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Existence and Uniqueness of the Equilibrium Point of Bidirectional Associative Memory Neural Networks With Fuzzy Logic

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--***ABSTRACT***--
In this paper, based on the theory of homotopic mapping and properties of M-matrix, by constructing proper vector Liapunov functions, the existence and uniqueness of the equilibrium point is investigated for a class of bidirectional associative memory neural networks with fuzzy logic and distributed delays. Without assuming the boundedness, monotonicity and differentiability of the activation functions, the new sufficient criterion for ascertaining the existence, uniqueness of the equilibrium point of such neural networks is obtained. The criterion is independent of the delays and is easy to test in practice.

***KEYWORD*：** *bidirectional associative memory neural networks, existence and uniqueness of the equilibrium point, fuzzy logic, distributed delays*

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I. INTRODUCTION

Bidirectional associative memory (BAM) neural networks known as an extension of the unidirectional autoassociator of Hopfield[1] was first introduced by Kosto[2]. It is composed of neurons arranged in two layers. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnection among neurons in the same layer. Through iterations of forward and backward propagation information flows between the two layers, which performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Due to the BAM neural networks has been used in many fields such as pattern recognition, image processing, and automatic control. Therefore, the BAM neural networks have attracted great attention of many researchers.One can refer to the articles [3-5] for detailed discussion on these aspects.

When a neural network is employed as an associative memory, the existence of many equilibrium points is a necessary feature. However, in applications to parallel computation and signal processing involving solution optimization problems, it is required that there be a well-defined computable solution for all possible initial states. From a mathematical viewpoint, this means that the network should have a unique equilibrium point that is globally asymptotically or exponential stable.

In this paper, we investigate a kind of delayed BAM neural networks with fuzzy logic which integrates fuzzy logic into the structure of traditional BAM neural networks with distributed delays. Our objective is to study the existence of unique equilibrium point of the kind of BAM neural networks. Without assuming the boundedness, monotonicity and differentiability of activation functions, by using M-matrix theory, we present the new condition ensuring existence, uniqueness of the equilibrium point for the class of BAM networks with fuzzy logic and distributed delays.

II. NOTATION AND PRELIMINARIES

For convenience, we introduce some notations. $x=(x_1,\cdots,x_n)^{\mathrm{T}}\in R^n$ denotes a column vector. $|x|$ denotes the absolute-value vector given by $|x|=(|x_1|,\cdots,|x_n|)^{\mathrm{T}}$, $\|x\|$ denotes a vector norm defined by $\|x\|=(x_1^2+x_2^2+\cdots+x_n^2)^{1/2}$. For matrix $A=(a_{ij})_{n\times n}$, A^{T} denotes the transpose of A , A^{-1} denotes the inverse of A , $[A]^s$ is defined as $[A]^s=(A^{\mathrm{T}}+A)/2$, and $|A|$ denotes absolute-value matrix given by $|A|=(|a_{ij}|)_{n\times n}$, $\|A\|$ denotes a matrix norm defined by $\|A\|=(\max\{\lambda:\lambda$ is an eigenvalue of $A^{\mathrm{T}}A\})^{1/2}$. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively.

The dynamical behavior of bidirectional associative memory neural networks with fuzzy logic and distributed time delays can be described by the following nonlinear differential equations:

$$\frac{du_i(t)}{dt} = -\alpha_i u_i(t) + \sum_{j=1}^{m} a_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(v_j(s))ds + \bigwedge_{j=1}^{m} b_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(v_j(s))ds$$

$$+ \bigvee_{j=1}^{m} c_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(v_j(s))ds + I_i,\ i = 1,2,\cdots,n; \tag{1a}$$

$$\frac{dv_j(t)}{dt} = -\beta_j v_j(t) + \sum_{i=1}^{n} d_{ji} \int_{-\infty}^{t} L_{ji}(t-s) g_i(u_i(s))ds + \bigwedge_{i=1}^{n} w_{ji} \int_{-\infty}^{t} L_{ji}(t-s) g_i(u_i(s))ds$$

$$+ \bigvee_{i=1}^{n} z_{ji} \int_{-\infty}^{t} L_{ji}(t-s) g_i(u_i(s))ds + J_j,\ j = 1,2,\cdots,m, \tag{1b}$$

where $\alpha_i > 0$, $\beta_j > 0$ for $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$ denote the passive decay rates; a_{ij}, d_{ji} for $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$ are the synaptic connection strengths; b_{ij}, w_{ji} and c_{ij}, z_{ji} for $i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; f_j for $j = 1,2,\cdots,m$ and g_i for $i = 1,2,\cdots,n$ denote the propagational signal functions ; I_i for $i = 1,2,\cdots,n$ and J_j for $j = 1,2,\cdots,m$ are the exogenous inputs. The initial conditions associated with (1) are of the form

$$u_i(s) = \phi_i(s),\ s \le 0,\ i = 1,2,\cdots,n,$$
$$v_j(s) = \psi_j(s),\ s \le 0,\ j = 1,2,\cdots,m,$$

where ϕ_i and ψ_j are bounded and continuous on $(-\infty, 0]$.

In the following, we let

$$u = (u_1,\cdots,u_n)^{\mathrm{T}},\ v = (v_1,\cdots,v_m)^{\mathrm{T}},\ \alpha = \mathrm{diag}(\alpha_1,\cdots,\alpha_n),\ \beta = \mathrm{diag}(\beta_1,\cdots,\beta_m),$$
$$A = (a_{ij})_{n\times m}, B = (b_{ij})_{n\times m}, C = (c_{ij})_{n\times m}, D = (d_{ji})_{m\times n}, W = (w_{ji})_{m\times n}, Z = (z_{ji})_{m\times n},$$
$$f(v) = (f_1(v_1),\cdots,f_m(v_m))^{\mathrm{T}}, g(u) = (g_1(u_1),\cdots,g_n(u_n))^{\mathrm{T}}, \phi = (\phi_1,\cdots,\phi_n)^{\mathrm{T}}, \psi = (\psi_1,\cdots,\psi_m)^{\mathrm{T}}.$$

For system (1), we make the following assumptions:

Assumption (A) For each $i \in [1,n]$, $j \in [1,m]$, $f_j : R \to R$ and $g_i : R \to R$ are globally Lipschitz continuous with Lipschitz constant $F_j > 0$, $G_j > 0$, i.e., $| f_j(y_j) - f_j(v_j) | \le F_j | y_j - v_j |$ for all y_j, v_j, and $| g_i(x_i) - g_i(u_i) | \le G_i | x_i - u_i |$ for all x_i, u_i.

We let $F = \mathrm{diag}(F_1, F_2,\cdots,F_n)$, $G = \mathrm{diag}(G_1, G_2,\cdots,G_m)$.

Assumption (B) For each $i \in [1,n]$, $j \in [1,m]$, $K_{ij} : [0,\infty) \to [0,\infty)$, $L_{ji} : [0,\infty) \to [0,\infty)$ are piecewise continuous on $[0,\infty)$ and satisfy

$$\int_0^{\infty} \mathrm{e}^{\lambda s} K_{ij}(s)\mathrm{d}s = p_{ij}(\lambda), \int_0^{\infty} \mathrm{e}^{\rho s} L_{ji}(s)\mathrm{d}s = h_{ji}(\rho), i = 1,2,\cdots,n, j = 1,2,\cdots,m,$$

where $p_{ij}(\lambda)$ and $h_{ji}(\rho)$ are continuous functions in $[0,\delta)$ and $[0,\sigma)$, respectively. $\delta > 0$, $\sigma > 0$ and $p_{ij}(0) = 1$, $h_{ji}(0) = 1$.

Lemma 1[6]. Let Ω be a $(n+m)\times(n+m)$ matrix with non-positive off-diagonal elements. Then the following statements are equivalent:

(i) Ω is an M-matrix,
(ii) The real parts of all eigenvalues of Ω are positive,
(iii) There exists a vector $\xi > 0$, such that $\Omega\xi > 0$,
(iv) Ω is nonsingular and all elements of Ω^{-1} are nonnegative,
(v) There exists a positive definite $(n+m)\times(n+m)$ diagonal matrix Q such that matrix $\Omega Q + Q\Omega^{\mathrm{T}}$ is positive definite.

Lemma 2[7]. Suppose $(x, y) \in R^{n\times m}$ and $(u, v) \in R^{n\times m}$ are two states of system (1), then

$$|\bigwedge_{i=1}^{n} w_{ji} g_i(x_i) - \bigwedge_{i=1}^{n} w_{ji} g_i(u_i)| \le \sum_{i=1}^{n} |w_{ji}||g_i(x_i) - g_i(u_i)|,$$

$$|\bigvee_{i=1}^{n} z_{ji} g_i(x_i) - \bigvee_{i=1}^{n} z_{ji} g_i(u_i)| \le \sum_{i=1}^{n} |z_{ji}||g_i(x_i) - g_i(u_i)|, \; i = 1,2,\cdots,n;$$

$$|\bigwedge_{j=1}^{m} b_{ij} f_j(y_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(v_j)| \le \sum_{j=1}^{m} |b_{ij}||f_j(y_j) - f_j(v_j)|,$$

$$|\bigvee_{j=1}^{m} c_{ij} f_j(y_j) - \bigvee_{j=1}^{m} c_{ij} f_j(v_j)| \le \sum_{j=1}^{m} |c_{ij}||f_j(x_j) - f_j(v_j)|, \; j = 1,2,\cdots,m.$$

Lemma 3[8]**.** If $H(x) \in C^0$ satisfies the following conditions, then $H(x)$ is a homeomorphism of R^{n+m}.

(1) $H(x)$ is injective on R^{n+m},

(2) $\lim_{\|x\|\to\infty} \|H(x)\| \to \infty$.

III. EXISTENCE AND UNIQUENESS OF THE EQUILIBRIUM POINT

In the section, we study the existence and uniqueness of the equilibrium point of (1). We firstly study the nonlinear map associated with (1) as follows:

$$\begin{cases} H_i(u_i) = -\alpha_i u_i + \sum_{j=1}^{m} a_{ij} f_j(v_j) + \bigwedge_{i=1}^{m} b_{ij} f_j(v_j) + \bigvee_{i=1}^{m} c_{ij} f_j(v_j) + I_i, \; i = 1,2,\cdots,n, \\ H_{n+j}(v_j) = -\beta_j v_j + \sum_{i=1}^{n} d_{ji} g_i(u_i) + \bigwedge_{i=1}^{n} w_{ji} g_i(u_i) + \bigvee_{i=1}^{n} z_{ji} g_i(u_i) + J_j, \; j = 1,2,\cdots,m. \end{cases} \tag{2}$$

Let $H(u,v) = (H_1(u_1), H_2(u_2), \cdots, H_n(u_n), H_{n+1}(v_1), H_{n+2}(v_2), \cdots, H_{n+m}(v_m))^{\mathrm{T}}$. It is known that if there exists a point (u^*, v^*) such that $H(u^*, v^*) = 0$, then, the point (u^*, v^*) is the equilibrium in (1). So in order to investigated the existence and uniqueness of the equilibrium in (1), we firstly investigate the existence and uniqueness of the solution for nonlinear equation (2). If map $H(u,v)$ is a homeomorphism on $R^{n\times m}$, then there exists a unique point (u^*, v^*) such that $H(u^*, v^*) = 0$, i.e., systems (1) have a unique equilibrium (u^*, v^*). Based on the Lemma 3, we get the conditions of the existence and uniqueness of the equilibrium for system (1) as follows.

Theorem 1 Assume that the Assumption (A) is satisfied, if Ω is an M-matrix, then, for every pair of input (I, J), systems (1) have a unique equilibrium (u^*, v^*). Ω is defined as

$$\Omega = \begin{bmatrix} \alpha & -[|A| + |B| + |C|]F \\ -[|D| + |W| + |Z|]G & \beta \end{bmatrix}.$$

Proof. In order to prove that systems (1) have a unique equilibrium point (u^*, v^*), it is only need to prove that $H(u,v)$ is a homeomorphism on $R^{n\times m}$. In the following, we shall prove that map $H(u,v)$ is a homeomorphism through two steps.

In the first step, we prove that $H(u,v)$ is an injective on $R^{n\times m}$. Suppose, for purposes of contradiction, that there exist $(x, y) \in R^{n\times m}$, $(u,v) \in R^{n\times m}$ with $(x, y) \ne (u,v)$ such that $H(x, y) = H(u,v)$. From (2), by Assumption (A) and Lemma 2, we get

$$|H_i(x_i) - H_i(u_i)|$$

$$= |-\alpha_i(x_i - u_i) + \sum_{j=1}^{m} a_{ij}[f_j(y_j) - f_j(v_j)] + \bigwedge_{j=1}^{m} b_{ij} f_j(y_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(v_j) + \bigvee_{j=1}^{m} c_{ij} f_j(y_j) - \bigvee_{j=1}^{m} c_{ij} f_j(v_j)|$$

$$\ge \alpha_i |x_i - u_i| - \sum_{j=1}^{m} |a_{ij}||f_j(y_j) - f_j(v_j)| - |\bigwedge_{j=1}^{m} b_{ij} f_j(y_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(v_j)| - |\bigvee_{j=1}^{m} c_{ij} f_j(y_j) - \bigvee_{j=1}^{m} f_j(v_j)|$$

$$\geq \alpha_i |x_i - u_i| - \sum_{j=1}^{m} |a_{ij}||f_j(y_j) - f_j(v_j)| - \sum_{j=1}^{m} |b_{ij}||f_j(y_j) - f_j(v_j)| - \sum_{j=1}^{m} |c_{ij}||f_j(y_j) - f_j(v_j)|$$

$$\geq \alpha_i |x_i - u_i| - \sum_{j=1}^{m} |a_{ij}| F_j |y_j - v_j| - \sum_{j=1}^{m} |b_{ij}| F_j |y_j - v_j| - \sum_{j=1}^{m} |c_{ij}| F_j |y_j - v_j|$$

$$= [\alpha_i, \quad -\sum_{j=1}^{m} (|a_{ij}| + |b_{ij}| + |c_{ij}|) F_j] \begin{bmatrix} |x_i - u_i| \\ |y_j - v_j| \end{bmatrix}, \ i = 1,2,\cdots,n . \quad (3a)$$

Similar to the above, we have

$$|H_{n+j}(y_j) - H_{n+j}(v_j)| \geq [-\sum_{i=1}^{n} (|d_{ji}| + |k_{ji}| + |l_{ji}|) G_i, \quad \beta_j] \begin{bmatrix} |x_i - u_i| \\ |y_j - v_j| \end{bmatrix}, \ j = 1,2,\cdots,m . \quad (3b)$$

From (3), we get

$$|H(x,y) - H(u,v)| \geq \begin{bmatrix} \alpha & -(|A| + |B| + |C|) F \\ -(|D| + |W| + |Z|) G & \beta \end{bmatrix} \begin{bmatrix} |x-u| \\ |y-v| \end{bmatrix} = \Omega \begin{bmatrix} |x-u| \\ |y-v| \end{bmatrix}.$$

By the supposition that $H(x,y) = H(u,v)$, we obtain $\Omega \begin{bmatrix} |x-u| \\ |y-v| \end{bmatrix} \leq 0$. Since Ω is an M-matrix, from Lemma 1, we know that all elements of Ω^{-1} are non-negative. Hence, $|x-u| = 0$, $|y-v| = 0$ i.e., $x = u$, $y = v$, which is a contradiction. So map $H(u,v)$ is injective.

In the second step, we prove that $\lim_{\|(u,v)\| \to \infty} \|H(u,v)\| \to \infty$. If $f(v)$ and $g(u)$ are bounded, it is easy to verify that when $\|(u,v)\| \to +\infty$, $\|H(u,v)\| \to +\infty$. In the following, we will discuss the case that $f(v)$ or $g(u)$ is unbounded. Let $\overline{H}(u,v) = H(u,v) - H(0,0)$ .To prove that $H(u,v)$ is a homeomorphism, it suffices to show that $\overline{H}(u,v)$ is a homeomorphism. Because of Ω is an M-matrix, from Lemma 1, there exists a positive define diagonal matrix $Q = \mathrm{diag}(q_1, q_2, \cdots, q_{n+m})$, such that

$$[Q\Omega]^s \leq -\varepsilon E_{n+m} < 0 \quad (4)$$

for sufficiently small ε>0. E_{n+m} is the identity matrix. By Assumption (A), Lemma 2 and (4), we get

$$[u^{\mathrm{T}}, v^{\mathrm{T}}] Q \overline{H}(u,v)$$

$$= \sum_{i=1}^{n} u_i q_i \{-\alpha_i u_i + \sum_{j=1}^{m} a_{ij}[f_j(v_j) - f_j(0)] + \bigwedge_{j=1}^{m} b_{ij} f_j(v_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(0)$$

$$+ \bigvee_{j=1}^{m} c_{ij} f_j(v_j) - \bigvee_{j=1}^{m} c_{ij} f_j(0)\} + \sum_{j=1}^{m} v_j q_{n+j} \{-\beta_j v_j + \sum_{i=1}^{n} d_{ji}[g_i(u_i) - g_i(0)]$$

$$+ \bigwedge_{i=1}^{n} w_{ji} g_i(u_i) - \bigwedge_{i=1}^{n} w_{ji} g_i(0) + \bigvee_{i=1}^{n} z_{ji} g_i(u_i) - \bigvee_{i=1}^{n} z_{ji} g_i(0)\}$$

$$\leq \sum_{i=1}^{n} q_i \{-\alpha_i u_i^2 + |u_i| [\sum_{j=1}^{m} |a_{ij}||f_j(v_j) - f_j(0)| + |\bigwedge_{j=1}^{m} b_{ij} f_j(v_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(0)|$$

$$+ |\bigvee_{j=1}^{m} c_{ij} f_j(v_j) - \bigvee_{j=1}^{m} c_{ij} f_j(0)|]\} + \sum_{j=1}^{m} q_{n+j} \{-\beta_j v_j^2 + |v_j| [\sum_{i=1}^{n} |d_{ji}||g_i(u_i) - g_i(0)|$$

$$+ |\bigwedge_{i=1}^{n} w_{ji} g_i(u_i) - \bigwedge_{i=1}^{n} w_{ji} g_i(0)| + |\bigvee_{i=1}^{n} z_{ji} g_i(u_i) - \bigvee_{i=1}^{n} z_{ji} g_i(0)|]\}$$

$$\leq \sum_{i=1}^{n} q_i \{-\alpha_i u_i^2 + |u_i| [\sum_{j=1}^{m} |a_{ij}||f_j(v_j) - f_j(0)| + \sum_{j=1}^{m} |b_{ij}||f_j(v_j) - f_j(0)|$$

$$+\sum_{j=1}^{m}|c_{ij}||f_j(v_j)-f_j(0)|]\} +\sum_{j=1}^{m}q_{n+j}\{-\beta_j v_j^{\,2}+|v_j|[\sum_{i=1}^{n}|d_{ji}||g_i(u_i)-g_i(0)|$$

$$+\sum_{i=1}^{n}|w_{ji}||g_i(u_i)-g_i(0)|+\sum_{i=1}^{n}|z_{ji}||g_i(u_i)-g_i(0)|]\}$$

$$\le \sum_{i=1}^{n}q_i\{-\alpha_i u_i^{\,2}+|u_i|[\sum_{j=1}^{m}|a_{ij}|F_j|v_j|+\sum_{j=1}^{m}|b_{ij}|F_j|v_j|+\sum_{j=1}^{m}|c_{ij}|F_j|v_j|]\}$$

$$+\sum_{j=1}^{m}q_{n+j}\{-\beta_j v_j^{\,2}+|v_j|[\sum_{i=1}^{n}|d_{ji}|G_i|u_i|+\sum_{i=1}^{n}|w_{ji}|G_i|u_i|+\sum_{i=1}^{n}|z_{ji}|G_i|u_i|]\}$$

$$=[|u|^{\mathrm{T}},|v|^{\mathrm{T}}]C\begin{bmatrix}-\alpha & 0\\ 0 & -\beta\end{bmatrix}\begin{bmatrix}|u|\\|v|\end{bmatrix}+[|u|^{\mathrm{T}},|v|^{\mathrm{T}}]Q\begin{bmatrix}0 & (|A|+|B|+|C|)F\\ (|D|+|W|+|Z|)G & 0\end{bmatrix}\begin{bmatrix}|u|\\|v|\end{bmatrix}$$

$$=[|u|^{\mathrm{T}},|v|^{\mathrm{T}}]Q\begin{bmatrix}-\alpha & (|A|+|B|+|C|)F\\ (|D|+|W|+|Z|)G & -\beta\end{bmatrix}\begin{bmatrix}|u|\\|v|\end{bmatrix}$$

$$=[|u|^{\mathrm{T}},|v|^{\mathrm{T}}][Q\Omega]^{s}\begin{bmatrix}|u|\\|v|\end{bmatrix}\le -\varepsilon\|(u,v)\|^2 . \tag{5}$$

Using Schwartz inequality, from (5), we get

$$\varepsilon\|(u,v)\|^2 \le \|C\|\|(u,v)\|\|\overline{H}(u,v)\|,$$

namely, $\dfrac{\varepsilon\|(u,v)\|}{\|C\|}\le\|\overline{H}(u,v)\|$. So when $\|(u,v)\|\to+\infty$, $\|\overline{H}(u,v)\|\to+\infty$, i.e., $\|H(u,v)\|\to+\infty$.

From the above two steps, according to Lemma 3, we know that for any pair of input (I,J) , map $H(u,v)$ is a homeomorphism on R^{n+m} . Hence system (1) has a unique equilibrium point.The proof is completed.

Above, we analyze the existence and uniqueness of the equilibrium point of system(1). Further, suppose Assumption (A) and Assumption (B) hold, then it is easy to prove the stability of the equilibrium point of system (1), if Ω is an M-matrix.

IV. CONCLUSION

In this paper, by using M-matrix theory and Liapunov functions, we present a new condition ensuring existence, uniqueness of the equilibrium point of bidirectional associative memory neural networks with fuzzy logic and distributed time delays. The result is applicable to both symmetric and nonsymmetric interconnection matrices, and all continuous non-monotonic neuron activation functions. It is easy to test the conditions of the criterion in practice.

REFERENCE

[1]. J. Hopfield. Neuron with graded response have collective computational properties like those of two-state neurons. Proc NatlAcad Sci USA, 1984, 81:3088–92.

[2]. B. Kosko. Aadaptive bi-directional associative memories. Appl. Opt., 1987, 26: 4947-4960.

[3]. X. Mao, X. Wang, H. Qin. Stability analysis of quaternion-valued BAM neural networks fractional-order model with impulses and proportional delays. Neurocomputing, 2022, 509(14), 206–220.

[4]. C. Huang, J. Wang, X. Chen, et al. Bifurcations in a fractional-order BAM neural network with four different delays. Neural Networks, 2021,141:344–354.

[5]. C. Huang, H. Wang, H. Liu , et al. Bifurcations of a delayed fractional-order BAM neural network via new parameter perturbations. Neural Networks, 2023, 168:123–142.

[6]. A. N. Michel, J. A. Farrell, W. Porod. Qualitative analysis of neural networks. IEEE Trans. Circuits Syst., 1989, 36: 229-243.

[7]. T. Huang. Exponential stability of fuzzy cellular neural networks with distributed delay. Physics Letters A, 2006,351:48–52.

[8]. J. Zhang, Y. Yang. Global stability analysis of bidirectional associative memory networks with time delay, Int. J. Circuit Theory Appl.,2001, 29: 185–196.