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A Special Note on Multi-Objective Optimization of Engineering Structures

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--***ABSTRACT***--
A common procedure in solving multi-objective structural optimization problems consists mainly of two phases. The first phase is to generate an approximate Pareto-optimal front of non-dominated solutions, while the second is to apply an appropriate technique, such as TOPSIS to rank the potential solutions and select one by the decision maker. In this way, there is no guarantee to find a single optimal solution as the best with respect to all design objectives because improving one of them usually deteriorate another. This paper introduces a novel approach to overcome such uncertainty by determining the optimal relative importance of each objective based on making the attained optimization gains in all objectives too close to each other. As a practical application of optimizing engineering structures, the developed mathematical model is implemented to solve the benchmark problem of a two-bar truss structure constructed from thin-walled circular tubes. Optimal solutions are obtained by applying a robust hybrid genetic algorithm and sequential quadratic programming.

KEYWORDS;- *Optimum structures – multi-objective optimization –Pareto front – TOPSIS method – truss design*

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I. INTRODUCTION

A real structural engineering problem includes several design objectives to be minimized (or maximized) simultaneously. These objectives are often in conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front for multi-objective optimization (*MOO*) problems [1]. The major obstacle is that these optimal solutions are non-dominant to each other but are superior to the rest of solutions in the design space. This means that it is not possible to find a single solution to be superior to all other solutions with respect to all objectives. The change in the design variables in Pareto front could not lead to the improvement of all objectives simultaneously. Consequently such a change will lead to deterioration of at least one design objective. There are many techniques for obtaining approximate Pareto-optimal solutions, such as *TOPSIS* method, in which the attained optimal solutions are ranked and one of them may be selected as the best by the decision maker [2, 3].

The most popular method for solving *MOO* problems is known as the weighting method or weighted sum technique because of its ease in application [4]. Weights are usually selected randomly for a specific design problem. Some researchers have argued that it is difficult to obtain proper weights for certain problem, and therefore, preferred to use the fuzzy theory [5], where the objective functions are characterized with values between zero and one (pseudo-goal). The pseudo-goal has a membership function one if the design is optimum and zero if the design is not optimum. However, a designer cannot differentiate between the improvement gains of each objective function.

In this paper a novel technique for *MOO* based on the weighted sum method is presented. It is called the multi-objective balancing (*MOB*) technique, in which the percentage improvement of each objective function is recorded with respect to the optimal values of the weighting factors, which ensure fair level of importance among all conflicting design goals. The method can hopefully overcome the problems of both of the Pareto-optimal and fuzzy techniques by searching for a design point that guarantee simultaneous and balanced improvements in all objective functions or at least improvement in some of them with certain degree without degrading others. Such a new approach can be implemented to several engineering applications dealing with design optimization of mechanical elements and structures. A practical case study considers structural multidisciplinary optimization of a two-bar truss structure constructed from thin-walled circular tubes, which was considered by many investigators as a very basic benchmark problem [3, 5, 6, and 7].

II. OPTIMIZATION MODEL

Two major conflicting design objectives are considered in this study: the minimum structural mass and the maximum buckling load subject to weight, strength and deflection as well as side constraints imposed on cross-sectional dimensions. In order to build a naturally scaled optimization model with all variables and parameters have the same order of magnitude, the various variables are normalized with respect to a known baseline design (refer to Appendix A).Therefore, the overall objective function can be defined by a weighted sum of the dimensionless structural mass and the critical buckling load, where the selected design variables encompass the dimensionless mean diameter and wall thickness of the truss member cross sections. The various dimensionless quantities are defined in the following:

Mean diameter	$\widehat{D} = D/D_o$	$(= x_1)$
Wall thickness	$\hat{h} = h/h_o$	$(= x_2)$
Structural mass	$\widehat{M} = M/M_o$	$(F_1 = x_1 * x_2)$
Critical buckling force	$\widehat{F}_{cr} = F_{cr}/F_{cro}$	$(F_2 = -x_1^3 * x_2)$
Applied stress	$\hat{\sigma} = \sigma/\sigma_o$	$(= (x_1 * x_2)^{-1})$
Vertical deflection	$\hat{\delta} = \delta/\delta_o$	$(= (x_1 * x_2)^{-1})$

The baseline design parameters are denoted by subscript 'o'. Details are given in Appendix A, Table A-1.

The normalized multi-objective optimization model of the present truss problem is cast in the following: Find the design variables vector $\vec{x} = (x_1, x_2)$ which minimizes:

$$F(\vec{x}) = \alpha * F_1(\vec{x}) + (1 - \alpha) * F_2(\vec{x}); \ \alpha = [0, 1] \quad (1)$$

Subject to the constraints

Mass :	$F_1 - 1.0 \leq 0.0$	(2a)
Buckling :	$F_2 + 1.0 \leq 0.0$	(2b)
Stress :	$(\sigma/\sigma_{allow}) - 1.0 \leq 0.0$	(2c)
Deflection:	$(\delta/\delta_{allow)}) - 1.0 \leq 0.0$	(2d)
Lower and upper limits:	$(\vec{x})^L \leq \vec{x} \leq (\vec{x})^U$	(2e)

where α is a weighting factor to be determined iteratively based on achieving balanced improvements in all the selected design objectives.

III. OPTIMIZATION METHOD

One of the most robust techniques is genetic algorithm (*GA*), which is classified as a global, non-gradient optimization algorithm based on the process of natural selection. It has been improved in several ways to make it faster and more efficient. Another powerful technique is the sequential quadratic programming (*SQP*), which is a gradient-local method widely used in many engineering applications. However, a global version is available in the *MATLAB* optimization toolbox. The combination of both *GA* and *SQP* are found to be more powerful than the single use of each of them [8]. The hybrid algorithm (*GA-SQP)*, which is implemented in this study, works as follows:

a. Initial values for the design variables are randomly generated.
b. Optimization is executed using *GA* to determine the best solution for the optimization problem.
c. The best solution of *GA* is employed in the *SQP* method to search for a better feasible solution.

IV. RESULTS AND DISCUSSIONS

The baseline design, to which the attained optimum designs are compared, shall have the same layout, material properties and subjected to the same vertical load *P* (refer to Appendix A).It is appropriately designed to have conservative strength satisfying all design requirements.

The attained optimal solutions for continuous variation of the weighting factor α are given in Table 1 and plotted in Fig.1. Three domains based on the range of the weighting factor **α**can be observed:

a. **α=0.0 to 0.835:** Buckling dominates producing the strongest truss designs with the maximum stability gain of 77.76%. The structural mass cannot be minimized below its baseline value resulting in zero mass saving. Therefore, the *MOO* model is equivalent to a single optimization model in the form: $F(\vec{x}) = F_2 \ (= -\widehat{F}_{cr})$.
b. **α=0.0.835 to 0.836:** Favorable transition range, where a remarkable change occurs in both mass and buckling load. It can be observed that the shape of the overall objective function level curves inside the defined design space change very significantly, as depicted in Fig. 2. This range represents a real *MOO*

problem. It encompasses the balanced optimum point *"BOP"* $\alpha_{opt} = 0.8358$, at which the achieved optimization gains are nearly equal (mass saving ≅ stability gain = 13.82%), as shown in Fig.1.

c. **α=0.836 to 1.0:** Structural mass dominates with maximum mass saving of 17.45%. The critical buckling load cannot be maximized above its baseline value. This case is equivalent to a single-objective optimization model in the form: $F(\vec{x}) = F_1 \ (= \widehat{M})$.

α	$\vec{x} = (x_1, x_2)$	$F_1 = x_1 x_2$	$F_2 = x_1^3 x_2$	Mass Saving %	Stability Gain %
0.0 – 0.835	(1.333, 0.75)	1.0	1.7776	0.0	77.76
0.8358	**(1.143, 0.75)**	**0.8575**	**1.1425**	**13.82**	**13.81**
0.836 – 1.0	(1.1006, 0.75)	0.8255	1.0	17.45	0.0

Table 1. Optimal solutions for continuous variation of the weighting factorα.

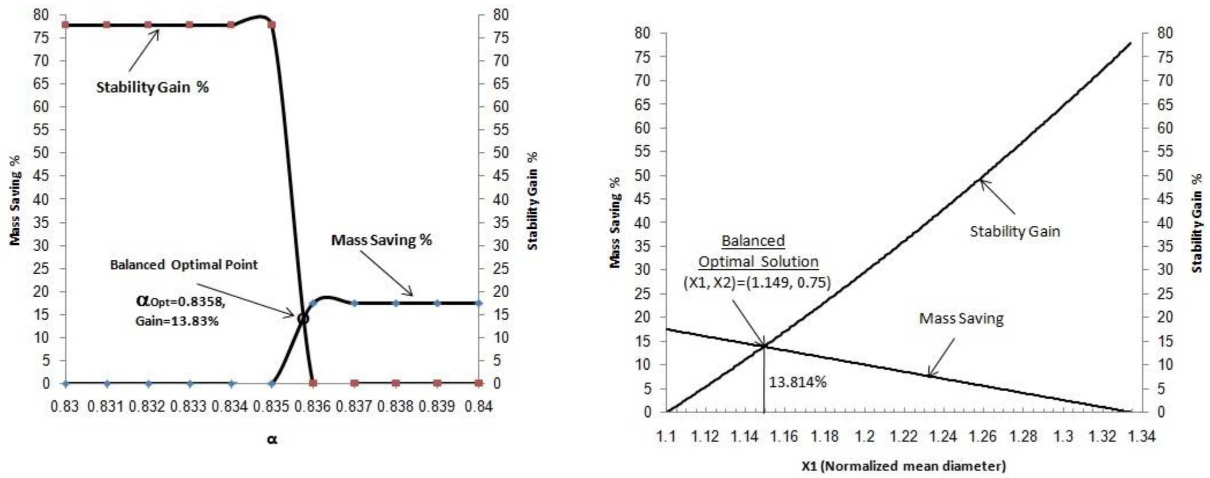
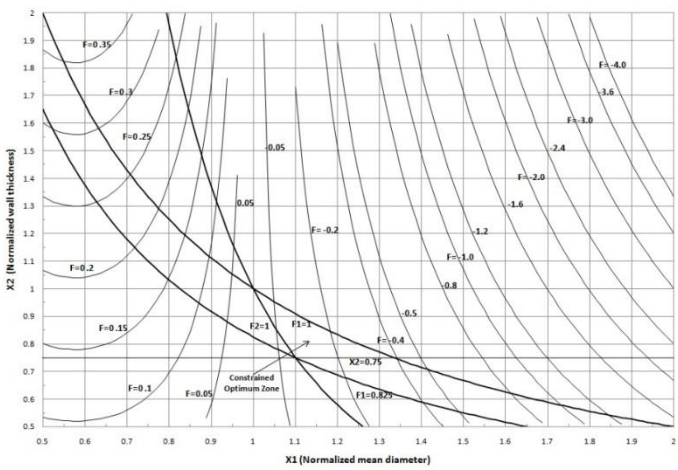


Fig.1. Balanced multi-objective optimization of two-bar truss structure
Mass saving $=(1.0 - \widehat{M})x100\%$**, Stability gain** $=(\widehat{F}_{cr} - 1.0)x100\%$

(a)



(b)

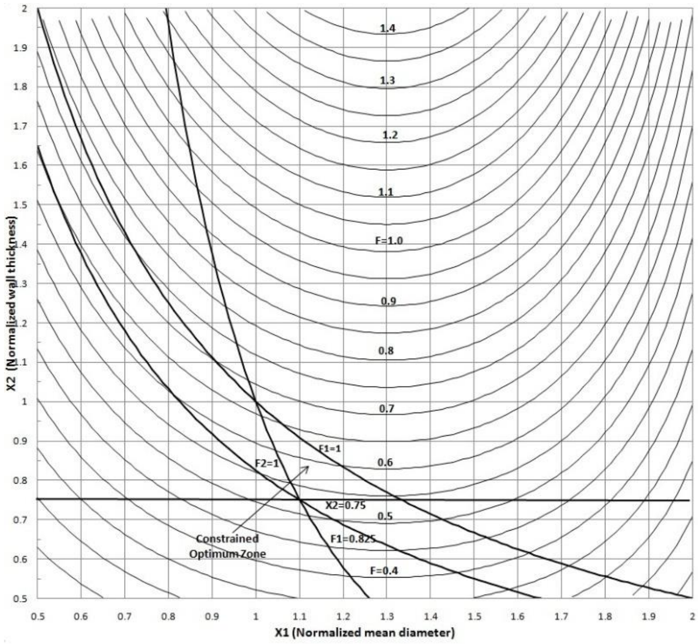


Fig.2. Design spaces at the bounds of the weighting factor favorable range: (a) $\alpha = 0.385$ and (b) $\alpha = 0.386$.

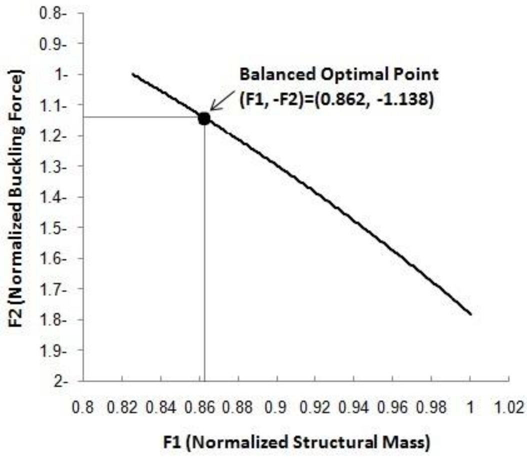


Fig.3.Balanced optimal solution located on Pareto-front curve.
Design objectives space ($F_1 = \widehat{M}, F_2 = -\widehat{F}_{cr}$)

Figure3 depicts the calculated Pareto-optimal front within the favorable range of the design objectives space. The attained balanced optimal point (*BOP*), which represents a unique *MOO*, is seen to occur at a dimensionless structural mass = 0.862 and dimensionless buckling load = -1.138, ensuring balanced improvements in both objectives.

V. CONCLUSION

In this paper, a novel approach is given for achieving balanced multi-objective optimization of structural mass and buckling strength, which are crucial in most thin-walled structures. The developed methodology eliminates the uncertainty in ranking and selecting a solution from the set of non-dominated Pareto-optimal solutions and guarantees the determination of the best unique design point that result in equal optimization gains for all conflicting design objectives. In addition, the proposed approach eliminates the complication in *TOPSIS* method for determining the required ideal and nadir solutions [2]. Current work considers balanced structural multidisciplinary optimization of thin-walled composite beams, plates and shells.

Appendix A: Base Line Design of a Two-Bar Truss Structure

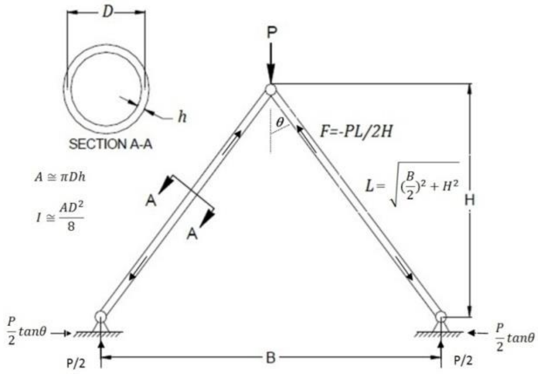


Fig. A-1. Symmetrical two-bar truss structure under vertical load *p*.

Parameter	Notation	Numerical Value
<u>Layout and cross-sectional parameters</u> Truss span Truss height Mean diameter Wall thickness	 *B* *H* D_o * h_o	 1.50 m 0.75 m $5x10^{-2}$ m $0.4\ x10^{-2}$ m
<u>Material of construction</u> Mass density Modulus of elasticity Allowable stress	*High strength steel (ASTM A514)* ρ *E* σ_{allow} (=0.6 σ_{yield})	 8050 Kg/m^3 200 GPa 410 MPa
Allowable deflection	δ_{allow}	$0.635x10^{-2}$ m
Applied load	*P*	300 KN
Axial compressive force	*F* (=*PL/2H*)	212.13 KN
Critical buckling force	$F_{cr,o}$ $(=\pi^2 EI_o/L^2)$	344.3 KN
Applied stress	σ_o $(=F/A_o)$	338 MPa
Maximum deflection	δ_o $(=PL^3/2A_oH^2E)$	$0.25\ x\ 10^{-2}$ m
Structural mass	M_o $(=2\rho A_o L)$	10.725 Kg

* $(\hat{D},\hat{h})^L = (0.75, 0.75)$, $(\hat{D},\hat{h})^U = (1.75, 1.75)$

Table A-1: Baseline design parameters

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship and/or publication of this technical note.

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