

Modelling Annual Maxima Series of River Niger with EasyFit Software

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-----ABSTRACT-----

Modelling annual maxima series (AMS) yields a compact and smoothed quantile relation for extrapolation to flood flow larger than those historically observed. The derived quantile relation obtained becomes a systematic, consistent and reliable tool for accurate estimation of design flood for design of water resources projects, flood plain management amongst others. In this study, the AMS of 62 years for Niger River at Lokoja were fitted to Generalized Extreme Value (GEV), Log Pearson type III(LP3) and Pearson type III(P3) distributions with probability weighed moments/Linear –moments (PWM/L-moments), and Method of moments (MOM) respectively.

The Kolmogorov-Smirnov (K-S) and Anderson-Darling (A-D) goodness of fit (GOF) tests module of Easyfit Software was used for selection of the best - fit probability distribution model. The result of the GOF tests show GEV is the best model, seconded by LP3 and thirdly P3. Furthermore, the three models; GEV,LP3 and P3 were used in quantile estimation for various return periods between 10- and 100- year, and the computed quantile estimates were consistent with the GOF outcome; GEV>LP3>P3. Consequently, the GEV model was used in constructing 90% confidence interval of the study. The paper is part of the on-going studies, aimed at finding a uniform and consistent probability distribution model of flood flow for Nigeria Rivers. It importance lies in its ability to provide accurate design flood estimates for Design of water infrastructure projects, insurance applications and flood plain management.

KeyWords:Design flood, Modelling, Parameter estimation, Goodness of fit tests, Probability distribution functions.

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I. INTRODUCTION

Floodsareamong the Earth's most common and most destructive natural hazards, affecting human lives and properties directly and indirectly around the world; Prieto [1]. Societies had confronted the negative impacts of flooding in river basins by implementing large investments through design and construction of water infrastructure projects. One of the key parameters for optimum designof water infrastructureprojects is the design flood. The design flood is estimated using the quantile relation of best – fit distribution, expressing the magnitude of the random variable in terms of its population parameters and the exceedance probability or return period. The suffering borne by societies globally due to flooding and it related impacts may be found in the following literature; UNISDR [2], (1996), Berz [3], Panda and Amarattunga [4], Cunhaet al. [5], NWS [6], Hoyois and Sapir[7], Feyen et al.[8], Kundzewicz et al. [9] and Bilau [10]. A review of the cited literature show that accurate estimation of design flood is crucial to optimum performance of infrastructure located in flood-prone areas, where huge national expenditure are at risk due to flooding. Besides floods cause deaths, property damage, poverty, degradation of environmental quality, huge economic losses, destroy social life annually across the globe. While engineers cannot prevent the occurrence of floods, they should develop structural and non-structural strategies to reduce the risk of large economic losses, social vulnerability, environmental damage and loss of life according to IFMRC [11]. The goal frequency analysis is to fit geophysical data to a continuous distribution function in order to derive a relationship between the magnitude of the extreme event, population parameters and its exceedance probability (or return period). The derived quantile relation is subsequently used for extrapolation to higher return periods for estimation of design flood. Furthermore, the quantile relation becomes a compact and smoothened representation of the frequency distribution revealed by the annual maxima series, and a systematic procedure for extrapolation to flood discharges longer than observed series, according to NAP [12]. Accurate estimates of the magnitude and frequency of flood flows are needed for the design and operation of water infrastructural projects,

floodplaindemarcation and management, and for the design of transportation infrastructure such as bridges and roads.

The estimation of design flood involves; (i) Data choice and screening (ii) distribution fitting and quantile estimates (iii) goodness-of-fit tests (iv)assessment of uncertainty. The uncertainty in parameter and quantile estimates is usually accounted for by defining confidence intervals.

Flood events can be analyzed using either the Annual maximum series or partial duration series. The annual maxima series is based on the maximum peak flow for each year. The partial duration series is obtained by taking all flood peaks equal to or greater than a predefined flood base that may cause damage. If more than one flood per year must be considered, a partial duration series may be appropriate. However, the annual maximum series is adopted in this paper, because it is consistent with the occurrence of annual floods in Nigeria wherein one damaging fluvial flood event occurs annually. The application of probability distribution model to annual flood flow presupposes screening and application of non-parametric tests of randomness, independence, homogeneity and stationarity. It is only when empirical evidence was found to rule the non-parametric tests, that the available datais considered fit for flood frequency analysis.

The probability distribution functions adopted in this study area: Log-Pearson Type III (LP3), Generalized Extreme Value (GEV) and Pearson type III (P3). These distributions have been recommended for at site flood frequency analysis in many countries. For example, Pearson Type III (P3) is the recommended probability distribution in Germany, LP3, is the recommended standard of USA, Germany and Australia. GEV as standard probability distribution in the United Kingdom, Bangladesh etc. Most countries in Europe had adopted GEV as their standard probability distribution. The World Meteorological Organization (WMO) conducted a global survey in 1984 on the use of flood frequency methods and found GEV, EVI, LN2, P3, LP3 and EV2, as the widely used probability distribution. Ologhadien [13] undertook a comparative evaluation of probability distribution models of flood flow in the Lower Niger Basin and found GEV the best –fit-distribution, seconded by P3, and thirdly, LP3 using statistical GOF tests; Dmod, RRMSE, NSE, RSR and PPCC. Thus, the three candidate distributions selected have shown satisfactory performance in the lower Niger River Basin.

The three most important parameters estimation methods in civil engineering practice are (i) Method of Moments (MOM); (ii) Method of maximum likelihood (MLE) and (iii) probability, Weight Moments (PWMs) and L-moments. A brief review of parameter estimation methods by AMEC [14] shows that the best parameter estimation method to use should be based on the type of distribution and size of annual maxima series. For example, the MOM is recommended when sample size (N) \leq 25. MLE is best for two-parameter log-normal (LN2) distribution. LP3 distribution is applied to flood frequency analyses when the shape factor (γ)>1.0, such that 1/ β (scale) >0.0. PWM/L – moment is preferred to GEV distribution when N>50 years. The Easyfit software adopted in this study uses MOM for P3 and LP3, while PWMs/L – moments for GEV distribution.

Goodness – of – fit (GOF) tests can be applied to evaluate whethera given probability distribution can be a smoothed representation of the flood frequency distribution revealed by the available data. Three main kinds of model selection techniques may be found in Chen et al.[15]. These are (i) hypothesis tests based GOF and Information – based criteria, and statistics based GOF. Examples at hypothesis tests based GOF are Kolmogorov – Smirnov (K-S) test, Anderson –Darling (AD) test, Probability Plot Correlation Coefficient (PPCC), Chi-square test and log-likelihood tests. Information-based criteria include the Akaike Information Criterion, Akaike Information criterion –second order variant (AIG) and Bayesian Information criterion (BIC). The statistical GOF test are (i) Relative Root Mean Square (RRMSE), (ii) Nash-Sutcliffe efficiency (NSE), (iii) percent bias (PBIAS), (iv) ratio of the root mean square error to the standard deviation of measured data (RSR). Each category of GOF tests has its own characteristicsand applicable scope, see Rahman et al.[16]. Consequently, the results of these tests are not always in agreement, see Chen et al.[15]. The hypothesis tests based GOF using K-S and AD test are adopted in the paper.

The quantile relation derived are subjected to some uncertainty, which increases with decreasing exceedance probabilities. The uncertainties in quantile relation were estimated using the standard error of estimated quantiles by constructing appropriate confidence interval of flood quantiles for various return periods. The relationship between flood magnitude and its return period is of great importance to avoid damages. The exceedance probability that such flood is an essential input in the design of hydraulic structure and riskestimation.

The objectives of the paper is to model the annual maxima series of Rivers Niger at Lokoja using the Easyfit software built –in GOF, module to evaluate which probability distribution best model the annual maxima series. The research leading to this paper is part of a series of studies to find a unified probability distribution that may be applied consistently across Nigeria.

II. Study Area and Descriptive Data

The annual maxima series of River Niger at Lokoja hydrological station was used for the study. The hydrological station is located at Longitude 07°49' and Latitude 06°44' at an elevation of 45.77m above mean sea

level. The station is situated on the confluence of River Niger and Benue with a catchment area of 750,790 km². This AMS data was collated from the hydrological year book of Nigeria inland water Authority (NIWA), Lokoja, Nigeria. The record length is 62years (1955-2016). The descriptive statistics is presented in Table 1, while the location map is shown in Figure 1. Figure 2 shows the plot of the annual maxima series of the Niger River at Lokoja, Nigeria.



Figure 1: Location of Hydrological Station at Lokoja, Nigeria: Source Mahe et al.[17]



III. Methodology

3.1 Probability Distribution Functions

Three PDFs have been adopted to model the annual maxima series. Their choice is based on the results of previous studies, e.g. Ologhadien [13] which found Generalized Extreme Value (GEV), Log-Pearson Type III (LP3), and Pearson Type III (P3) to produce better results in modelling annual maxima series in the lower Niger Basin. In the cited literature, the three probability distributions were selected on the basis of statistical goodness – of –fit tests, namely: modified index of agreement (Dmod), relative root mean square error (RRMSE), Nash – Sutcliffe efficiency (NSE), percent bias (PBIAS), ratio of RMSE and standard deviation of the measurement (RSR) were used to identify the identify the best – fit distribution(s). The probability density functions and quantile relations of the selected distributions are presented in Table 2 These distributions have been adopted consensus probability distributions inother countries.

DistributionProbability Density FunctionsQuantile relationshipLog Pearson (LP3)
$$f(x) = \frac{1}{\alpha x \Gamma(\beta)} \left[\frac{\log(x) - \gamma}{\alpha} \right]^{\beta-1} e^{-\left[\frac{\log(x) - \gamma}{\alpha} \right]}$$
 $Z_{\tau} = L_n Q_r = \mu_z + K_\tau \sigma_z$ Pearson Type III (P3) $f(x) = \frac{1}{\alpha x \Gamma(\beta)} \left[\frac{(x) - \gamma}{\alpha} \right]^{\beta-1} e^{-\left[\frac{(x) - \gamma}{\alpha} \right]}$ $Q_T = \alpha \beta + \gamma + KT \sqrt{\sigma^2 \beta}$ Generalized Extreme
Value (GEV) $f(x) = \frac{1}{\alpha} \left[1 - k \left(\frac{x - u}{\alpha} \right) \right]^{1/k-1} e^{-\left[1 - k \left(\frac{x - u}{\alpha} \right) \right]^{1/k}}$ $Q_T = \beta + \frac{\alpha}{k} \left[1 - \left\{ -Log \left(1 - \frac{1}{T} \right)^K \right\} \right]$

Where β , α , and k are location, scale, and shape parameters

Table 2: Probability density functions and quantile relations

3.2 Parameter Estimation

The main methodsused in civil engineering practice for estimation of the parameters of the probability distribution functions are method of moments, maximum likelihood method probability weighted moments/L-moments. Different softwaresuses different parameter estimation methods. The Easy fit software amongst others uses MOM for LP3 and P3 distributions and PWM/L moment for GEV distribution. Accordingly, the Easy fit software has in-built parameter estimation methods for performing these tasks.

3.2.1 Probability Weighted Moments/L-Moments

The L-oments are computed from the probability weighted moments, first by arranging the data in ascending order, and then the PWMS are calculated as follows:

$$\beta_0 = n^{-1} \sum_{j=1}^n x_j$$

(1)

(4)

$$\beta_1 = n^{-1} \sum_{j=2}^n x_j \left[(j-1)/(n-1) \right]$$
(2)

$$\beta_2 = n^{-1} \sum_{j=3}^n x_j \left[(j-1)(j-2) \right] / \left[(n-1)(n-2)(n-3) \right]$$
(3)

$$\beta_3 = n^{-1} \sum_{j=4}^n x_j \left[(j-1)(j-2)(j-3) \right] / \left[(n-1)(n-2)(n-3) \right]$$

Where n is the data length, x is the data value, i is the rank of the value of n in ascending order. The unbiased L-moment estimators are obtained by replacing the PWMs in Equations 1 - 4, by their sample estimates in Equations 5 - 8 as follows:

$$\lambda_1 = \beta_o \tag{5}$$

$$\lambda_2 = 2\beta_1 - \beta_o \tag{6}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta \tag{7}$$

$$\lambda_4 = 20\beta_3 - 30\beta_1 + \beta_0 \tag{8}$$

The L-moment measure of location and L-moment ratio measures of scale, skewness and kurtosis are: $Location(mean) = \lambda_1$ (9)

$$L-coefficien \ tof \ variation \ (L-CV) = \tau_2 = \frac{\lambda_2}{\lambda_1}$$
(10)

$$L - Skweners(\tau_3) = \frac{\lambda_3}{\lambda_2}$$
(11)

$$L = Kurtosis(\tau_4) = \frac{\lambda_4}{\lambda_2}$$
(12)

The parameters of the GEV distribution are computed using L-moments and the relationship,see Hosking et al.[18] and Kamal et al. 19] as follows;

$$k = 7.8590c + 2.9554c^2$$
(13)

$$\alpha = k\lambda_2 / \left[\Gamma(1+k)(1-2^{-k}) \right]$$
(14)

$$\beta = \lambda_1 + (\alpha/k) [\Gamma(1+k) - 1]$$
⁽¹⁵⁾

 $c = [2/(\tau_3 + 3)] - \ln(2)/\ln(3)$ and $\Gamma(1+k)$ is the classical gamma function

The GEV parameters (location, scale and shape) are computed using the annual maxima series and the sample L-moment estimators in Equations 5–8, in the following order by Millington et al. [20] and Kamal et al.[19].

- i. Arrange the annual maxima series in ascending order.
- ii. Compute the 4 PWMs (β_0 , β_1 , β_3 and β_4 in Equations 5 8)
- iii. Compute the 4 L-moments($\lambda_1, \lambda_2, \lambda_3$ and λ_4 using the 4 PWMS.
- iv. Compute the shape parameter (k) using Equation 13
- v. Compute the location parameter (α) using Equation 14.
- vi. Compute the location parameter (β) using Equation 15.
- vii. The GEV parameters k, α and β computed above are now substituted in the quantile relation in Table 2.

3.2.2 Method of Moments

Using the annual maxim a series $\{X_i, i = 1, 2, ..., N\}$, estimators of the product moments can be calculated:

$$Mean(\hat{\mu}) = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{16}$$

Variance
$$\left(\hat{\sigma}^{2}\right) = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_{i} - \hat{\mu}\right)$$
 (17)

Coefficient of skewness
$$(\hat{\gamma}_2) = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \hat{\mu})^3}{\left[\frac{1}{n} \sum_{i=1}^n (x_i - \hat{\mu})^2\right]^{\frac{3}{2}}}$$

$$(18)$$

$$Kurtosis(\hat{\gamma}_{4}) = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_{i} - \hat{\mu})}{\left[\frac{1}{n} \sum_{i=1}^{n} (x_{i} - \hat{\mu})^{2}\right]^{2}}$$
(19)

The moment estimators of the distribution parameters are then obtained by replacing the theoretical product moments for the specified distribution by the sample moments. Equation for the moment estimators for the different distribution are given in standard texts, for example EVA [21].

3.3 **Goodness** – of – Fit Tests

The GOF tests are tools for evaluating the adequacy of candidate distributions to the observed annual maxima series. Therefore, in order to identify the best - fit probability distribution for modelling of AMS at Lokoja, two GOF tests; K-S and A-D were applied using the Easy fit software. A brief description of the two tests is presented below for completeness for details see, Hossain [22] and Sharma et al. [23].

Kolmogorov-Smirnov (K-S) Test 3.3.1

The K-S test uses empirical CDF and theoretical CDF to calculate test statistics. The K-S test statistics (D) is the maximum vertical different between empirical CDF $(P(x_n))$ and the theoretical CDF $(F(x_n))$ as follows:

$$D = \max |P(x_n) - F(x_n)|$$

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(20)

Where P(Xn) is empirical CDF of observed annual maxima series of n ordered observations, and F(Xn) is the theoretical CDF for each of the ordered, observations. When the test statistics (D) is smaller than the critical value of 0.01255 then the observed data is considered a good fit for the assumed distribution.

3.3.2 Anderson-Darling (A-D) test

The A-D test compares expected (theoretical) CDF to an observed CDF. The A-D test gives higher weight to the tails of distribution to be fitted than K-S test. The A-D test statistic (A^2) is:

$$A^{2} = -n - \sum_{k=1}^{n} \frac{2k-1}{n} \left\{ \ln F(Q_{k}) + \ln\{1 - F(Q_{n+1-k})\} \right\}$$
(21)

Where n is sample size, Q_1, \ldots, Q_n are observed data and F is CDF. If the test statistic (A²) is higher than critical value of 2.5018, the null hypothesis is rejected.Both K-S and A-D test were conducted at significance level of $\alpha = 0.05.$

3.4 **Uncertainty in Quantile Estimates**

Uncertainty is inherent in flood quantile estimates due to randomness of variables, sample size, selection of inappropriate probability distribution(s). The uncertainties are lumped in the form of confidence interval, see (Rao and Hamed [24]:

$$U_{T\beta} = \overline{Q} + K_U S_Q \tag{22}$$
$$I = \overline{Q} + K_U S_Q$$

$$L_{T\beta} - Q + K_L S_Q \tag{23}$$

Where U_{TB} and L_{TB} are upper confidence and lower confidence limits respectively K_U and K_L are the upper

and lower confidence limit factors which are functions of recurrence interval (T) and confidence level (β). \overline{Q} and Soare mean and standard deviation respectively of observed series. The confidence limit factors Ku and K_L are computed as follows:

(24)

$$K_{U} = \frac{K_{T} + \sqrt{K^{2}_{T} - pq}}{P}$$

$$K_{L} = \frac{K_{T} - K^{2}_{T} - pq}{P}$$
(25)

Where $P = 1 - [Za^2 / (2N - 1)]$ and $q = K^2_T - (Za^2 / N)$ K_T is frequency factor, Z_a is standard normal variable and N is sample size

RESULTSAND DISCUSSION IV.

The main results of this study are summarized in Tables 1, 3 and 4 and Figures 3-6. The entire analyses were executed in Microsoft Excel 2010 and Easy Fit software, version 5.6.

In Table1, the AMS are expressed in cubic metre per second. It may be observed that the data is negatively skewed, indicating it should be modelled with non-normal distribution. For the results of frequency analysis to be valid, some statistical hypothesis, namely, randomness and stationarity of the AMS must be satisfied. The in-house consultant of Nigeria Inland Waterway Authority (NIWA), Nigeria had performed independence and randomness of the data series using correlation coefficient (r) at lag-1 and Wald-Wolfowitz (WW) tests respectively. While stationarity of the AMS had been checked using Mann-Kendall test. These hypothesis had been verified and the AMS certified for flood frequently analysis in her hydrological handbooks. In order to determine the best-fit-distribution which in - turn gives the output in terms of return period. The Easy fit software compares the outcome of three GOF tests; K-S, A-D and chi-squared (C-S) and the results of the GOF tests are presented in Table 3 while the parameters and quantile equation of each distribution are presented in Table 4. The C-S test has lesser power and it was included for completeness. The results of GOF tests, in Table 3 shows that GEV is best-fit model, seconded by LP3, and thirdly P3. The quantile equations in Table4 had been used to compute the quantile estimates for various periods and the output values are given in Table5 and graphically displayed in Figure 3. The output values in Table 5 agree with ranking in Table 3 indicating that the best-fit distribution (GEV) also produce the highest outputs. Figures 4 and 5 show the PDF and CDF of the GEV distribution. One may observe that the PDF is slightly skewed to the left. This is because of the negative skewness coefficient. The estimate of 90% confidence interval of flood quantile for various return periods are presented graphically in Figure 6. It may be observed in Figure 6 that the quantile estimate (Q_T) values fall within the lower and upper limits. The present study is corroborated with similar studies on the Niger River, namely Ibeje [25], Ehiorobo and Akpejiori [26] and Ologhadien [13]. Ibeje conducted flood frequency analysis of Niger River at Shintaku and found LP3 the best fit distribution. While, Ehiorobo and Akpejiori [26] undertook frequency analysis of Niger River at Agenebode; but conversely found LN2 the best - fit distribution. Ologhadien [13] also conducted flood flow probability distribution Model selection on Niger/Benue basins in Nigeria and found GEV distribution. The results of the present study agrees with Ologhadien [13] and Ibeje [25], but disagrees with Ehiorobo and Akpejiori [26]. In contrast, Ibeje[25] and Ehiorobo and Akpejiori [26] did not include GEV and P3 as candidate distributions in their studies. The choice of GEV distribution agrees with Haktanir [27] who reported that GEV has a convincing relevance to the peak of floods, as most other probability distributions are not true depictions of flood peaks from the theoretical cause - effect standpoint.

#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Gen. Extreme Value	0.07077	1	0.19324	1	1.8836	2
2	Log-Pearson 3	0.07605	3	0.19696	2	1.2964	1
4	Pearson 5 (3P)	0.07383	2	0.2322	3	2.0292	3

	Table 3: Goodness of Fit Test Results				
l	Parameters	Quantile Equation			

#	Distribution	Parameters	Quantile Equation			
1	Gen. Extreme Value	k=-0.29994, σ=3871.0, μ=18192.0	$\begin{bmatrix} & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & $			
			$18192 - 12,906 \left 1 - \right - \left \log \frac{I - I}{T} \right \right $			
2	Log-Pearson 3	α=8.0153, β=-0.07342, γ=10.447	$Log Q_T = 9.86 + 0.208 K_T$			
4	Pearson 5 (3P)	α=306.56,β=2.0406E+7, γ=-47291.0	$Q_{\rm T} = 19515 + 3826K_{\rm T}$			
	Table 4: Estimated Parameters and Quantile Equations					

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R. Period	GEV	P3	LP3
10	30633.18	24234.08	24942.81
20	36741.46	25391.61	27201.5
30	40901.38	25939.55	28405.13
50	46883.49	26518.86	29784.43
75	52310.51	26902.74	30764.31
100	56574.22	27139.45	31396.63

 Table 5: Quantile Estimates for Various Return Periods



Figure 3: Quantile Estimates of GEV, P3 and LP3 against Return Periods.



Figure 4: PDF for GEV fitted AMS of Niger River, Lokoja



Figure 5: CDF for GEV fitted AMS of Niger Rivers, Lokoja





V. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this study, the annual maxima series of River Niger at Lokoja hydrological station are fitted to GEV, LP3 and P3 distributions. Parameter estimation using EasyFit software is carried out with MOM for LP3 and P3, PWM/L-moments for GEV. The EasyFit software in-built GOF tests module using K-S, A-D and C-S was applied. The test statistic of K-S, A-D and C-S are based on the lowest values of each of the test statistics. The results show that GEV distribution using PWM/L- moments is the best-fit probability distribution model of flood flow for River Nigeria at Lokoja. The results found in this study is useful for accurate estimation of design flow of water infrastructure projects and hydraulic structures in the lower Niger River Basin. The study recommend that GEV, LP3 and P3 should be considered candidate distributions for development of regional flood frequency analysis in the Lower River Niger Basin.

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