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Queue with Breakdowns and Interrupted Repairs

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ABSTRACT
This paper considers a queue, consisting of a Poisson input stream and a server. The server is subject to breakdowns. The times to failure of the server follows exponential distribution. The failed server requires repair at a facility, which has an unreliable repair crew. The repair times of the failed server follows exponential distribution, but the repair crew also subjects to breakdown when it is repairing. The times to failure of the repair crew is also assumed to be exponentially distributed. This paper obtains the steady-state performance of the queuewith server breakdowns and interrupted repairs.

Keywords: *Breakdown, interrupted repair, performance, queue*

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I. INTRODUCTION

In many real life situations, perfectly reliable servers are virtually nonexistent. It is well known that performance of unreliable system is heavily influenced by server breakdowns [1]. For this reason, queueing systems with servers subject to breakdowns and repairs have been studied extensively. Recently, Aissani[2], Choudhury and Deka[3], Choudhury and Ke[4], Do et al. [5], Lee [6], and others considered the unreliable queues wherein customers who find the server broken down should wait in the queue until the server is repaired. The failed server requires repair at a facility, which has a repair crew.

In realistic environments the repair crew is possible to break down when it is repairing. Therefore, considering an unreliable repair crew in a repairable system is also practical and imperative [7]. In this paper, we consider a queueing system with breakdowns and interrupted repairs. Within the framework of M/M/1 queue, the steady-state distribution of the system state and the number of customers is derived analytically by using probability generating function method.

II. MODEL

A single server queueing system is considered. Customers arrive from outside according to a Poisson process with rate λ. Service times of customers are independent of each other and have a common exponential distribution with rate μ. It is assumed that the server is subject to breakdowns. Failed server is repaired by a repair crew. Once the server is repaired, it is as good as new. In addition, the repair crew may function wrongly or fail sometimes when it is repairing. It can also be fixed. Once the repair crew is fixed, it can function again. We shall assume that all relevant random variables, such as the times to failure of the server, the repair times of the failed server, the times to failure of the repair crew, and the repair times of the failed repair crew, have exponential distributions with rates α, β, γ, and δ, respectively. We also make all the necessary independence assumptions on the inter-arrival times, the service times, the times to failure of the server, the repair times of the failed server, the times to failure of the repair crew, and the repair times of the failed repair crew.

Let $\mathrm{S(t)}$ be the state of the server and repair crew at time t:

$$\mathrm{S(t)} \equiv \begin{cases} 1 & \text{if the server is available at time t} \\ 2 & \text{if the repair crew is repairing the failed server at time t} \\ 3 & \text{if both the server and repair crew are failed at time t} \end{cases} \tag{1}$$

Define $\mathrm{N(t)}$ as the number of customers in system at time t. Then, the stochastic process $\{(\mathrm{S(t)}, \mathrm{N(t)}), \mathrm{t} \geq 0\}$ is a continuous time Markov chain with state space $\{(\mathrm{s}, \mathrm{n}), \mathrm{s} = 1,2,3, \mathrm{n} \geq 0\}$.

III. STEADY STATE ANALYSIS

The steady-state behavior of the system is examined in this section. In order to have a proper steady-state distribution, we assume that the system reaches a stationary state.

Let $p_{i,n} \equiv \lim_{t\to\infty} \mathrm{P}\{S(t) = i,\ N(t) = n\}$. The steady state equations governing the system are:

$$(\lambda + \alpha)p_{1,0} = \mu p_{1,1} + \beta p_{2,0} \tag{2}$$
$$(\lambda + \mu + \alpha)p_{1,n} = \lambda p_{1,n-1} + \mu p_{1,n+1} + \beta p_{2,n}, \quad n \geq 1 \tag{3}$$
$$(\lambda + \beta + \gamma)p_{2,0} = \alpha p_{1,0} + \delta p_{3,0} \tag{4}$$
$$(\lambda + \beta + \gamma)p_{2,n} = \lambda p_{2,n-1} + \alpha p_{1,n} + \delta p_{3,n}, \quad n \geq 1 \tag{5}$$
$$(\lambda + \delta)p_{3,0} = \gamma p_{2,0} \tag{6}$$
$$(\lambda + \delta)p_{3,n} = \lambda p_{3,n-1} + \gamma p_{2,n}, \quad n \geq 1 \tag{7}$$

Define partial probability generating functions as follows:

$$P_i(z) \equiv \sum_{n=0}^{\infty} p_{i,n} z^n \tag{8}$$

After some calculations, we obtain:

$$P_1(z) = \frac{\mu z(1-z)p_{1,0}}{\lambda z - (\lambda + \mu + \alpha) + \frac{\mu}{z} + \frac{\alpha\beta\,(\lambda+\delta-\lambda z)}{(\lambda+\beta+\gamma-\lambda z)(\lambda+\delta-\lambda z)-\delta\gamma}} \tag{9}$$

$$P_2(z) = \frac{\alpha(\lambda + \delta - \lambda z)}{(\lambda + \beta + \gamma - \lambda z)(\lambda + \delta - \lambda z) - \delta\gamma} P_1(z) \tag{10}$$

$$P_3(z) = \frac{\gamma}{\lambda + \delta - \lambda z} P_2(z) \tag{11}$$

From the normalization condition

$$P_1(1) + P_2(1) + P_3(1) = 1 \tag{12}$$

we have:

$$p_{1,0} = \frac{1}{1 + \frac{\alpha}{\beta}\left(1 + \frac{\gamma}{\delta}\right)} - \frac{\lambda}{\mu} \tag{13}$$

IV. NUMERICAL EXAMPLES

We provide 6 cases for illustration purposes andshow the effects of different system parameterson the steady-state performance:

Case (a):$\lambda \in [0.31, 0.57]$, $\mu = 1$, $\alpha = 0.5$, $\beta = 1$, $\gamma = 0.5$, $\delta = 1$
Case (b): $\lambda = 0.5$, $\mu \in [0.90, 1.50]$, $\alpha = 0.5$, $\beta = 1$, $\gamma = 0.5$, $\delta = 1$
Case (c): $\lambda = 0.5$, $\mu = 1$, $\alpha \in [0.01, 0.66]$, $\beta = 1$, $\gamma = 0.5$, $\delta = 1$
Case (d): $\lambda = 0.5$, $\mu = 1$, $\alpha = 0.5$, $\beta \in [0.76, 1.50]$, $\gamma = 0.5$, $\delta = 1$
Case (e): $\lambda = 0.5$, $\mu = 1$, $\alpha = 0.5$, $\beta = 1$, $\gamma \in [0.01, 0.99]$, $\delta = 1$
Case (f): $\lambda = 0.5$, $\mu = 1$, $\alpha = 0.5$, $\beta = 1$, $\gamma = 0.5$, $\delta \in [0.51, 1.50]$

The effects of various parameters on the mean number of customers in the system are shown in Figure 1.

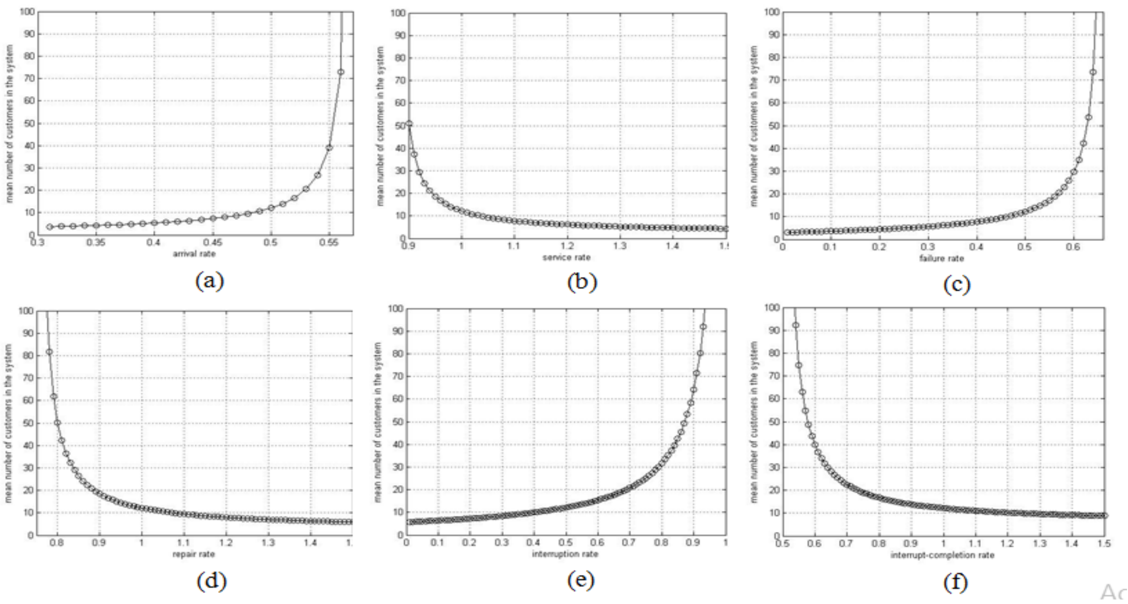

Figure 1. Mean number of customers in the system

V. CONCLUSION

We considered a queueing system, consisting of a Poisson input stream and a server. The server is subject to breakdowns. The failed server requires repair at a facility, which has an unreliable repair crew. The repair crew also subjects to breakdown when it is repairing. This paper obtained the steady-state performance of the queueing system with server breakdowns and interrupted repairs.

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