

# Strategic Optimization and Performance Analysis for Urban Electrical Transmission Networks

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## I. Introduction

This research focuses on a particular transmission network relevant to an urban area that is transforming swiftly. The selected network offers electricity to households, industries, and critical infrastructure, acting as an essential resource for the site. This case study was justified courtesy of its direct application to practical situations. Considering the nuances of this network is essential given the evolving energy needs and the requirement for a stable, sustainable power source. Through an in-depth analysis of this particular transmission system, critical information can be determined to solve emerging concerns that urban areas around the world are facing. This paper advances academic knowledge while also having practical implications that will direct future expansion plans and ensure the continuous supply of energy in critical urban areas.

## II. Literature Review

Chakrabarti and Halder's (2022) work provides vital insights for power system analysis and control, which are critical knowledge for understanding the operational components of electrical networks. The 2019 systematic review by Mosavi et al. on machine learning models in energy systems provides a sophisticated viewpoint by illustrating the developments in the utilization of machine learning methods for process optimization connected to energy. This systematic review is a relevant tool that provides examples of cutting-edge approaches that can be applied to energy systems research.

Additionally, a notable article on the estimation of daily dew point temperature using machine learning algorithms is presented by Qasem et al. (2019), illustrating the essentiality of machine learning in environmental parameter prediction. Their study highlights the flexibility and variety of machine learning protocols and how they might enhance the accuracy of environmental forecasts in energy systems. Additionally, Nosratabadi et al. (2019), who concentrate on incorporating sustainability concepts into energy system design, provide insightful information about sustainable business models. Their assessment provides a comprehensive viewpoint essential for forming progressive expansion plans within electrical power transmission networks, highlighting the importance of executing ecologically friendly practices and matching commercial objectives with sustainable development goals.

Furthermore, Fardad et al.'s (2018) study on the biodegradation of medicinal plant waste in anaerobic digestion reactors for biogas production exhibits a novel method for producing renewable energy. Through exploring eco-friendly biogas production techniques, these studies highlight the importance of environmentally friendly procedures in the energy industry. Together, these varied investigations add value to the body of literature and strengthen the groundwork for the second research study, centered on case studies and strategic growth planning in electrical power transmission networks.

## III. Results

### 3.1 Load Growth Prediction

Load growth prediction uses the known values of load from previous years. Load growth can be quite accurately approximated by an exponential function. To get the parameters of the function a minimum square method is used.

$$y = a \times b^x \quad (4.1)$$

$$\sum_{i=1}^n \ln y_i = n \times \ln a + \ln b \times \sum_{i=1}^n x_i \quad (4.2)$$

$$\sum_{i=1}^n x_i \times \ln y_i = \ln a \times \sum_{i=1}^n x_i + \ln b \times \sum_{i=1}^n x_i^2 \quad (4.3)$$

$$\ln a = \frac{\sum_{i=1}^n \ln y_i - \ln b \times \sum_{i=1}^n x_i}{n} \quad (4.4)$$

$$\ln b = \frac{\sum_{i=1}^n x_i \times \ln y_i - \frac{\sum_{i=1}^n \ln y_i - \ln b \times \sum_{i=1}^n x_i}{n} \times \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i^2}$$

$$\ln b \times \left( \sum_{i=1}^n x_i^2 - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n x_i}{n} \right) = \sum_{i=1}^n x_i \times \ln y_i - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n \ln y_i}{n}$$

$$\ln b = \frac{\sum_{i=1}^n x_i \times \ln y_i - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n \ln y_i}{n}}{\sum_{i=1}^n x_i^2 - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n x_i}{n}}$$

$$\ln b = \frac{\sum_{i=1}^n x_i \times \ln y_i - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n \ln y_i}{n}}{\sum_{i=1}^n x_i^2 - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n x_i}{n}}$$

### 3.2 Expansion Planning of a Given Transmission Network

#### 3.2.1. Description Of a Network

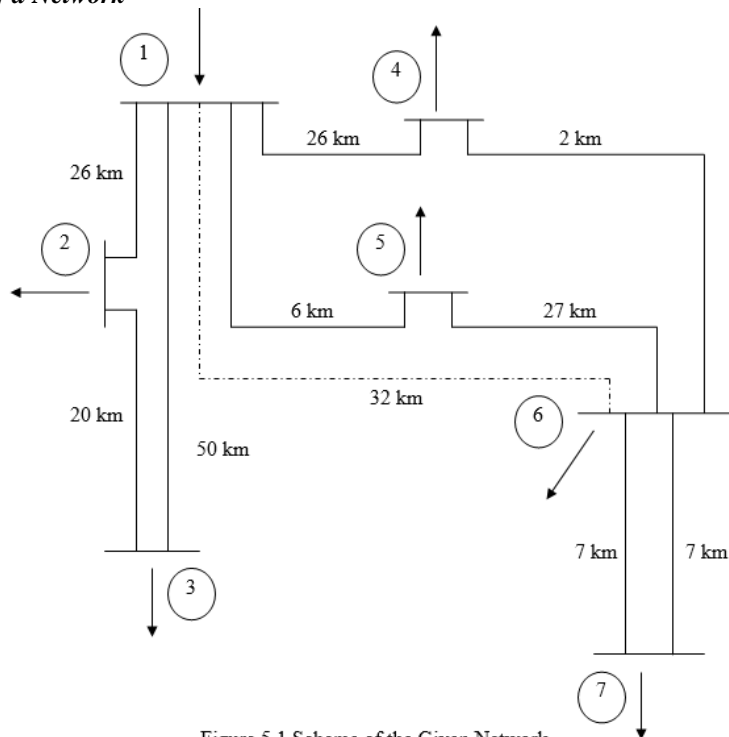


Figure 5.1 Scheme of the Given Network

Lines in all sections are of one type:

$$s = 185\text{mm}^2$$

$$R_k = 0.156 \ \Omega / \text{km}$$

$$X_k = 0.4 \ \Omega / \text{km}$$

$$B_k = 2.8 \ \mu\text{s} / \text{km}$$

$$G_k = 0$$

$$l_{12} = 26 \ \text{km}$$

$$l_{23} = 20 \ \text{km}$$

$$l_{13} = 50 \ \text{km}$$

$$l_{14} = 26 \ \text{km}$$

$$l_{15} = 6 \ \text{km}$$

$$l_{46} = 2 \ \text{km}$$

$$l_{56} = 27 \ \text{km}$$

$$l_{67} = 7 \ \text{km}$$

$$l_{67} = 7 \ \text{km}$$

Year	P <sub>2</sub> [MW]	P <sub>3</sub> [MW]	P <sub>4</sub> [MW]	P <sub>5</sub> [MW]	P <sub>6</sub> [MW]	P <sub>7</sub> [MW]
1997	38.0	28.9	19.8	24.4	24.4	43.3
1998	38.0	29.0	20.2	24.5	24.5	43.0
1999	39.0	29.4	20.3	24.9	24.9	44.1
2000	39.5	30.0	20.4	25.1	25.1	44.8
2001	40.0	30.3	20.6	25.5	25.5	45.3

Table 5.1 History of Loads: Active Power

Year	Q <sub>2</sub> [MVA <sub>r</sub> ]	Q <sub>3</sub> [MVA <sub>r</sub> ]	Q <sub>4</sub> [MVA <sub>r</sub> ]	Q <sub>5</sub> [MVA <sub>r</sub> ]	Q <sub>6</sub> [MVA <sub>r</sub> ]	Q <sub>7</sub> [MVA <sub>r</sub> ]
1997	2.82	4.80	4.82	5.80	7.80	9.60
1998	2.86	4.80	4.90	5.80	7.70	9.70
1999	2.93	4.90	4.93	6.00	7.80	9.80
2000	2.95	4.80	4.90	6.10	7.90	10.0
2001	3.00	5.20	5.10	6.00	8.00	10.1

Table 5.2 History of Loads: Reactive Power

### 3.3 Load Growth Prediction

Example of calculation for P<sub>2</sub>:

$$P_{2n} = a \times b^n$$

$$y = a \times b^x$$

$$y_{30} = a \times b^{30}$$

$$P_{2,30} = a \times b^{30}$$

$$\ln a = \frac{\sum_{i=1}^n \ln y_i - \ln b \times \sum_{i=1}^n x_i}{n}$$

$$\ln b = \frac{\sum_{i=1}^n x_i \times \ln y_i - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n \ln y_i}{n}}{\sum_{i=1}^n x_i^2 - \frac{\sum_{i=1}^n x_i \times \sum_{i=1}^n x_i}{n}}$$

$$n = 5$$

$$\sum_{i=1}^n x_i = 15$$

$$\sum_{i=1}^n x_i^2 = 55$$

$$\sum_{i=1}^n \ln y_i = 18.304$$

$$\sum_{i=1}^n x_i \times \ln y_i = 55.053$$

$$\ln b = \frac{55.053 - \frac{15 \times 18.304}{5}}{55 - \frac{15 \times 15}{5}} = 0.0141$$

$$\ln a = \frac{18.304 - 0.0141 \times 15}{5} = 3.618$$

$$a = 37.278$$

$$b = 1.014$$

$$P_{2n} = 37.278 \times 1.014^n$$

Year	P <sub>2</sub> [MW]	P <sub>3</sub> [MW]	P <sub>4</sub> [MW]	P <sub>5</sub> [MW]	P <sub>6</sub> [MW]	P <sub>7</sub> [MW]
1997	38.0	28.9	19.8	24.4	36.6	43.3
1998	38.0	29.0	20.2	24.5	37.0	43.0
1999	39.0	29.4	20.3	24.9	37.7	44.1
2000	39.5	30.0	20.4	25.1	38.0	44.8
2001	40.0	30.3	20.6	25.5	38.7	45.3
2002	40.6	30.7	20.8	25.7	39.2	45.8
2003	41.2	31.1	21.0	26.0	39.7	46.3
2004	41.7	31.5	21.2	26.3	40.3	46.9
2005	42.3	31.9	21.4	26.6	40.8	47.4
2006	42.9	32.3	21.6	26.9	41.4	48.0
2007	43.6	32.7	21.8	27.2	42.0	48.6
2008	44.2	33.1	22.0	27.5	42.6	49.1
2009	44.8	33.6	22.2	27.8	43.2	49.7
2010	45.4	34.0	22.3	28.1	43.8	50.3
2011	46.1	34.4	22.5	28.4	44.4	50.9
2012	46.7	34.9	22.7	28.8	45.0	52.5
2013	47.4	35.3	22.9	29.1	45.6	52.1
2014	48.1	35.8	23.2	29.4	46.3	52.7
2015	48.8	36.3	23.4	29.8	46.9	53.4
2016	49.5	36.7	23.6	30.1	47.6	54.0
2017	50.2	37.2	23.8	30.5	48.2	54.6
2018	50.9	37.7	24.0	30.8	48.9	55.3
2019	51.6	38.2	24.2	31.1	49.6	55.9
2020	52.3	38.7	24.4	31.5	50.3	56.6
2021	53.1	39.2	24.6	31.9	51.0	57.3
2022	53.8	39.7	24.9	32.2	51.7	57.9
2023	53.6	40.2	25.1	32.6	52.4	58.6
2024	55.4	40.7	25.3	32.9	53.1	59.3
2025	56.2	41.2	25.5	33.3	53.9	60.0
2026	57.0	41.8	25.8	33.7	54.6	60.7

Table 5.3 Given and Predicted Loads (1997–2026): Active Power

Year	Q <sub>2</sub> [MVA <sub>r</sub> ]	Q <sub>3</sub> [MVA <sub>r</sub> ]	Q <sub>4</sub> [MVA <sub>r</sub> ]	Q <sub>5</sub> [MVA <sub>r</sub> ]	Q <sub>6</sub> [MVA <sub>r</sub> ]	Q <sub>7</sub> [MVA <sub>r</sub> ]
1997	2.82	4.80	4.82	5.80	7.80	9.60
1998	2.86	4.80	4.90	5.80	7.70	9.70
1999	2.93	4.90	4.93	6.00	7.80	9.80
2000	2.95	4.80	4.90	6.10	7.90	10.0
2001	3.00	5.20	5.10	6.00	8.00	10.1
2002	3.05	5.14	5.10	6.15	8.02	10.2
2003	3.10	5.22	5.16	6.23	8.08	10.4
2004	3.15	5.31	5.22	6.30	8.14	10.7
2005	3.20	5.39	5.28	6.38	8.21	10.7
2006	3.24	5.48	5.34	6.45	8.27	10.8
2007	3.30	5.57	5.40	6.53	8.33	10.9
2008	3.35	5.66	5.46	6.61	8.40	11.1
2009	3.40	5.75	5.52	6.68	8.46	10.2
2010	3.45	5.84	5.58	6.76	8.53	11.4
2011	3.50	5.94	5.65	6.84	8.59	11.5

2012	3.56	6.03	5.71	6.93	8.66	11.8
2013	3.62	6.13	5.77	7.01	8.72	11.8
2014	3.67	6.23	5.84	7.09	8.79	12.0
2015	3.73	6.33	5.91	7.18	8.86	12.2
2016	3.79	6.43	5.97	7.26	8.93	12.3
2017	3.85	6.53	6.04	7.35	8.99	12.5
2018	3.91	6.64	6.11	7.44	9.06	12.6
2019	3.97	6.75	6.18	7.52	9.13	12.8
2020	4.03	6.86	6.25	7.61	9.20	13.0
2021	4.09	6.97	6.32	7.70	9.27	13.2
2022	4.16	7.08	6.39	7.80	9.34	13.3
2023	4.22	7.19	6.47	7.89	9.41	13.5
2024	4.29	7.31	6.54	7.98	9.49	13.7
2025	4.35	7.43	6.61	8.08	9.56	13.9
2026	4.42	7.55	6.69	8.17	9.63	14.1

Table 5.4 Given and Predicted Loads (1997–2026): Reactive Power

**IV. Costs Calculations**

The aim of this chapter is to find an optimal year for installing the tenth line. Generally, the line can be installed between the years 2002 and 2020. To compare these possible variants, for each them the total discounted costs during lines life expectancy (1997–2026) will be calculated.

$$C^A = C_{A,1-9} + C_{A,10}^A + \sum_{i=1}^{n_1-1} C_{MAINi}^A + \sum_{i=n_1}^n C_{MAINi}'^A + \sum_{i=1}^{n_1-1} C_{\Delta i}^A + \sum_{i=n_1}^n C_{\Delta i}'^A - C_{v,10}^A \tag{5.24}$$

$$C_{A,10}^A = C_{A,10} \times q^{-n_1} \tag{5.25}$$

$$C_{MAINi}^A = C_{MAIN} \times q^{-i} \tag{5.26}$$

$$C_{MAINi}'^A = C_{MAIN}' \times q^{-i} \tag{5.27}$$

$$C_{\Delta i}^A = C_{\Delta i} \times q^{-i} \tag{5.28}$$

$$C_{\Delta i}'^A = C_{\Delta iN}' \times q^{-i} \tag{5.29}$$

$$C_{\Delta i} = \Delta P_i \times c_{\Delta} \tag{5.30}$$

$$C_{\Delta i}' = \Delta P_i' \times c_{\Delta} \tag{5.31}$$

$$C_{v,10}^A = C_{A,10} \times \frac{n_1}{n} \times q^{-n} \tag{5.32}$$

- where
- $C^A$  are total discounted costs
  - $C_{A,1-9}$  is cost of acquisition of the original network (9 lines)
  - $C_{A,10}$  is cost of acquisition of the added line
  - $C_{A,10}^A$  is discounted cost of acquisition of the added line
  - $C_{\square\square\square\square}$  are maintenance costs of the original network (9 lines)
  - $C'_{\square\square\square\square}$  are maintenance costs of the expanded network (10 lines)
  - $C_{\square}$  are costs of losses of the original network (9 lines)
  - $C'_{\square}$  are costs of losses of the expanded network (10 lines)
  - $\square P$  are active power losses of the original network (9 lines)
  - $\square P'$  are active power losses of the expanded network (10 lines)
  - $c_{\square}$  are total specific costs of losses

$C_{v,10}^A$  is discounted value of the added line at the end of year  
 $n = 30$  (2026)  
 $n_1$  is year when the added line is installed ( $n_1 = 6 \div 23$ )

Given values:

- $p_{MAIN} = 3\%$
- $C_{AK} = 1.2 \times 10^6$  CZK/km
- $c_{\Delta} = 7000$  CZK/kW×yr
- $p = 5\%$  interest rate (for discounting)

No.	Year	$\Delta P$ [kW]	$C_{\Delta}$ [CZK]	$\Delta P'$ [kW]	$C'_{\Delta}$ [CZK]	$C_{MAIN}$ [10 <sup>6</sup> CZK]	$C'_{MAIN}$ [10 <sup>6</sup> CZK]
1	1997	3488	24416000	-	-	6.16	-
2	1998	3543	24801000	-	-	6.16	-
3	1999	3656	25592000	-	-	6.16	-
4	2000	3751	26257000	-	-	6.16	-
5	2001	3855	26985000	-	-	6.16	-
6	2002	3949	27643000	2992	20944000	6.16	7.31
7	2003	4052	28364000	3146	22022000	6.16	7.31
8	2004	4159	29113000	3229	22603000	6.16	7.31
9	2005	4266	29862000	3312	23184000	6.16	7.31
10	2006	4377	30639000	3399	23793000	6.16	7.31
11	2007	4492	31444000	3489	24423000	6.16	7.31
12	2008	4609	32263000	3580	25060000	6.16	7.31
13	2009	4715	33005000	3665	25655000	6.16	7.31
14	2010	4854	33978000	3770	26390000	6.16	7.31
15	2011	4981	34867000	3869	27083000	6.16	7.31
16	2012	5174	36218000	4013	28091000	6.16	7.31
17	2013	5246	36722000	4075	28525000	6.16	7.31
18	2014	5384	37688000	4182	29274000	6.16	7.31
19	2015	5526	38682000	4292	30044000	6.16	7.31
20	2016	5671	39697000	4405	30835000	6.16	7.31
21	2017	5821	40747000	4522	31654000	6.16	7.31
22	2018	5974	41818000	4641	32487000	6.16	7.31
23	2019	6132	42924000	4763	33341000	6.16	7.31
24	2020	-	-	4889	34223000	-	7.31
25	2021	-	-	5019	35133000	-	7.31
26	2022	-	-	5151	36057000	-	7.31
27	2023	-	-	5244	36708000	-	7.31
28	2024	-	-	5428	37996000	-	7.31
29	2025	-	-	5572	39004000	-	7.31
30	2026	-	-	5720	40040000	-	7.31

Table 5.17 Yearly Costs of Losses and Yearly Maintenance Costs

No.	Year	$C_{\Delta A}$	$C'_{\Delta A}$	$C_{MAIN A}$	$C'_{MAIN A}$
1	1997	23253333	-	5862857	-
2	1998	22495238	-	5583673	-
3	1999	22107332	-	5317784	-
4	2000	21601699	-	5064556	-
5	2001	21143454	15628735.3	4823387	5453342
6	2002	20627632	15650624.3	4593702	5193659
7	2003	20157765	15298600.1	4374954	4946342
8	2004	19704824	14944613.1	4166623	4710802
9	2005	19249311	14606838	3968212	4486478
10	2006	18809688	14279622.3	3779250	4272836
11	2007	18384656	13954345.7	3599286	4069368
12	2008	17965246	13605394.3	3427891	3875588

13	2009	17503256	13328743.3	3264658	3691037
14	2010	17161199	13027386.1	3109198	3515273
15	2011	16771623	12868810.8	2961141	3347879
16	2012	16591883	12445363	2820135	3188456
17	2013	16021687	12163951.7	2685842	3036625
18	2014	15660142	11889431	2557945	2892024
19	2015	15307781	11621387.2	2436138	2754308
20	2016	14961382	11361961.6	2320132	2623151
21	2017	14625825	11105676.8	2209649	2498239
22	2018	14295478	10854872.9	2104428	2379275
23	2019	13974823	10611454.1	2004217	2265976
24	2020	-	10374872.3	-	2158073
25	2021	-	10140697.2	-	2055307
26	2022	-	9832176.09	-	1957436
27	2023	-	9692537.84	-	1864224
28	2024	-	9475878.31	-	1775452
29	2025	-	9264353.04	-	1690906
30	2026	-	15628735.3	-	5453342

Table 5.18 Discounted Yearly Costs of Losses and Yearly Maintenance Costs

No.	Year	$C_{A,1-9}$	$C_{A,1-10}$	$\sum_{i=1}^{n-1} C_{MAINi}^A$	$\sum_{i=n_1}^n C_{MAINi}'^A$	$\sum_{i=1}^{n_1-1} C_{\Delta i}^A$	$\sum_{i=n_1}^n C_{\Delta i}'^A$	$C_{v,10}^A$	$C^A$
		[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]	[10 <sup>6</sup> CZK]
1	1997								
2	1998								
3	1999								
4	2000								
5	2001								
6	2002	205.00	28.60	26.652	80.702	110.60	308.03	1.777	<b>758.06</b>
7	2003	205.00	27.29	31.246	75.249	13.123	29.240	2.073	<b>760.54</b>
8	2004	205.00	25.99	35.621	70.055	15.139	27.675	2.369	<b>762.63</b>
9	2005	205.00	24.75	39.788	65.109	17.109	26.145	2.666	<b>764.73</b>
10	2006	205.00	23.57	43.756	60.398	19.034	24.651	2.962	<b>766.81</b>
11	2007	205.00	22.45	47.535	55.911	20.915	23.190	3.258	<b>768.89</b>
12	2008	205.00	21.38	51.1343	51.639	22.753	21.762	3.554	<b>770.96</b>
13	2009	205.00	20.36	54.562	47.569	24.550	20.366	3.850	<b>773.01</b>
14	2010	205.00	19394609.4	57.827	43.694	26.300	19.006	4.146	<b>775.03</b>
15	2011	205.00	18471056.57	60.936	40.003	28.017	17.673	4.442	<b>777.06</b>
16	2012	205.00	17591482.44	63.897	36.487	29.694	16.370	4.739	<b>779.08</b>
17	2013	205.00	16753792.8	66.717	33.139	31.353	15.083	5.034	<b>781.14</b>
18	2014	205.00	15955993.15	69.403	29.951	32.955	13.839	5.331	<b>783.12</b>
19	2015	205.00	15196183.95	71.961	26.914	34.521	12.623	5.627	<b>785.08</b>
20	2016	205.00	14472556.14	74.397	24.022	36.052	11.434	5.923	<b>787.02</b>
21	2017	205.00	13783386.8	76.717	21.268	37.548	10.271	6.219	<b>788.94</b>
22	2018	205.00	13127035.05	78.927	18.645	39.010	91.353	6.516	<b>790.84</b>
23	2019	205.00	12501938.14	81.031	16.147	40.440	80.247	6.812	<b>792.72</b>
24	2020		11906607.75		13.767		69.392		
25	2021		11339626.43		11.501		58.781		
26	2022		10799644.22		93.433		48.406		
27	2023		10285375.4		72.880		38.265		
28	2024		9795595.666		53.306		28.433		
29	2025		9329138.73		34.664		18.740		
30	2026		8884894.028		16.909		92.644		

Table 5.19 Total Discounted Costs

The series admittance of the added line:

$$\bar{Z}_{16} = (R_k + X_{ki}) \times l_{16}$$



then,  $\bar{y}_{16} = \frac{1}{\bar{Z}_{16}}$

The shunt admittance of the added line:

$\bar{y}_{160} = (G_k + B_{ki}) \times l_{16}$

Line	Length	Series Impedance	Series Admittance	Shunt Admittance	
i-j	$l_{i-j}$ [km]	$Z_{i-j}$ [ $\Omega$ ]	$y_{i-j}$ [S]	$y_{i-j,0}$ [S]	$y_{i-j,0}/2$ [S]
1-2	26	4.056+10.4i	0.03255-0.08346i	$7.28 \times 10^{-5}$ i	$3.64 \times 10^{-5}$ i
1-3	50	7.8+20i	0.01692-0.04340i	$1.4 \times 10^{-4}$ i	$7 \times 10^{-5}$ i
1-4	26	4.056+10.4i	0.03255-0.08346i	$7.28 \times 10^{-5}$ i	$3.64 \times 10^{-5}$ i
1-5	6	0.936+2.4i	0.14105-0.36166i	$1.68 \times 10^{-5}$ i	$8.4 \times 10^{-6}$ i
1-6	32	4.992+12.8i	0.02645-0.06781i	$8.96 \times 10^{-5}$ i	$4.48 \times 10^{-5}$ i
2-3	20	3.12+8i	0.04231-0.10850i	$5.6 \times 10^{-5}$ i	$2.8 \times 10^{-5}$ i
4-6	2	0.312+0.8i	0.42314-1.08498i	$5.6 \times 10^{-6}$ i	$2.8 \times 10^{-6}$ i
5-6	27	4.212+10.8i	0.03134-0.08037i	$7.56 \times 10^{-5}$ i	$3.78 \times 10^{-5}$ i
6-7	7	1.092+2.8i	0.12090-0.30999i	$1.96 \times 10^{-5}$ i	$9.8 \times 10^{-6}$ i

Table 5.11 Lines Series and Shunt Admittances

Elements of the nodal admittance matrix:

$Y_{11} = y_{12} + \frac{y_{120}}{2} + y_{13} + \frac{y_{130}}{2} + y_{14} + \frac{y_{140}}{2} + y_{15} + \frac{y_{150}}{2} + y_{16} + \frac{y_{160}}{2}$

$Y_{12} = Y_{21} = -\bar{y}_{12}$

$Y_{13} = Y_{31} = -\bar{y}_{13}$

$Y_{14} = Y_{41} = -\bar{y}_{14}$

$Y_{15} = Y_{51} = -\bar{y}_{15}$

$Y_{16} = Y_{61} = -\bar{y}_{16}$

$Y_{17} = Y_{71} = 0$

$Y_{22} = y_{12} + \frac{y_{120}}{2} + y_{23} + \frac{y_{230}}{2}$

$Y_{23} = Y_{32} = -\bar{y}_{23}$

$Y_{24} = Y_{42} = 0$

$Y_{25} = Y_{52} = 0$

$Y_{26} = Y_{62} = 0$

$Y_{27} = Y_{72} = 0$

$Y_{33} = y_{13} + \frac{y_{130}}{2} + y_{23} + \frac{y_{230}}{2}$

$Y_{34} = Y_{43} = 0$

$Y_{35} = Y_{53} = 0$

$Y_{36} = Y_{63} = 0$

$Y_{37} = Y_{73} = 0$

$Y_{44} = y_{14} + \frac{y_{140}}{2} + y_{46} + \frac{y_{460}}{2}$

$Y_{45} = Y_{54} = 0$

$Y_{46} = Y_{64} = -\bar{y}_{46}$

$$Y_{47} = Y_{74} = 0$$

$$Y_{55} = y_{15} + \frac{y_{150}}{2} + y_{56} + \frac{y_{560}}{2}$$

$$Y_{56} = Y_{65} = -\bar{y}_{56}$$

$$Y_{57} = Y_{75} = 0$$

$$Y_{66} = y_{16} + \frac{y_{160}}{2} + y_{46} \frac{y_{460}}{2} + y_{56} + \frac{y_{560}}{2} + y_{67} + \frac{y_{670}}{2}$$

$$Y_{67} = Y_{76} = -\bar{y}_{67}$$

$$Y_{77} = y_{67} + \frac{y_{670}}{2} + y_{67} + \frac{y_{670}}{2}$$

	Admittance	Magnitude	Angle
	Y [S]	Y [S]	$\alpha$ [rad]
Y <sub>11</sub>	0.2495-0.6396i	0.6865	-1.1988
Y <sub>12</sub>	-0.0326+0.0835i	0.0896	1.9427
Y <sub>13</sub>	-0.0169+0.0434i	0.0466	1.9427
Y <sub>14</sub>	-0.0325+0.0835i	0.0896	1.9427
Y <sub>15</sub>	-0.1410+0.3617i	0.3882	1.9427
Y <sub>16</sub>	-0.0264+0.0678i	0.0728	0
Y <sub>17</sub>	0	0	0
Y <sub>22</sub>	0.0749-0.1919i	0.2060	-1.1988
Y <sub>23</sub>	-0.0423+0.1085i	0.1165	1.9427
Y <sub>24</sub>	0	0	0
Y <sub>25</sub>	0	0	0
Y <sub>26</sub>	0	0	0
Y <sub>27</sub>	0	0	0
Y <sub>33</sub>	0.0592-0.1518i	0.1629	-1.1987
Y <sub>34</sub>	0	0	0
Y <sub>35</sub>	0	0	0
Y <sub>36</sub>	0	0	0
Y <sub>37</sub>	0	0	0
Y <sub>44</sub>	0.4557-1.1684i	1.2541	-1.1989
Y <sub>45</sub>	0	0	0
Y <sub>46</sub>	-0.4231+1.0850i	1.1646	1.9427
Y <sub>47</sub>	0	0	0
Y <sub>55</sub>	0.1724-0.4419i	0.4744	-1.1989
Y <sub>56</sub>	-0.0313+0.0804i	0.0863	1.9427
Y <sub>57</sub>	0	0	0
Y <sub>66</sub>	0.7227-1.8530i	1.9890	-1.1989
Y <sub>67</sub>	-0.2418+0.6200i	0.6655	1.9427
Y <sub>77</sub>	0.2418-0.6200i	0.6654	-1.1989

Table 5.12 Elements of the Nodal Admittance Matrix for 10 lines

$$[Y] = \begin{bmatrix} 0.6865 & 0.0896 & 0.0466 & 0.0896 & 0.3882 & 0.0728 & 0 \\ 0.0896 & 0.2060 & 0.1165 & 0 & 0 & 0 & 0 \\ 0.0466 & 0.1165 & 0.1629 & 0 & 0 & 0 & 0 \\ 0.0896 & 0 & 0 & 1.2541 & 0 & 1.1646 & 0 \\ 0.3882 & 0 & 0 & 0 & 0.4744 & 0.0863 & 0 \\ 0.0728 & 0 & 0 & 1.1646 & 0.0863 & 1.9890 & 0.6655 \\ 0 & 0 & 0 & 0 & 0 & 0.6655 & -1.1989 \end{bmatrix}$$

$$[\alpha] = \begin{bmatrix} -1.1988 & 1.9427 & 1.9427 & 1.9427 & 1.9427 & 0 & 0 \\ 1.9427 & -1.1988 & 1.9427 & 0 & 0 & 0 & 0 \\ 1.9427 & 1.9427 & -1.1987 & 0 & 0 & 0 & 0 \\ 1.927 & 0 & 0 & -1.1989 & 0 & 1.9427 & 0 \\ 1.9427 & 0 & 0 & 0 & -1.1988 & 1.9427 & 0 \\ 0 & 0 & 0 & 1.9427 & 1.9427 & -1.1989 & 1.9427 \\ 0 & 0 & 0 & 0 & 0 & 1.9427 & -1.1989 \end{bmatrix}$$

## V. Conclusion

Ultimately, the case study generated important data on the transmission network under investigation, providing a thorough grasp of its functionality and room for expansion. The paper recommends execution of the suggested expansion plan, designed to solve the unique constraints faced by the network and meet future demand based on the research findings. Nevertheless, there are significant limitations to this paper, primarily because of the extent of the data and the modeling assumptions. Future research could focus on improving data confidentiality and implementing sophisticated modeling protocols to enhance the field further. Additionally, exploring how renewable energy sources might be incorporated into the enlarged network provides a viable direction for future research, ensuring an efficient and sustainable power source for metropolitan surroundings.

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