The International Journal of Engineering and Science (IJES)
|| Volume || 12 || Issue || 11 || Pages || PP 65-75 || 2023 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Thermal radiative MHD Flow, heat and mass transfer over vertical plate with internal heat generation and chemical reaction

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ABSTRACT
In this paper, we investigated the magnetohydrodynamics (MHD) boundary layer flow with heat and mass transfer over a moving vertical plate in the presence of magnetic field, thermal radiation, heat absorption/injection, chemical reaction and a convective heat exchange at the surface with the surrounding. Partial differential equation describing the problem is formulated and the similarity variable is used to transform the system of partial differential equations, describing the problem under consideration, into a boundary value problem of coupled ordinary differential equations, and an efficient numerical technique with help of Maple is implemented to solve the reduced system. The results are presented graphically and tabularly and the conclusion is drawn that the flow velocity, temperature and concentration and other quantities of physical interest are significantly influenced by these parameters and the results are compared with some related existing works in the literature. The results are in excellent agreement. The research revealed that velocity profiles decrease monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition. It was discovered that both the Nusselt number and the Sherwood number increases while the magnitude of the local skin friction decreases with an increase in the internal heat generation parameter.

Keywords**: **Magnetohydrodynamics, Thermal Radiation, Injection, Nusselt, Sherwood

Date of Submission: 06-11-2023 Date of acceptance: 20-11-2023

I. INTRODUCTION

The magnetohydrodynamic (MHD) heat and mass transfer processes over a moving surface are of interest engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched into a cooling system; the fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymer fluids like polyethylene oxide and polyisobutylene solution in cetane, having better electromagnetic properties, are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. In a pioneering work, Sakiadis (1961a, b) investigated the boundary layer flow induced by a moving plate in a quiescent ambient fluid. Thereafter, various aspects of the problem have been investigated by many authors. A comprehensive review on the subject of the above problem has been made by many researchers (Fang, 2003; Fang and Lee, 2005; White, 2006; Magyari, 2008). Makinde (2010) investigated on MHD heat and mass transfer over a moving vertical plate with a convective surface boundary conditions. The similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media as explained by Chamkha and Khaled (2000). Seddeek (2001) studied the thermal radiation and buoyancy effects on MHD free convection heat generation flow over an accelerating permeable surface with temperature-dependent viscosity. Postelnicu (2004) numerically studied the influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media by considering the Soret and Dufour effects. Makinde (2005) carried out a numerical study on the effect of thermal radiation on boundary layer flow with heat and mass transfer past a moving vertical porous plate. Cortell (2007a) theoretically examined the flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species. The numerical results show that the effect of destructive chemical reaction is to diminish the concentration boundary layer and this phenomenon is quite the opposite when a generative reaction is present. In a similar study, Cortell (2007b) examined the flow and mass diffusion of chemical species with first and higher order reactions of two electrically conducting viscoelastic fluids over a porous stretching sheet with an applied magnetic field. He reported that an increase in the order of chemical reaction will produce a decrease in the concentration boundary layer thickness when the reaction rate is negative, and the opposite trend is true for the case of reaction rate is positive. The effect of thermal radiation on heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field was reported in Makinde and Ogulu (2008). Bataller (2008) investigated the effect of thermal radiation on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Rajeswari et al. (2009) studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a stationary vertical porous surface in the presence of suction with power law surface temperature and concentration. Their numerical results reveal that the temperature of the fluid decreases and the concentration of the fluid increases with an increase in buoyancy parameter, however, a slight error were observed in their velocity profiles due to technical error in their model formulation and this invariably affects all other results in their paper. Aziz (2009) reported a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. The paper demonstrates that a similarity solution is possible if the convective heat transfer associated with the hot fluid on the lower surface of the plate is proportional to the inverse square root of the axial distance. In the present study, our objective is to extend the recent work of Makinde (2008) to include internal heat generation and thermal radiation in the energy equation and chemical reaction in the mass equation over a moving vertical plate with a convective surface boundary condition. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables, and these have been solved numerically using Rungen Kutta scheme implemented with the aid Maple. The effects of embedded parameters on fluid velocity, temperature, and concentration have been shown graphically and tabularly. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies. Also, our results were validated by comparing with Makinde (2010).

MATHEMATICAL MODEL

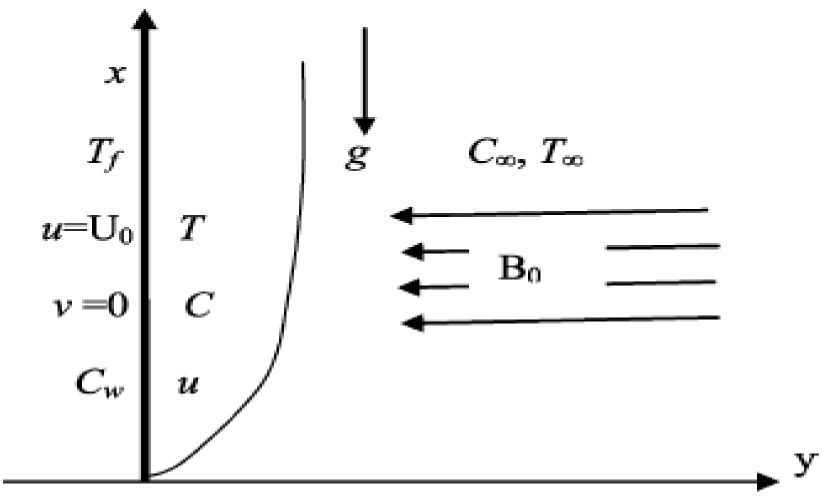


Fig 1: Problem Geometry

Let us consider a boundary layer flow with heat and mass transfer over amoving vertical plate in a stream of cold fluid at temperature $T\infty$ in the presence of magnetic field. We assume the left surface of the plate is heated by convection from a hot fluid at temperature Tf which provides a heat transfer coefficient hf . The cold fluid on the right side of the plate is assumed to be Newtonian, electrically conducting and its property variations due to temperature and chemical species concentration are limited to fluid density. In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected (see Figure 1). Since the magnetic Reynolds number is very small for most fluid used in industrial applications,we assume that the induced magnetic field is negligible. Let the *x*-axis be taken along the direction of plate and *y*axis normal to it. If *u*, *v*, *T*, and *C* are the fluid *x*-component of velocity, *y*-component of velocity, temperature and concentration, respectively, then under the Boussinesq and boundary layer approximations, the governing equations for this problem can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta_T\left(T-T_\infty\right)++g\beta_c\left(C-C_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sigma\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr(C-C_\infty) \quad (4)$$

Where v is the kinematic viscosity, C∞ is the free stream concentration, U_O is the plate velocity, is the thermal diffusivity and D is the mass diffusivity, β_T is the thermal expansion coefficient, β_C is the solutal expansion coefficient, C_p is the heat capacity, ρ is the fluid density, g is gravitational acceleration, and σ is the fluid electrical conductivity, q_r is the radiative heat flux, Q_0 is the heat generation coefficient, chemical reaction. The boundary conditions at the plate surface and far into the cold fluid may be written as:

$$u(x,0)=U_0 \qquad v(x,0)=0, \qquad -k\frac{\partial T}{\partial y}(x,0)=h_1[T_1-T(x,0)] \quad (5)$$

$$C_w(x,0)=Ax^b+C_\infty, \quad u(x,\infty)=0, \quad T(x,\infty)=T_\infty, C(x,\infty)=C_\infty,$$

where L is the plate characteristic length, C_w is the species concentration at the plate surface, is the plate surface concentration exponent, and k is the thermal conductivity coefficient. The stream function ψ, satisfies the continuity Equation (1) automatically with

$$u=\frac{\partial \psi}{\partial y} \text{ and } v=-\frac{\partial \psi}{\partial x}$$

A similarity solution of Equations (1)–(6) is obtained by defining an independent variable and a dependent variable f in terms of the stream function as:

$$\eta=y\sqrt{\frac{U_0}{vx}}, \qquad \phi(\eta)=\frac{C-C_w}{C_w-C_\infty} \quad (6)$$

The dimensionless temperature and concentration are given as:

$$\phi(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \qquad \phi(\eta)=\frac{C-C_w}{C_w-C_\infty} \quad (7)$$

After introducing Equations (6)–(8) into Equations (1)–(5), we obtain:

$$f'''+\frac{1}{2}ff''-Mf'+Gr\theta+Gr\phi=0 \quad (8)$$

$$\theta''+\frac{1}{2}\Pr f\theta'+Q\theta-Ra\theta'=0 \quad (10)$$

$$\phi''+\frac{1}{2}Scf\phi'+Kc\phi=0 \quad (11)$$

$$f(0)=0, \quad f(0)=1, \quad \theta'=Bi[\theta(0)-1], \quad \phi(0)=1 \quad (12)$$

$$f'(\infty)=0, \qquad \theta(0)=0, \qquad \phi(0)=0 \quad (13)$$

Where the prime symbol represents the derivative with respect to η

$$M=\frac{\sigma B_0^2 x}{\rho U_0}, \quad Gc=\frac{gB_T(T_f-T_\infty)}{U_0^2}, \quad Gc=\frac{gB_T(C_w-C_\infty)}{U_0^2},$$

$$Bi=\frac{h_f}{k}\sqrt{\frac{vx}{U_0}}, \ \Pr=\frac{v}{\alpha}, \ \Pr=\frac{v}{D} \ Ra=\frac{16\sigma T_\infty^3}{3k^*Cp}, \quad Q=\frac{v^2 Q_0}{\sigma\rho C_p}, \quad Kc=\frac{vkr}{U_0^2}$$

Where M is the magnetic parameter, Gr is the local thermal Grashof number , Gr is the local solutal Grashof number , Bi is the local convective heat transfer parameter , Pr is the Prandtl and Sc is the Schmidt number, Ra is the thermal radition, Qis the internal heat generation and kc is the chemical reaction

The set of Equations (9)–(11) under the boundary conditions (12) and (13) have been solved numerically by applying the Nachtsheim and Swigert (1965) shooting iteration technique together with Runge–Kutta sixth-order integration scheme. From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are, respectively, proportional to, are also worked out and their numerical values are presented in a tabular form

II. Result and discussion

Table 1. Computation showing $f''(0)$, $\theta'(0)$, and $\phi'(0)$ for various values of embedded parameter when compared with Makinde (2010)

						$f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
Bi	Gr	Gc	M	Pr	Sc	Makinde (2010)	Present work	Makinde (2010)	Present work	Makinde (2010)	Present work
0.1	0.1	0.1	0.1	0.72	0.62	−0.402271	−0.402262	0.078635	0.078634	0.3337425	0.3337434
1.0	0.1	0.1	0.1	0.72	0.62	−0.352136	−0.352140	0.273153	0.273152	0.3410294	0.3410293
10	0.1	0.1	0.1	0.72	0.62	−0.329568	−0.329556	0.365258	0.365258	0.3441377	0.3441376
0.1	0.5	0.1	0.1	0.72	0.62	−0.322212	−0.322220	0.079173	0.079174	0.3451301	0.3451303
0.1	1.0	0.1	0.1	0.72	0.62	−0.231251	−0.231260	0.079691	0.079692	0.3566654	0.3566653
0.1	0.1	0.5	0.1	0.72	0.62	−0.026410	−0.026421	0.080711	0.080713	0.3813954	0.3813953
0.1	0.1	1.0	0.1	0.72	0.62	0.3799184	0.3799183	0.082040	0.082042	0.4176697	0.4176696
0.1	0.1	0.1	1.0	0.72	0.62	−0.985719	−0.985720	0.074174	0.074172	0.2598499	0.2598498
0.1	0.1	0.1	5.0	0.72	0.62	−2.217928	−2.217930	0.066156	0.066158	0.1806634	0.1806633
0.1	0.1	0.1	0.1	1.00	0.62	−0.407908	−0.407910	0.081935	0.081938	0.3325180	0.3325181
0.1	0.1	0.1	0.1	7.10	0.62	−0.421228	−0.421225	0.093348	0.093346	0.3305618	0.3305613
0.1	0.1	0.1	0.1	0.72	0.78	−0.411704	−0.411703	0.078484	0.078483	0.3844559	0.3844558
0.1	0.1	0.1	0.1	0.72	2.63	−0.453094	−0.453092	0.077915	0.077917	0.7981454	0.7981453

Table 1. showing Computation of local Skin friction $f''(0)$, local Nusselt number $\theta'(0)$ and local Sherwood number $\phi'(0)$ when compared with makinde(2010). It is interesting to note that there is excellent agreement with our result and Makinde (2010). This once again justify the accuracy of our result.

Effects of Parameter Variation on Velocity Profiles

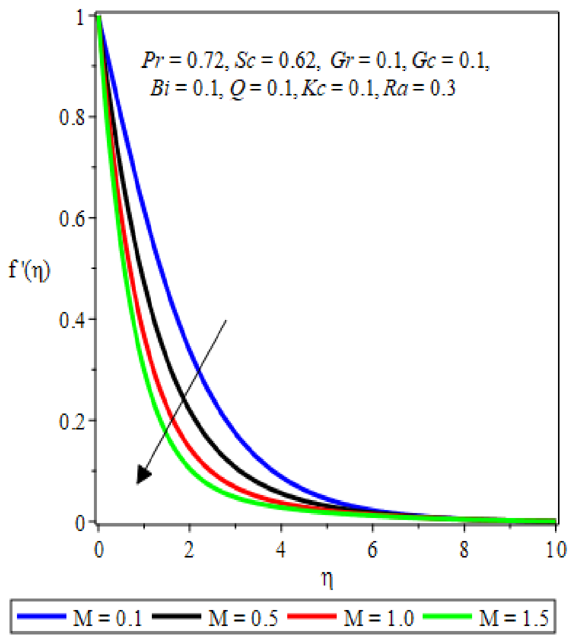


Fig 2: Influence of M on velocity profile

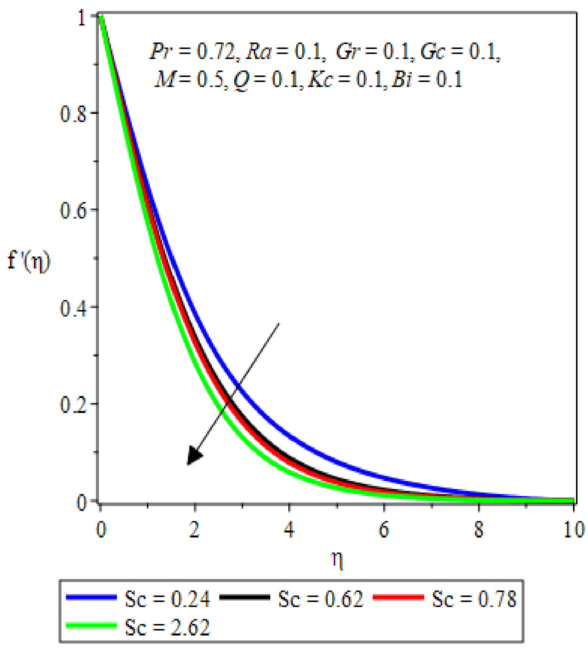


Fig 3: Influence of on Sc on velocity profile

The numerical results for the velocity profiles are displayed in Figures 2–9. The effects of magnetic field parameter (*M*) on the velocity field are shown in Figure 2. It is seen from this figure that the velocity profiles decrease monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition. Moreover, it is interesting to note in Figure 2 that the effect of increasing magnetic field parameter (*M*) is to decrease the value of the velocity profiles throughout the boundary layer. The presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. This result qualitatively agrees with the results obtain in Makinde (2010), since the magnetic field exerts retardisng force on the free convection flow. Similar trend of decrease in fluid velocity profile is observed in fig 3 with increase in Schmitdt number.

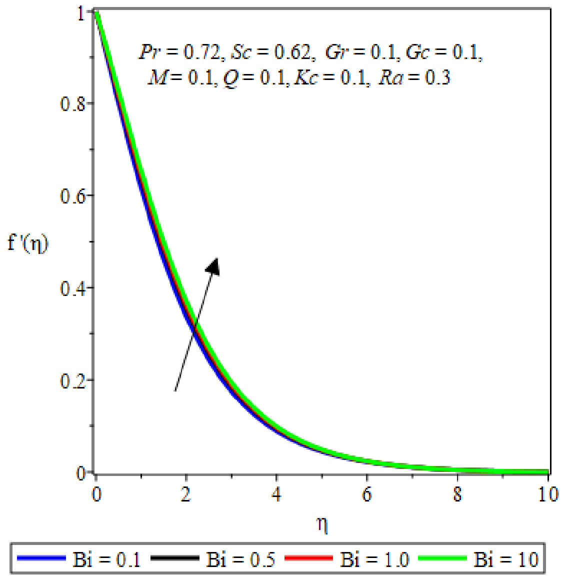


Fig 4:Influence of Bi on velocity profile

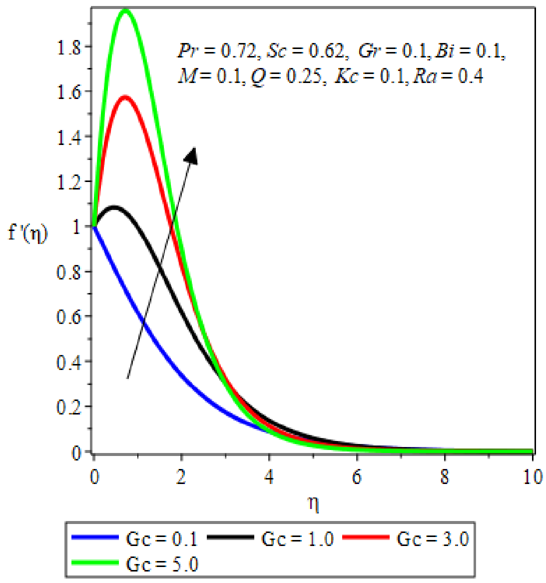


Fig 5: Influence of Gc on velocity profile

In Figure 4, a slight increase in the velocity profiles within the boundary layer is observed with and increase in the convective heat exchange parameter (*Bi*). Since the fluid on the right surface of the plate is heated up by the hot fluid on the left side of the plate, making it to become lighter

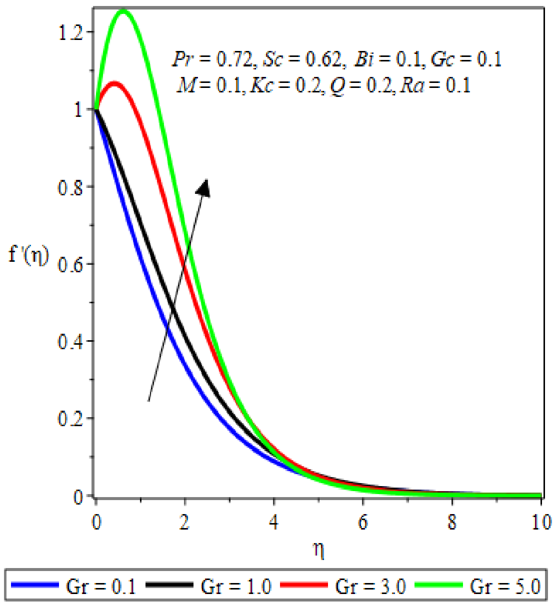


Fig 6: Influence of Gr on velocity profile

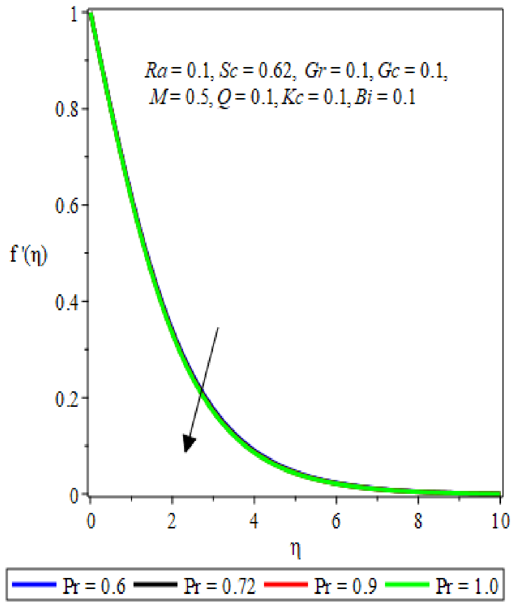


Fig 7:Influence of $\Pr$ on velocity profile

and flow faster. The effects of both thermal Grashof Gr and solutal Grashof Gc numbers are shown in Figures 5 and 6. As the Grashof number increases, the fluid velocity increases, reaching its peak value within the boundary layer and then decreases monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition. From these figures, it is

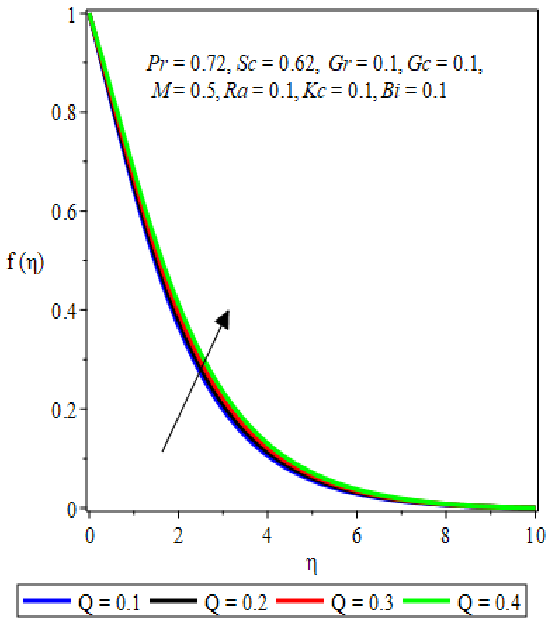


Fig 8: Influence of Q on velocity profile

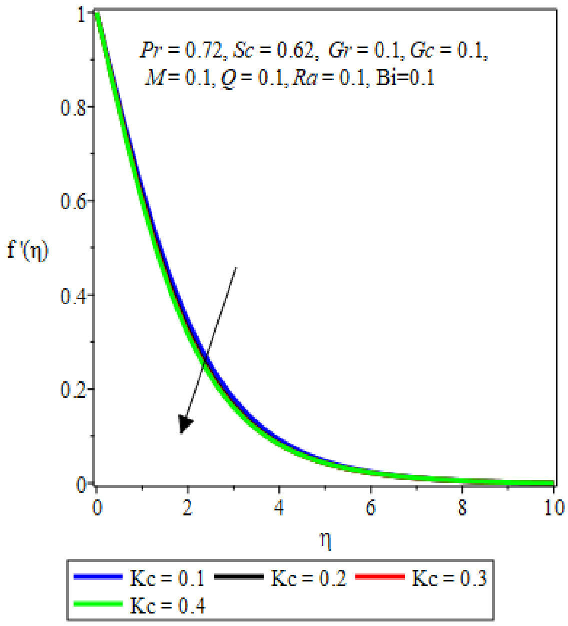


Fig 9: Influence of Kc on velocity profile

obvious that the buoyancy. This result also in agreement with Makinde(2010). In figure 7, increase in Prandtl number resulted in small decrease in velocity profiles while opposite effect is observed is observed in Figure 7 in which increase in heat absorption lead to increase in velocity profile. In figure 9, increase in chemical reaction correspond to decrease in fluid velocity because the particle in the fluid tend to closely packed together and resulted in slow down the fluid movement .In figure 10, increase in thermal radiation result in increase in thermal radiation.

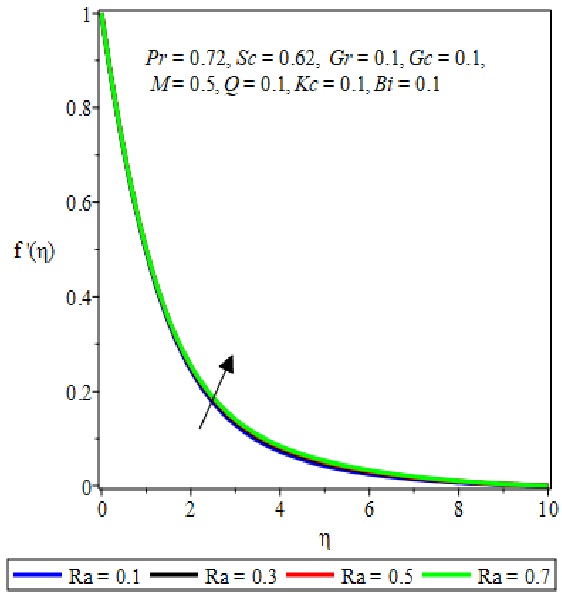


Fig 10: Influence of Ra on velocity profile

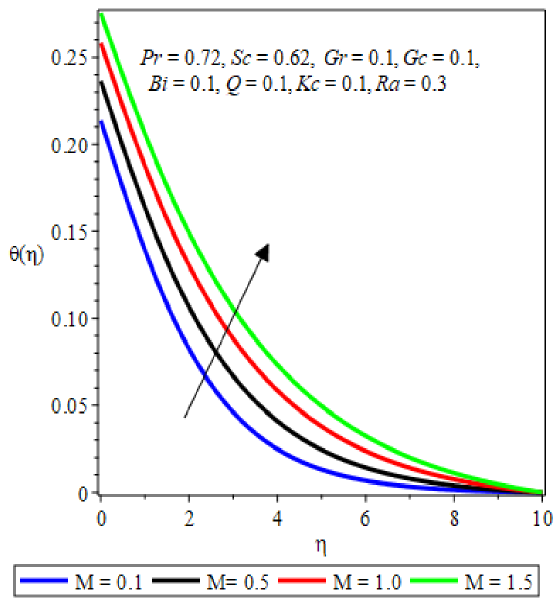


Fig 11: Influence of M on temperature profile

Effects of Parameter Variation on Temperature Profiles

The numerical results for the temperature profiles are shown in Figures 11–19. It is seen from these figures that the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary conditions. It is noteworthy that the thermal boundary layer thickness increases with an increase in the intensity of the magnetic field parameter (M), plate surface convective heat exchange parameter (Bi) and Schmidt number (Sc). Increase in the intensity of buoyancy forces (Gr, Gc) and the Prandtl number (Pr) causes a decrease in the fluid temperature leading to leading to a decaying thermal boundary layer thickness. The results in here agreed with the one reported recently in Makinde

(2010). Increase in thermal radiation parameter Ra, heat source parameter Q and chemical reaction parameter increases and enhance fluid temperature.

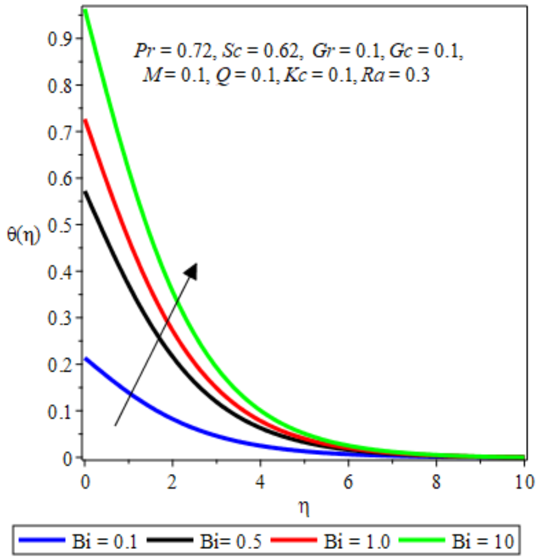


Fig 12: Influence of Sc on temperature profile

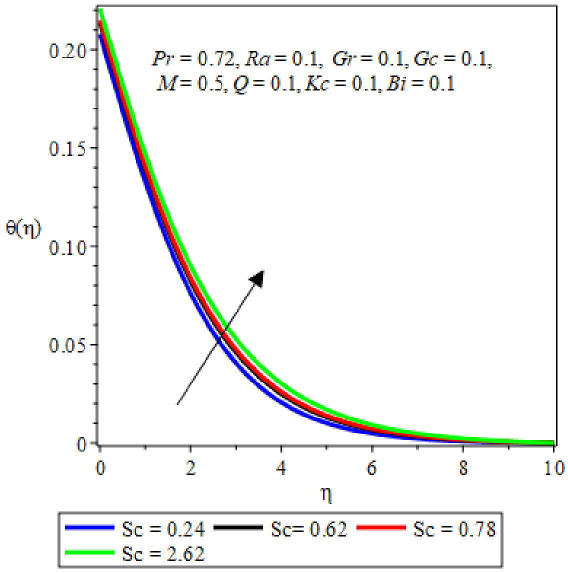


Fig 13: Influence of Bi on temperature profile

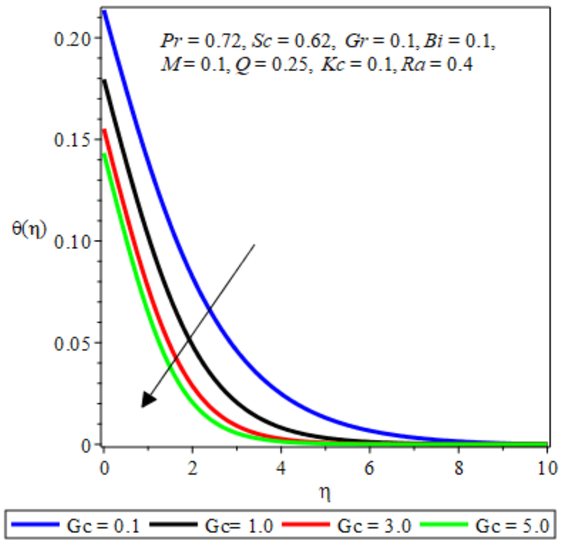


Fig 14: Influence of Gr on temperature profile

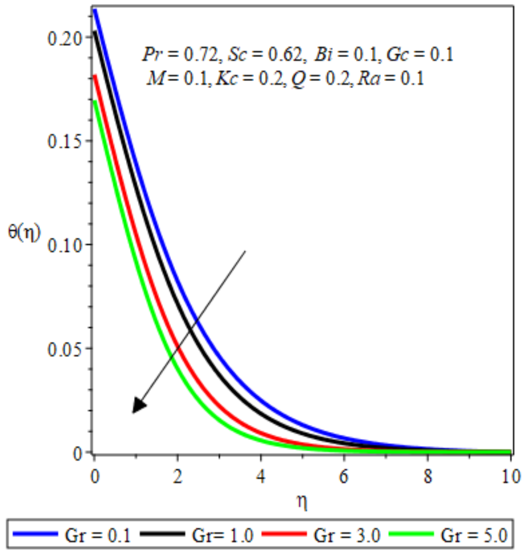


Fig 15: Influence of Gc on temperature profile

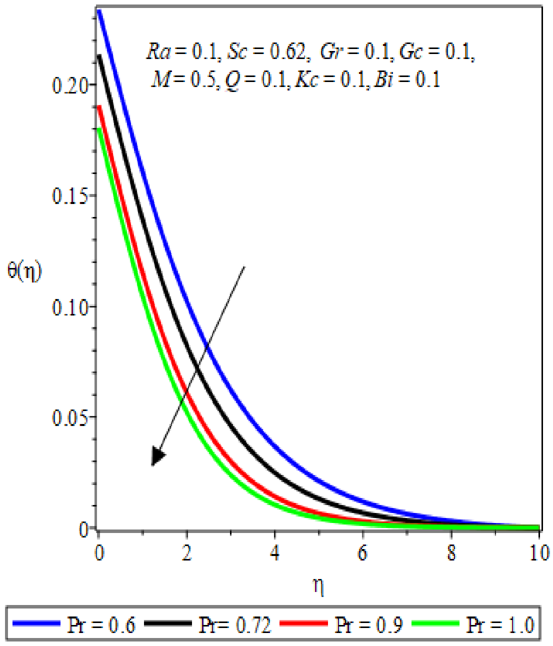


Fig16:Influence of Ra on temperature profile

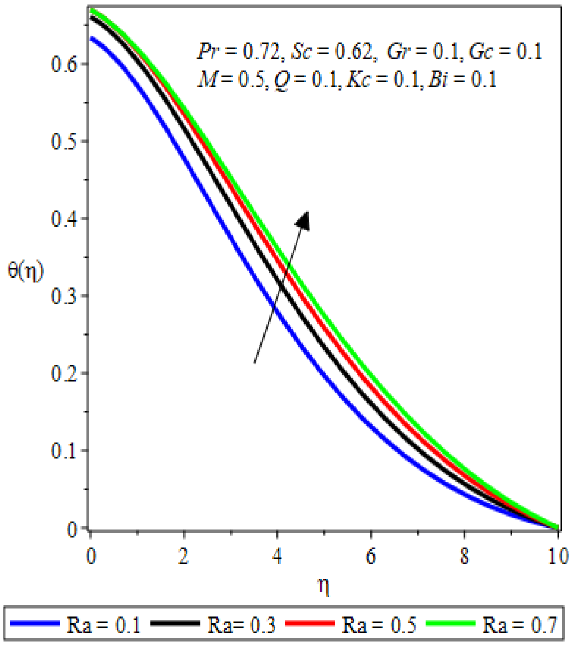


Fig17:Influence of Ra on temperature profile

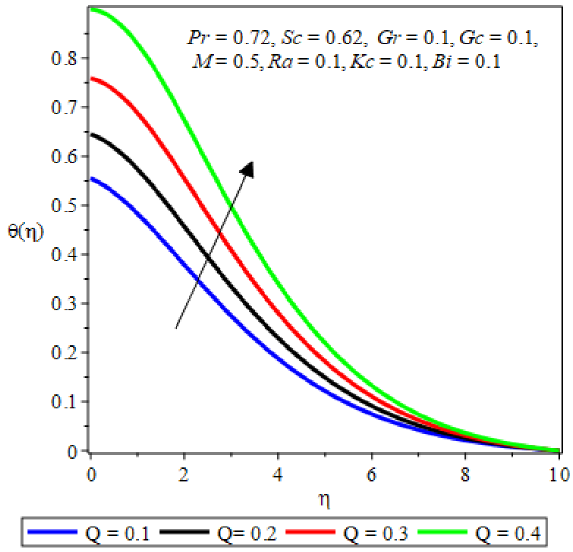


Fig18:Influence of Q on temperature profile

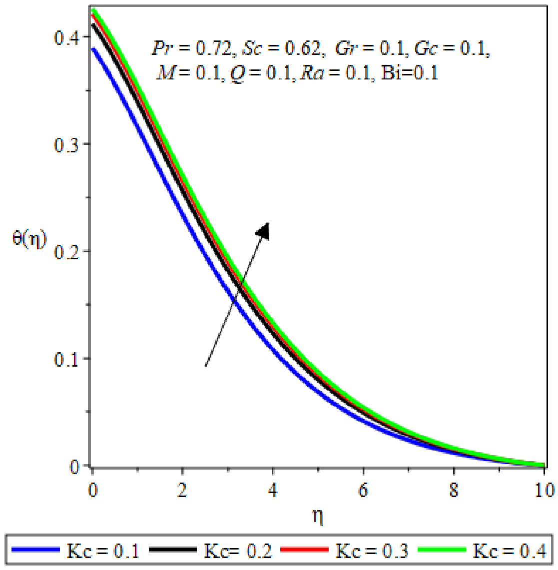
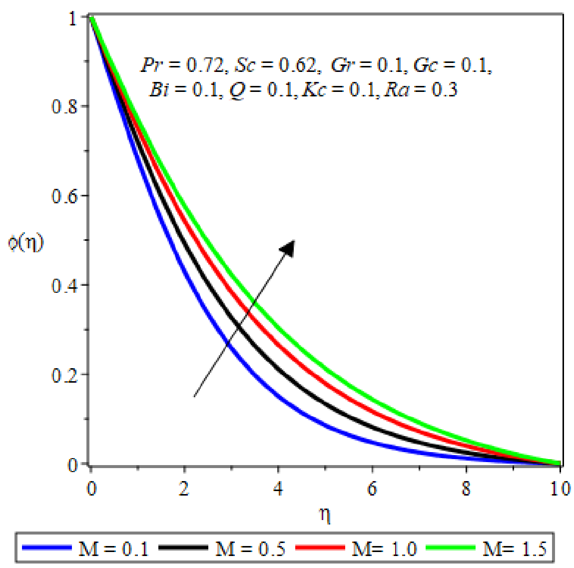


Fig19:Influence of Kc on temperature profile

Effects of Parameter Variation on Concentration Profiles

Concentration profiles for different values physical parameters in the boundary layer are shown in Figures 20–28. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the



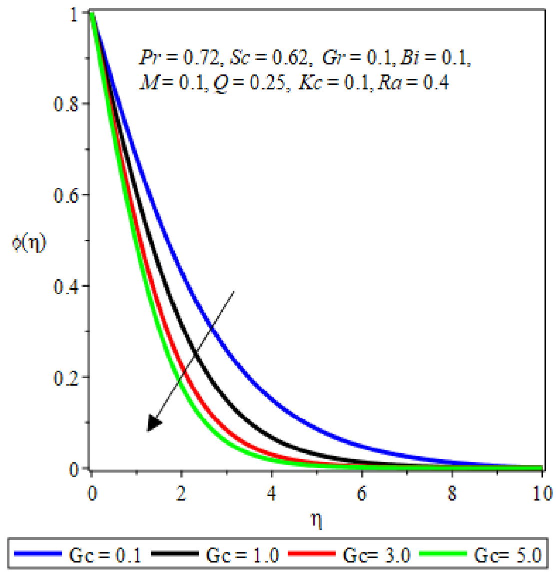
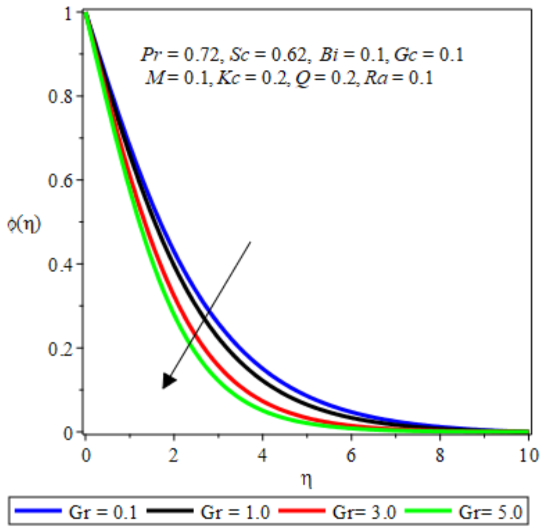


Fig 20: Influence of M on concentration profile   Fig 21: Influence of Gc on concentration profile

concentration boundary layer thickness increases with an increase in the magnetic field intensity (*M*). An increase in the values of thermal and solutal Grashof numbers (*Gr*, *Gc*) due to buoyancy forces causes a decrease in the concentration boundary layer thickness. Moreover, as the Schmidt



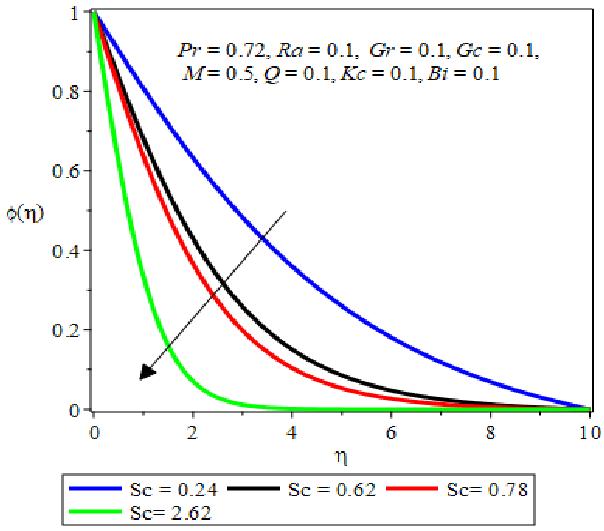


Fig 22: Influence of Sc on concentration profile   Fig 23: Influence of $\Pr$ on concentration profile

number (Sc) increases due to a decrease in the chemical species molecular diffusivity, the thickness of the concentration boundary layer decreases. The result are also in agreement with Makinde (2010). Finally, In figures 25, 26, 27 and 28, increase in thermal radiation Ra, heat source , chemical reaction and convective parameter resulted in decrease concentration profiles of the fluid.s

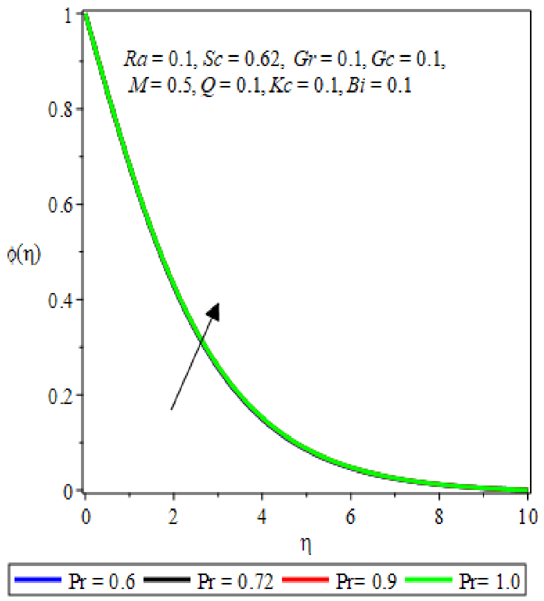


Fig 24: Influence of $\Pr$ on concentration profile

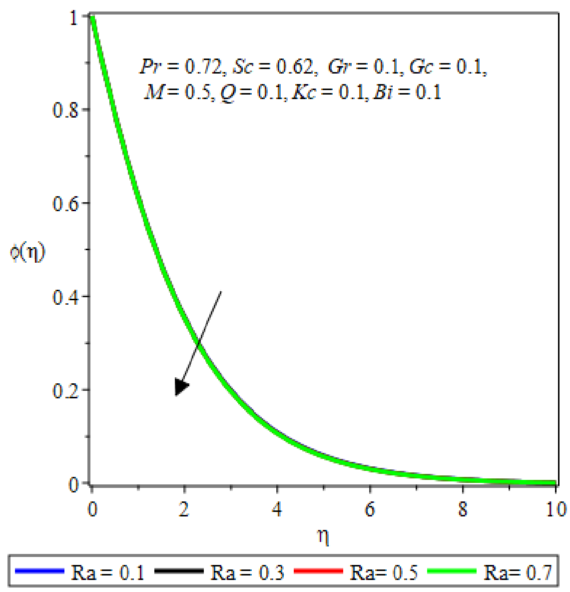


Fig 25: Influence of Ra on concentration profile

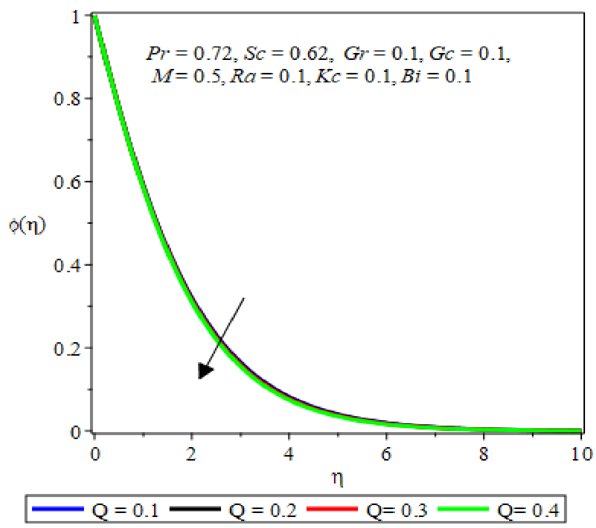


Fig 26: Influence of Q on concentration profile

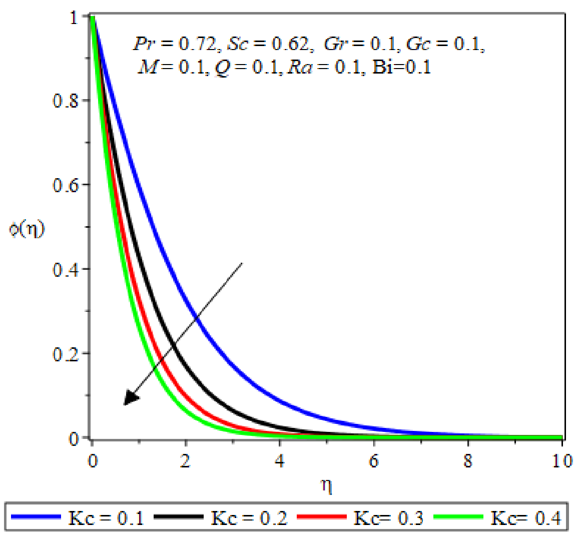


Fig 27: Influence of Kc on concentration profile

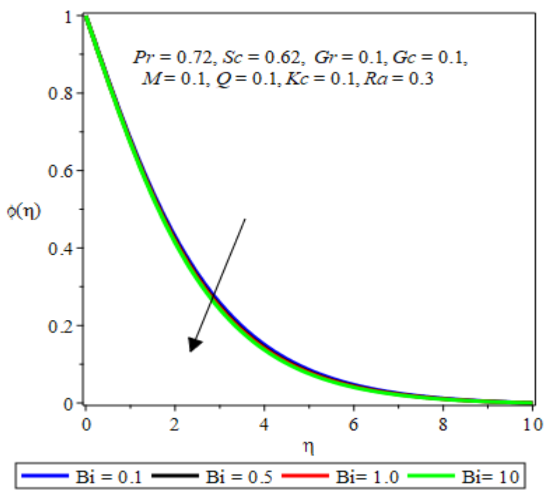


Fig 28: Influence of Bi on concentration profile

III. CONCLUSION

In this paper, the hydromagnetic boundary layer flow with heat and mass transfer over a moving vertical plate with internal heat generation, thermal radiation and chemical reaction. The governing equations are transformed via similarity variable to a system of nonlinear ordinary differential equations. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It was found that both the local Nusselt number and the local Sherwood number increases while the magnitude of the local skin friction decreases with an increase in the convective heat exchange (Bi) at both sides of the plate surface. An increase in magnetic field intensity causes an increase in the concentration and thermal boundary layer thickness whiles the velocity boundary layer thicknesses decreases.

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