the appliances. Appliances with a low automation level correspond to an aggregated unsupervised service, while appliances with a high automation level continuously providing services correspond to modulable services. Appliances with high automation level with discrete start-ups correspond to shift-able services.

2.2.1 Shift-able services
A shift-able service depicts an activity that is required at some time and execution of which has a given duration that is assumed to be shorter than the planning horizon. Typically, the act of washing dishes is a shift-able service. A shift-able service *i* is characterized by the following input data:
- *$P(i)$, required power in execution [W]*
- *$ef(i)$, $lf(i)$, the earliest and latest requested end times respectively [s]*
- *$d(i)$, the execution time [s]*

The end time $f(i)$ is the decision variable associated with the shift-able service. Preemption is not available.
The earliest and latest requested end times are given by the user or predicted from the user's behavior. Such prediction could be provided by a statistical tool from measurements [14]. Shift-able services are associated with appliances that could be automated and without a large number of start-ups, typically a few times a day.

2.2.2 Modulable services
Modulable services depict services that are potentially continuously delivered throughout the planning horizon. Typically, room heating and refrigerating services are modulable services. Let us assume a modulable service *i*. It is characterized by:
- $P(i)$, required power in execution [W]
- $T_{min}(i,t)$, $T_{max}(i,t)$ the minimum and maximum satisfactory controlled parameters, respectively, at time t [°C]

The set point $T_{in}(i,t)$ is the decision variable associated with the modulable service *i*. This modulable set point corresponds to a variable amount of energy allocated to the modulable service as depicted in section 3. For example the user requests a temperature in his room within the satisfactory interval [18,20]°C. The optimization problem aims at setting the best temperature each time to minimize energy cost complying with the variable values defined by intervals. The thermal storage ability of the building is then used to attain this optimum value. In this paper only one type of modulable service is addressed. This type is defined by a dynamic thermal model depicted in section 3.3.

2.2.3 Unsupervised service
From a practical point of view, not all activities in housing and the derived energy consumption can be considered as services to be scheduled. Lighting is one of the best examples of unsupervised service, as switching on a light is an act that is totally dependent on the inhabitant's presence in a room, a parameter that is neither controllable nor predictable. In this case, there is no point in precisely scheduling activities that cannot be controlled and/or predicted. These activities are merged into one unsupervised service as otherwise we would have a large number of shift-able services considerably increasing the optimization problem without any benefit to the computed schedule. The unsupervised service is defined by the power $P_u(t)$ consumed at each time t given as data for the optimization problem. Section 3.4 will show that the unsupervised service is a given consumption that reduces the power available for the modulable services and the shift-able services. Thanks to the global control architecture (anticipative + reactive) in which the optimization process takes place, there is no need for very accurate forecasting of $P_u(t)$.

2.3 HESP complexity
In this section the HESP restricted to the shift-able services will be demonstrated to be NP-Complete due to the time windows in which the tasks have to be executed. Let us use a reduction of HESP for the partition problem. The partition problem defined as follows is known as NP-complete [15]:

Instance: n integer number a_1, a_2,…,a_n and $B = \sum_{i=1}^{n} \frac{a_i}{2}$

Question QP: does a set A such that $\sum_{i \in A} a_i = B$ exist?

Let us consider the following instance of HESP:
Instance: one power supply service with a constant available power P, n shift-able services $i \in \{1,...,n\}$ such that $P(i,t) = P$ (so HESP becomes a set of tasks to be successively scheduled on one resource), a_i the processing time of i, $[0,2B+1]$ its required time window (it is easy to compute $0=ef(i)-d(i)$ and $2B+1=lf(i)$) and one shift-able service $n+1$ such that $P(n+1,t)=P$, processing time equals 1 and the required time window is $[B,B+1]$.
Question QHESP: does a schedule of the tasks with total duration $(2B+1)$ exist?
Proposal: Questions *QP* and *QHESP* have the same answer.
Proof:
- If the instance of partition is a yes-instance, then schedule the tasks of set A first in any order, then schedule task $n+1$ followed by the remaining tasks achieving a feasible schedule of length $2B+1$.

If the scheduling problem is feasible, then task $n+1$ is scheduled at time $[B,B+1]$ and uses the entire resource. Therefore it partitions the set of tasks in 2 subsets. The tasks before time B, whose total processing times amount to B, define the set A of partition problem.

III. HESP MODEL
The first paragraph under each heading or subheading should be flush left, and subsequent paragraphs should have a five-space indentation. A colon is inserted before an equation is presented, but there is no punctuation following the equation. All equations are numbered and referred to in the text solely by a number enclosed in a round bracket (i.e., (3) reads as "equation 3"). Ensure that any miscellaneous numbering system you use in your paper cannot be confused with a reference [4] or an equation (3) designation.
Let $H = \{1,2,…,T\}$ be the planning horizon consisting of T time periods with a length Δ. At every planning period k, the amount of energy allocated to every service has to be decided. For shift-able services this decision is a scheduling decision. When the end time of the service is decided, then the associated consumed energy can be computed at each planning period. For modulable services the amount of allocated energy at each period is the decision variable. Energy costs and resource availability are assumed to be constant over a length Δ of a planning period. In this case, in the approach developed in this paper Δ is a data item given by the variation of the resource. Duration of the shift-able services and length Δ are independent. In the formulation of the energy management problem proposed in [12,16], execution of the services are synchronized with the planning period. In this paper we propose an MILP formulation of the optimization problem.

3.1. Power supply service model
Based on the problem description of a power supply service i the following constraint can be written:

$$E(i,k) \le P(i,k)\Delta \quad \forall k \in \{1,...,T\} \tag{1}$$

where
- $P(i,k)$ stands for the maximum available power [W]
- $E(i,k)$ the energy supplied during the time window $[k\Delta,(k+1)\Delta]$ [Wh]

This constraint aims at converting available power into a maximum amount of energy per planning period.

3.2. Shift-able services model
Let us recall that $d(i)$ and $P(i)$ denote, respectively, the duration and the power consumption related to i the shift-able service, and $f(i)$ the end time to be scheduled with respect to the user's request given by the time window $[ef(i),lf(i)]$. Firstly, the scheduled end time $f(i)$ is constrained in a time window:

$$ef(i) \le f(i) \le lf(i) \tag{2}$$

According to [17] and energy reasoning, the consumption duration $d(i,k)$ of a service i during a planning period k is given by (see figure 1):

$$d(i,k) = \max\{0, \min[f(i),(k+1)\Delta] - max[f(i) - d(i), k\Delta]\} \tag{3}$$

Therefore, the consumption energy $E(i,k)$ of service i during a planning period k is given by:

$$E(i,k) = d(i,k)P(i) \tag{4}$$

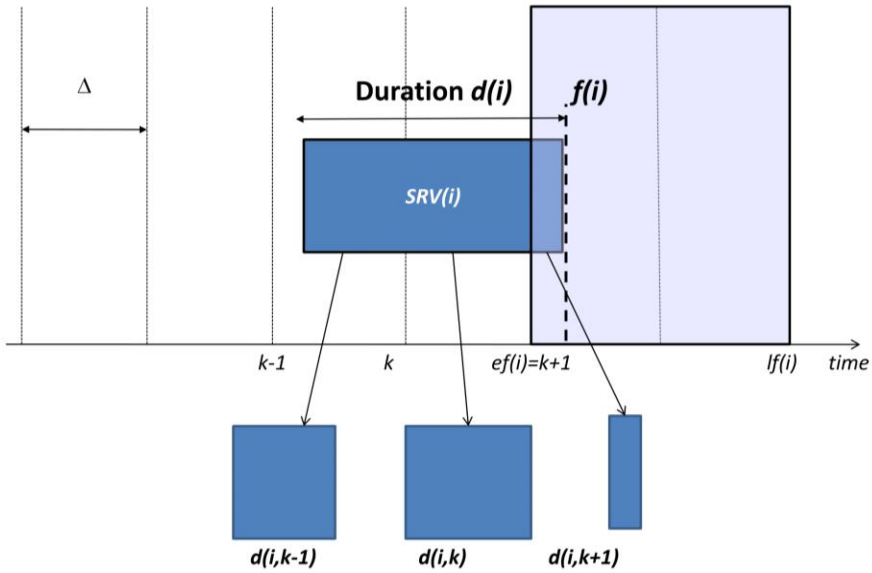

Figure 1. Scheduling of shift-able services

However, the model contains nonlinear min and max functions in the expression of $d(i,k)$. Let us now introduce the binary variables $\delta_1(i,k)$ and $\delta_2(i,k)$ defined by:

$$\begin{aligned} &\delta_1(i,k) = 1 \text{ if and only if } (f(i) - k\Delta \le 0) \\ &\delta_2(i,k) = 1 \text{ if and only if } (f(i) - d(i) - k\Delta \le 0) \end{aligned} \tag{5}$$

The following linear constraints can be written to assign the binary variables:

$$\begin{aligned} &f(i) - k\Delta \le [lf(i) - k\Delta][1 - \delta_1(i,k)] \\ &f(i) - k\Delta \ge \Delta + [ef(i) - k\Delta - \Delta]\delta_1(i,k) \\ &f(i) - d(i) - k\Delta \le [lf(i) - d(i) - k\Delta][1 - \delta_2(i,k)] \\ &f(i) - d(i) - k\Delta \ge \Delta + [ef(i) - d(i) - k\Delta - \Delta]\delta_2(i,k) \end{aligned} \tag{6}$$

where ε is a given small number. Therefore, min and max functions in the definition of $d(i,k)$ become:

$$\begin{aligned} &\min\{f(i),(k+1)\Delta\} = [1 - \delta_1(i,k+1)](k+1)\Delta + \delta_1(i,k+1)f(i) \\ &\max\{f(i) - d(i),\ k\Delta\} = [1 - \delta_2(i,k)][f(i) - d(i)] + \delta_2(i,k).k\Delta \end{aligned}$$

Two variables $z_1(i,k)$ and $z_2(i,k)$ have to be introduced to obtain the linear formulation of the min and max functions. The following set of constraints is used to define these variables:

$$\begin{aligned}
&z_1(i,k) \le lf(i)\delta_1(i,k)\\
&z_1(i,k) \ge ef(i)\delta_1(i,k)\\
&z_1(i,k) \le f(i) - ef(i)\left[1-\delta_1(i,k)\right]\\
&z_1(i,k) \ge f(i) - lf(i)\left[1-\delta_1(i,k)\right]\\
&z_2(i,k) \le lf(i)\delta_2(i,k)\\
&z_2(i,k) \ge ef(i)\delta_2(i,k)\\
&z_2(i,k) \le f(i) - ef(i)\left[1-\delta_2(i,k)\right]\\
&z_2(i,k) \ge f(i) - lf(i)\left[1-\delta_2(i,k)\right]
\end{aligned} \tag{7}$$

The duration $d(i,k)$ can be evaluated by $d(i,k) = \max\left\{0, d'(i,k)\right\}$
where

$$d'(i,k) = z_1(i,k+1) + z_2(i,k) - f(i) - (k+1)\Delta.\delta_1(i,k+1) - \delta_2(i,k)\left[d(i) - k\Delta\right] + (k+1)\Delta + d(i) \tag{8}$$

A linear formulation of the remaining maximum function in $d(i,k)$ is necessary. Let us introduce the binary variable $\delta_0(i,k)=1$ if and only if $d'(i,k)>0$, assigned by the following constraints:

$$\begin{aligned}
&d'(i,k) \le \Delta.\delta_0(i,k)\\
&d'(i,k) \ge \Delta - (T\Delta + \varepsilon)\left[1-\delta_0(i,k)\right]
\end{aligned} \tag{9}$$

Then the following set of constraints must be added to assign $d(i,k)$:

$$\begin{aligned}
&d(i,k) \le \Delta\delta_0(i,k)\\
&d(i,k) \ge 0\\
&d(i,k) \le d'(i,k) + T\Delta\left[1-\delta_0(i,k)\right]\\
&d(i,k) \ge d'(i,k)
\end{aligned} \tag{10}$$

Equations (2) to (10) model the time shifting of a shift-able service in which $d(i,k)$, $d'(i,k)$, $\delta_0(i,k)$, $\delta_1(i,k)$, $\delta_2(i,k)$, $z_1(i,k)$, $z_2(i,k)$ are intermediate variables used to compute the physical decision variables $f(i)$ and $E(i,k)$. The number of decision variables can be reduced by setting $E(i,k)$ to 0 outside the requested time window as follows:

$$E(i,k) = 0, \forall k \in \left[0, \frac{ef(i) - d(i)}{\Delta}\right] \cup \left[\frac{lf(i)}{\Delta} + 1, T\right] \tag{11}$$

3.3. Modulable services model

This paper addresses the modulable services that can be modeled by a first-order dynamic. This typically concerns heating services. Such services include room heating and refrigeration. Every modulable service that can be depicted through the following dynamic equations can be taken into account. In order to highlight the models, without loss of generality, the room heating service will be used as a reference. In buildings, thermal phenomena are continuous phenomena. A number of models are available for the thermal behavior of a HVAC system. A first-order state space thermal model relevant for control purposes is proposed in [18]. It is ideally adapted to local control. The second-order model based on an electrical analogy proposed in [19] may be preferred for planning layers as it is more adapted to the dynamics of indoor temperature. However, model parameters depend on the characteristics of the building such as the thermal capacities of the indoor environment and the housing envelope, the thermal resistances, the equivalent surface of the windows. This type of highly physical model is not really suited to energy management and planning. A behavioral model is preferred as it is easier to identify for each application case. In the model concerned, the only parameters useful are those accounting for the dynamics between temperature and energy consumption. For a room heating service *i*, it yields:

$$\frac{\partial\left[T_{\text{in}}(i,t)-T_{\text{out}}(i,t)\right]}{\partial t}=-\frac{1}{\tau(i)}\left[T_{\text{in}}(i,t)-T_{\text{out}}(i,t)\right]+\frac{G(i)}{\tau(i)}P(i,t)+\frac{G_s(i)}{\tau(i)}\phi_s(i,t) \quad (12)$$

This model provides a fairly accurate of indoor temperature dynamic variations where:

- T_{in}, T_{out} respective indoor and outdoor temperatures [°C]
- P, power consumed by the thermal generator [W]
- ϕ_s , power generated by solar radiance [W]
- G,Gs gains of the first-order dynamic from heating power and solar radiance respectively
- τ constant time of the first-order dynamic

The previous equation can be integrated on the discretized planning period [$k\Delta$,(k+1)Δ]. In automatic control theory, one of the most commonly used formulations for integration of such an equation is based on the following hypothesis: the variables not controlled are assumed to be constant throughout the planning period. *Tin* is the only controlled variable. The operator $(1-e^{\frac{-\Delta}{\tau(i)}})$ known as zero-order hold is used for constant parameters. The difference [$T_{\text{in}}(i,t)-T_{\text{out}}(i,t)$] is integrated, and the final value of T_{out} is assumed to be equal to its initial value. It yields:

$$T_{\text{in}}(i,k+1)-T_{\text{out}}(i,k)=e^{\frac{-\Delta}{\tau(i)}}\left[T_{\text{in}}(i,k)-T_{\text{out}}(i,k)\right]+(1-e^{\frac{-\Delta}{\tau(i)}})\left[G(i)E(i,k)+G_s(i)\phi_s(i,k)\right] \quad (13)$$

This equation modeling the service i can be written as a constraint as follows:

$$T_{\text{in}}(i,k+1)-e^{\frac{-\Delta}{\tau(i)}}T_{\text{in}}(i,k)=(1-e^{\frac{-\Delta}{\tau(i)}})\left[T_{\text{out}}(i,k)+G(i)E(i,k)\ +G_s(i)\phi_s(i,k)\right] \quad (14)$$

The temperature is also constrained in an interval:

$$T_{\min}(i,k)\le T_{\text{in}}(i,k)\le T_{\max}(i,k) \quad (15)$$

In this model of modulable services, $T_{\text{out}}(i,k)$, $\phi_S(i,k)$, $T_{\text{in}}(i,0)$, $\tau(i)$, $G_s(i)$ and $G(i)$ are the data of the optimization problem. $T_{\text{in}}(i,k)$ and $E(i,k)$ are the decision variables.

3.4. Energy balance

A constraint modeling the production/consumption balance must also be added. Generally speaking, this constraint can be written:

$$\forall k\in\{1,...,T\},\sum_{j\in I_s}E(j,k)=\sum_{i\in I_E}E(i,k)+P_{\text{u}}(k)\Delta \quad (16)$$

where $Pu(k)$ stands for the power consumed by the unsupervised service, I_s the set of indexes of the power supply services, and I_E the set of indexes of the end-user services.

3.5. Optimization criteria

In the basic problem, optimization is achieved through the economic criterion. The optimization problem consists in minimizing the cost of energy consumption. The objective function to be minimized can be written as:

$$J=\sum_{j\in I_s}\sum_{k=1}^{T}C(j,k)E(j,k) \quad (17)$$

The objective function and the cost parameters can be adjusted to every pricing policy. Power supply services can be seen as different resources associated with a vector of supplies rather than a matrix. Indeed every end-user service needs a given quantity of the energy resource irrespective of the supplier.

3.6. Comparison with the energy scheduling problem and RCPSP

The HESP restricted to the shift-able services can be compared to both the Cumulative Scheduling Problem (CuSP) defined in [20] as a sub-problem of the RCPSP (Resource Constrained Project Scheduling Problem), and the Energy Scheduling Problem (EnSP) defined in [12]. In the HESP, CuSP and EnSP there is no precedence constraint, and only one resource is concerned at a time. In the CuSP, activities have to be processed without interruption on a given resource of capacity B. A resource requirement, a time window and an execution time are associated with each activity, just as in the HESP. However, in the CuSP, capacity B is constant throughout the planning horizon. The specificity of the EnSP relies on modulation of the resource requirement as a decision variable. The EnSP is also defined over a discretized planning horizon. In the proposed formulation of the HESP, the decision variable is the allocated energy at each period under resource constraint. Energy is also adjustable throughout the planning horizon for modulable services. The HESP restricted to modulable services is equivalent to the EnSP without any time constraints. Nevertheless, in the HESP, the energy allocation bounds are given by the physical phenomena to be managed as shown in section 3.3.

In the scientific literature, continuous time formulations of scheduling problems are available [21,22,23]. However, these results concern scheduling problems with disjunctive resource constraints. Instead of computing the starting time of tasks, the aim is to determine the execution sequence of tasks on disjunctive shared resources. In the energy management problem, the issue is not limited to determining this sequence because a number of services can be achieved at the same time. Both discrete and continuous time models are available for the RCPSP [24,25,26]. However, in such models, availability of the renewable resource is a given data item that is constant throughout the planning horizon. In the HESP, it is a given data item that is constant only over a planning period.

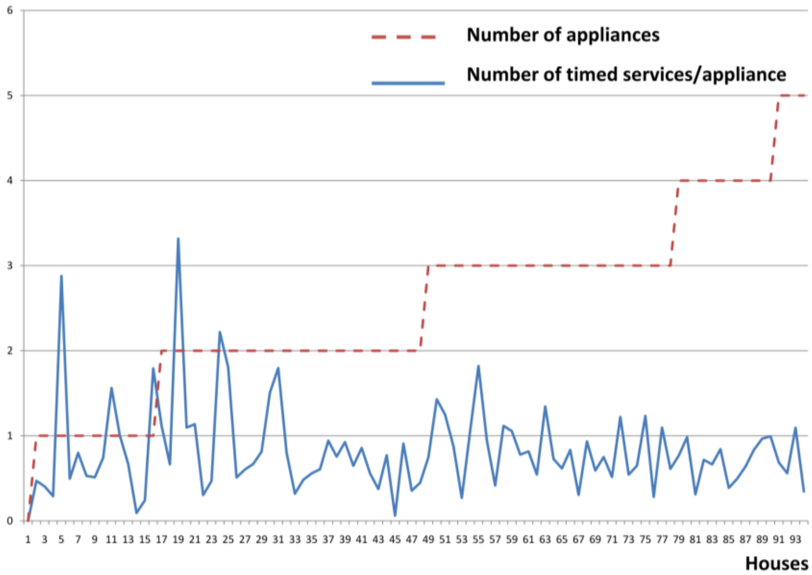

Figure 2. Number of shift-able services per house

Exact MILP formulations are available for the resource-constrained project scheduling problem RCPSP. A number of MILP formulations have been proposed. The basic discrete-time formulation has been proposed in [27,28]. Only one type of binary variable has been defined. These variables are indexed by both activities and time, and model the starting time of the activities. In these models, starting times are synchronized with planning periods. Continuous-time formulations are also proposed in the literature in [24,29]. In these models, starting times are continuous variables. Precedence relations are modeled through binary variables. Event-based models have more recently been proposed in [25,30]. In these formulations, continuous variables are introduced to represent the occurrence times of events and binary variables are used to decide whether an activity starts or ends at the event e. The model proposed in this paper can be related to the continuous-time formulation because the end time is a continuous variable, and the binary variables are introduced to deduce energy consumption in every *a priori* fixed length of the planning period.

3.7. Model size

Table 1: Model sizing per service type

Scenario	Modulable service	Shift-able service	Energy balance
Constraints	3T	20T + 1	T
Continuous variables	3T	5T	T
Integer variables	0	3T	0

The HESP leads to an optimization problem comprising power supply services and a set of modulable and shift-able services depending on the number of supervised appliances in the house. Table 1 gives the number of constraints, continuous variables and integer variables used by each modulable and shift-able service as well as the added continuous variables and constraints corresponding to the energy balance for a given length TΔ of the planning horizon.
Such discrete time formulation of the HESP restricted to shift-able services comprises some 20 times more constraints and 3 times more integer variables than the classic discrete time formulation of the RCPSP. This is due to non-synchronization of the planning period and execution of the shift-able services. In the HESP, time discretization is imposed by the power supply fluctuations. From a practical point of view and assuming the very specific application field involved in these models, data size is not particularly large. In point of fact, the length of the planning period Δ is typically one hour and the planning horizon is 24 hours. However, the number of services that can be requested in households is limited. Only appliances that can be controlled and for which the user can describe the associated requirements are concerned. Typically, washing machines, dishwashers and electric ovens are addressed. Lighting appliances and communication devices requiring the user's presence are not assumed to be supervised.

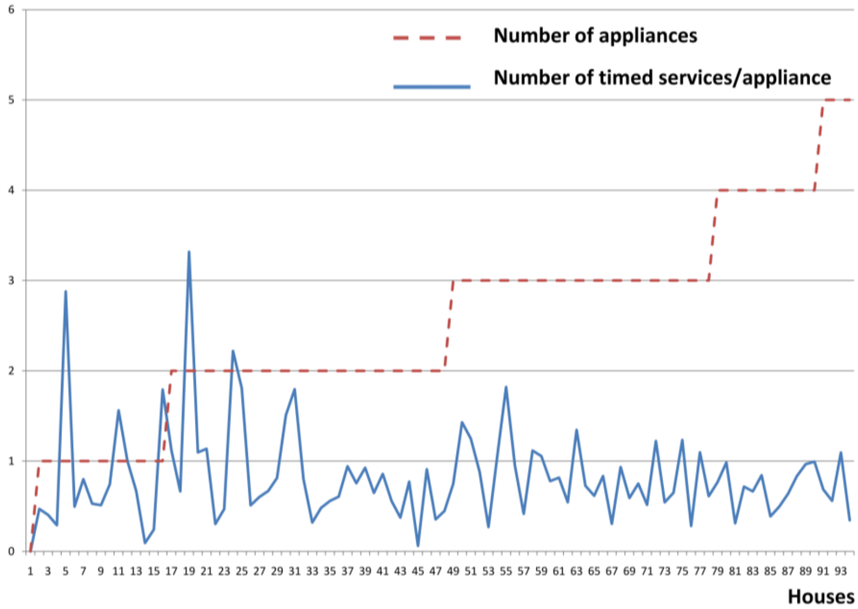

Figure 3. Number of shift-able services per appliance

The project *Residential Monitoring to Decrease Energy Use and Carbon Emissions in Europe (REMODECE)* provides an energy database on residential consumption. This database is studied in [14] and stores the characteristics of residential electrical consumption by country. The *IRISE* project is a part of the *REMODECE* project dealing only with some 100 houses in France. One database is available for every house in which information is recorded every 10 minutes over a one year period for each appliance in the associated house. The energy consumed at every planning period by every appliance is given in this database. Based on this information we can find out the number of services requested each day. The average number of shift-able services each day out of 93 representative houses is 2 with a minimum value equal to 0 and a maximum value equal to 29. The average and maximum numbers of shift-able services for each house are given in figure 2. Houses are ordered in increasing number of addressed appliances. Figure 3 shows that a house has 2.5 appliances on average and 0.8 requests for shift-able services per appliance. In these houses there are 1 to 6 modulable services to be managed with an average of 2.3. It is thus easy to deduce that the typical size of an

HESP is around (47T+2) constraints and 10T integer variables. Thus for T=24 the basic HESP can be sized to 1130 constraints and 240 integer variables. However the number of integer variables can be drastically reduced to only those variables that satisfy the time window constraints thanks to equation (23).

IV. PROBLEM EXTENDED TO COMFORT SATISFACTION

The basic energy management problem can be generalized to take into account the user's comfort as an optimization criterion as well as energy cost. In this multi-criteria problem, the best compromise is sought between comfort and cost. The user's comfort is defined by a satisfaction indicator quantifying service quality achievement. In this paper, service quality is quantified by the difference between a preferred value defined by the user and the optimized value.

3.8. Modulable services

The addressed modulable services control a physical variable such as temperature. Therefore user satisfaction can be quantified by the difference between an expected preferred value and the optimized value. Let us define $T_{\mathrm{opt}}(i,k)$ as the preferred temperature at each planning period. According to the comfort standard 7730, [31] proposes typical models for thermal comfort that depend on type (office, room, etc.) and quality (humidity and air velocity) of the environment. These models are based on an aggregated criterion known as the predictive mean vote (PMV) modeling the deviation from a neutral environment. In this case, for a given type and quality of the environment, a discomfort index $D(i,k)$ can be computed for the modulable service i at each planning period k from the following equation:

$$D(i,k)=\begin{cases}\dfrac{T_{\mathrm{opt}}(i,k)-T_{\mathrm{in}}(i,k)}{T_{\mathrm{opt}}(i,k)-T_{\mathrm{min}}(i,k)} & \text{if} \quad T_{\mathrm{in}}(i,k)\le T_{\mathrm{opt}}(i,k)\\[2ex] \dfrac{T_{\mathrm{in}}(i,k)-T_{\mathrm{opt}}(i,k)}{T_{\mathrm{max}}(i,k)-T_{\mathrm{opt}}(i,k)} & \text{if} \quad T_{\mathrm{in}}(i,k)> T_{\mathrm{opt}}(i,k)\end{cases} \tag{18}$$

where $T_{\mathrm{opt}}(i,k)$ stands for the requested temperature, and $T_{\mathrm{min}}(i,k)$ and $T_{\mathrm{max}}(i,k)$ stand, respectively, for the minimum and maximum acceptable temperatures.

3.9. Shift-able services

The user expects shift-able services such as washing to be finished at a given preferred time denoted by $f_{\mathrm{opt}}(i)$. As such, service quality achievement depends on the amount of time it is shifted from this preferred value. Just as for modulable services, a dissatisfaction criterion for a service i is defined as follows:

$$D(i)=\begin{cases}\dfrac{f(i)-f_{\mathrm{opt}}(i)}{lf(i)-f_{\mathrm{opt}}(i)} & \text{if} \quad f(i)> f_{\mathrm{opt}}(i)\\[2ex] \dfrac{f_{\mathrm{opt}}(i)-f(i)}{f_{\mathrm{opt}}(i)-ef(i)} & \text{if} \quad f(i)\le f_{\mathrm{opt}}(i)\end{cases} \tag{19}$$

where $f_{\mathrm{opt}}(i)$ stands for the requested end time, and $ef(i)$ and $lf(i)$ stand, respectively, for the minimum and maximum acceptable end times.

3.10. Linear dissatisfaction model

Dissatisfaction of a shift-able service is a particular case of the dissatisfaction of a modulable service. Consequently, without loss of generality, we only show the equations for dissatisfaction incurred by modulable services.

Let $\delta_D(i,k)$ be a binary variable such that $\delta_D(i,k)=1$ if and only if $T_{\mathrm{in}}(i,k)\le T_{\mathrm{opt}}(i,k)$. The following linear constraints can be written to assign the binary variable:

$$\begin{aligned}&T_{\mathrm{in}}(i,k)-T_{\mathrm{opt}}(i,k)\le\left[T_{\mathrm{max}}(i,k)-T_{\mathrm{opt}}(i,k)\right]\left[1-\delta_{\mathrm{D}}(i,k)\right]\\&T_{\mathrm{in}}(i,k)-T_{\mathrm{opt}}(i,k)\ge\varepsilon+\left[T_{\mathrm{min}}(i,k)-T_{\mathrm{opt}}(i,k)-\varepsilon\right]\delta_{\mathrm{D}}(i,k)\end{aligned} \tag{20}$$

where ε is a given small number.
Then the discomfort index can be written via the following constraints:

$$D(i,k)=\delta_{\mathrm{D}}(i,k)\frac{T_{\mathrm{opt}}(i,k)-T_{\mathrm{in}}(i,k)}{T_{\mathrm{opt}}(i,k)-T_{\mathrm{min}}(i,k)}+\left[1-\delta_{\mathrm{D}}(i,k)\right]\frac{T_{\mathrm{in}}(i,k)-T_{\mathrm{opt}}(i,k)}{T_{\mathrm{max}}(i,k)-T_{\mathrm{opt}}(i,k)} \quad (21)$$

The variable $z_D(i,k)$ defined by the following inequalities must be added to obtain the linear formulation of the discomfort index:

$$D(i,k)=\frac{\delta_{\mathrm{D}}(i,k)T_{\mathrm{opt}}(i,k)-z_{\mathrm{D}}(i,k)}{T_{\mathrm{opt}}(i,k)-T_{\mathrm{min}}(i,k)}+\frac{T_{\mathrm{in}}(i,k)-z_{\mathrm{D}}(i,k)-\left[1-\delta_{\mathrm{D}}(i,k)\right]T_{\mathrm{opt}}(i,k)}{T_{\mathrm{max}}(i,k)-T_{\mathrm{opt}}(i,k)} \quad (22)$$

$$\begin{aligned}
z_{\mathrm{D}}(i,k) &\le T_{\mathrm{max}}(i,k)\delta_{\mathrm{D}}(i,k)\\
z_{\mathrm{D}}(i,k) &\ge T_{\mathrm{min}}(i,k)\delta_{\mathrm{D}}(i,k)\\
z_{\mathrm{D}}(i,k) &\le T_{\mathrm{in}}(i,k)-T_{\mathrm{min}}(i,k)\left[1-\delta_{\mathrm{D}}(i,k)\right]\\
z_{\mathrm{D}}(i,k) &\ge T_{\mathrm{in}}(i,k)-T_{\mathrm{max}}(i,k)\left[1-\delta_{\mathrm{D}}(i,k)\right]
\end{aligned} \quad (23)$$

3.11. Optimization criteria

The extended HESP is a bi-criteria optimization problem. Depending on the user's requests, a compromise between cost and comfort has to be formulated. This is generally the case when energy cost is variable, as the higher cost corresponds to peak consumption periods. In this paper focused on problem modeling, a very simple aggregation approach has been implemented. The corresponding objective function to be minimized is depicted by the following equation:

$$J=\sum_{j\in \mathrm{I}_S}\sum_{k=1}^{T}C(i,k)E(j,k)+\frac{\beta}{\sum_{i_m}\alpha(i_m)+\sum_{i_s}\alpha(i_s)}\left[\sum_{i_m}\sum_{k=1}^{T}\alpha(i_m)D(i,k)+\sum_{i_s}\alpha(i_s)D(i)\right] \quad (24)$$

Modulable services are identified by indexes i_m, and shift-able services are identified by indexes i_s. Parameters α depict the priority between end-user services, while parameter β depicts the relative importance granted by the user to cost criteria and discomfort criteria. The parameter β helps us understand the distinction between the thrifty user and the comfort addict user.

3.12. Extended model size

The dissatisfaction proposed in this paper results in adding to the basic formulation $5T$ constraints, $2T$ continuous variables and T binary variables for every end-user service.

V. COMPUTATIONAL RESULTS

The HESP has been implemented and solved with the IBM ILOG CPLEX Optimizer 11.1 in a computer with XP SP3, 3.45 GB of RAM and an Intel Core 2 Duo 2.4GHz. A power supply service is assumed with a constant available power P on the planning horizon T = 24. Several sets of 100 instances of 10 shift-able services each have been randomly generated. Five parameters have been studied in order to show their impact on problem hardness: energy ratio, power ratio, time ratio, dissatisfaction, and energy price.

- *RE*, the *energy ratio*, defines the ratio of the total amount of available energy required to execute the 10 shift-able services. The total amount of available energy is *P.T*. In this case the total amount of energy consumed by the 10 shift-able services is assumed to equal *RE.P.T*. The values 0.3, 0.6 and 0.9 have been studied for the parameter *RE*. They are denoted by 3, 6 and 9, respectively, in the results.
- *RP*, the *power ratio*, defines the maximum ratio of the available power required for execution of each service. The power of each service is assumed to be less than or equal to *RP.P.T*. From the maximum available power *P* the power ratio defines whether or not, the services can be executed in parallel. The values 0.3, 0.5 and 0.8 have been studied for the parameter *RP*. They are denoted by 3, 5 and 8, respectively, in the results.

The sets of 100 random instances of 10 shift-able services to be scheduled have been built using the energy ratio and the power ratio as follows. Firstly, the amount of energy *RE.P.T* is randomly divided into 10 parts $E(i)$ associated with the 10 services. Then 10 values $P(i)$ of power, smaller than *RP.P*, are generated. The execution time $d(i)$ of every service is derived from $E(i)$ and $P(i)$. Finally a feasible value of $fopt(i)$ is randomly generated $[fopt(i) \geq d(i)]$.

- *RT*, the *time ratio*, defines the maximum length of the end time windows of the services. From the planning horizon *T*, the length of the end time window of each service is shorter than or equal to $RT.[T-d(i)]$. The time ratio defines the ratio of the available planning horizon that can be used to schedule a service. The values 0.2, 0.6 and 1 have been studied for the parameter *RT*. They are denoted by 2, 6 and 1, respectively, in the results.
- *Dissatisfaction*. For the basic HESP without a dissatisfaction indicator, the end time window is centered around $fopt(i)$. Scenarios without dissatisfaction are denoted by 0 in the results. When dissatisfaction is involved, feasible values of $ef(i)$ and $lf(i)$ are randomly generated under the constraint of the length of the time window $[ef(i) \leq fopt(i) \leq lf(i)]$ and $[lf(i)-ef(i)] \leq RT.[T-d(i))]$. Scenarios with dissatisfaction are denoted by 1 in the results.
- *Price*. Three scenarios have been studied for energy cost. The constant scenario is denoted by *C*. The scenario denoted by *PS* has a low cost in slack periods $k=\{1,2,3,4,5,6,23,24\}$ and a cost 4 times greater than the low cost in the peak periods $k \in \{7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22\}$. The scenario denoted by 3D has three randomly generated costs. For each instance, three values of the energy cost are generated satisfying minimum and maximum constraints given in Figure 4 as well as the associated time periods.

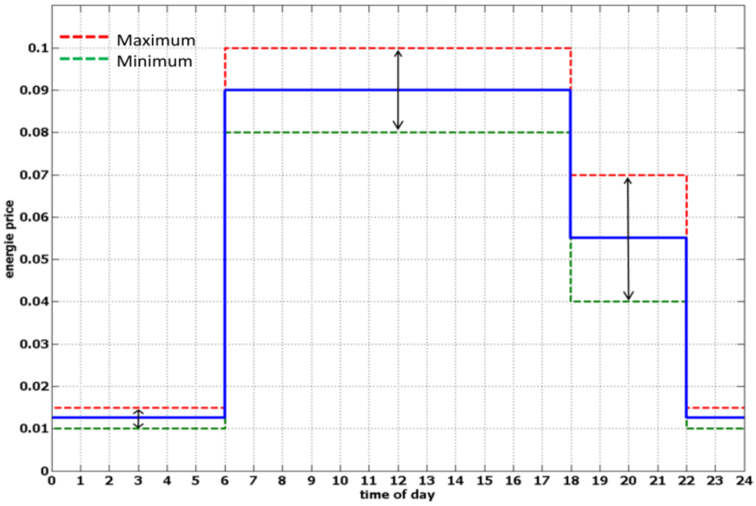

Figure 4. Minimum and maximum values of energy price for the 3D scenario

Table 2 shows the computational results of the HESP obtained from 11 sets of values for the 5 parameters. 100 random instances of 10 shift-able services have each been generated for every assumed set of parameters. Each set of instances is denoted by the corresponding values of the parameters *RE−RP−RT−Price−Dissatisfaction*. A maximum execution time of 15 minutes (900 seconds) has been chosen. The ratio of instances solved up to the optimality, the maximum solving time for the entirely solved instances, and the mean solving time obtained over the solved instances for the not entirely solved instances are given in Table 2.

Table 2: Shift-able services solving

Scenario	Solved to optimality	Maximum time (s)	Mean time(s)
351*C*0	100%	6	2.39
651*C*0	100%	57	7.16
951*C*0	93%	-	124.3
931*C*0	97%	-	3.51
981*C*0	94%	-	115.77
952*C*0	100%	1	0.04
956*C*0	100%	156	4.26
951*PS*0	88%	-	182.64
9513*D*0	88%	-	183.26
551*C*0	100%	38	5.84
551*C*1	94%	-	335.44

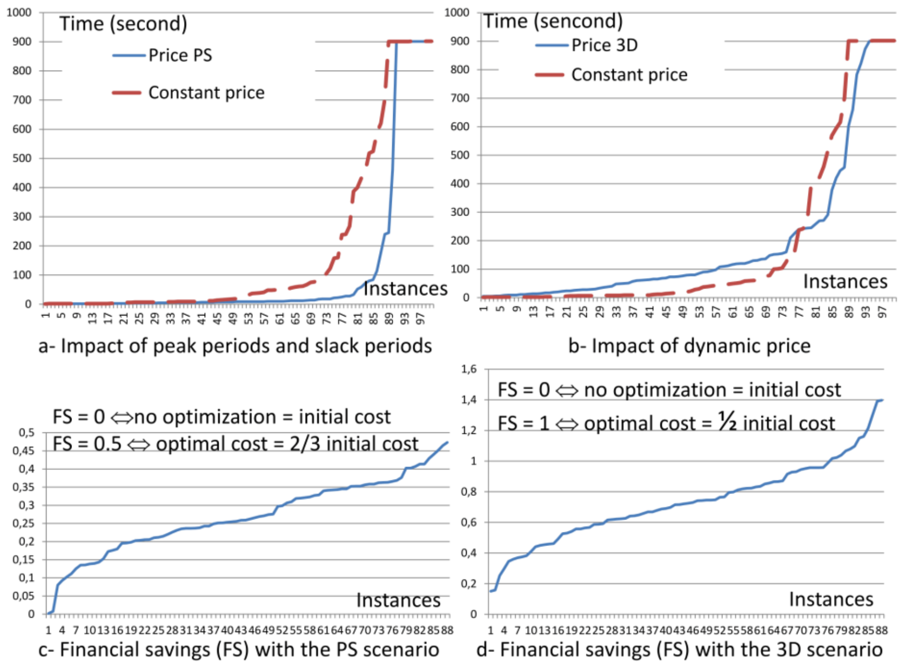

Figure 5. Impact of energy price

In instances with a constant energy price the HESP consists in finding an admissible solution. It can be observed that the power ratio has a greater impact than the energy ratio and the length of the time window. 100% of instances are solved to optimality in 2 out of 3 cases for the parameters *RE* and *RT*. However, no studied value of *PT* can solve to optimality 100% of instances. The impact of the dissatisfaction indicator has been studied for a rather easy set of parameters ($RE-RP-RT-Price=551C$). It can be observed that the dissatisfaction indicator has an impact on the solving properties of the HESP. Figures 5-a and 5-b show that energy price can structure the solution space. The solving times for prices *PS* and 3*D* do not vary greatly from the constant price solving time. Figures 5-c and 5-d show the potential financial savings of the optimization model. For each instance of the 88% of instances solved to optimality, the solution provided by the first iteration of the optimizer (denoted as the first solution attained) and the optimal solution have been stored. The cost of the first solution attained (c_{FR}) and the cost of the optimal solution (c_O) were compared. The difference between these two costs was then divided by the optimal cost. The financial saving was been estimated via the following indicator $FS = \left(c_{FR} - c_O\right) / c_O$. The optimization formulation of the HESP prevents an increase of 50% cost for rates based on peak periods and slack periods. Financial savings are as much as 140% for the assumed dynamic price limited to 3 different periods. These results show the potential financial savings achieved by the HESP in the context of a dynamic energy market.

VI. CONCLUSION

This paper explains how operational research can provide solutions for the home energy management problem. The common features and differences with the Resource Constrained Project Scheduling Problem, the Cumulative Scheduling Problem and the Energy Scheduling Problem have been discussed. The complexity analysis of the HESP has been conducted. Also, a formulation of the HESP has been proposed, designed to schedule all the household services i.e. modulable, shift-able and unsupervised services. An enhanced problem that takes into account user comfort expectations has also been proposed. Numerical results show that, using the IBM ILOG CPLEX Optimizer, most HESP instances can be solved within a few seconds. Exact solving of the HESP using the MILP formulation would appear pertinent for one house. However, heuristics is required to cope with a multi-residential application. Finally, differences are frequently observed between prediction and reality. This issue can be managed either by a reactive adjustment mechanism or by a robust scheduling approach minimizing the risk of incorrect prediction by taking into account a statistical prediction model. The linear formulation of the HESP could prove useful for implementing robust approaches based on available results concerning robust linear programming.

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