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The Asymptotic Behaviour of Laminar Forced Convection for Herschel-Bulkley Fluid in a Circular Duct with Viscous Dissipation Effects

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ABSTRACT
The asymptotic behavior of laminar forced convection in a circularduct, for a Herschel-Bulkley fluid with constantproperties, is analyzed by taking into account the viscous dissipation effects. The axial heat conduction in the fluid isneglected. The asymptotic temperature field and the asymptotic value of the Nusselt number are determined for every boundary condition that allows a fully developed region. Comparisons with other existing solutions for Newtonian and non-Newtonian cases are presented.

Keywords: *Asymptotic behavior, Herschel-Bulkley fluid, variable wall heat flux, laminar forced convection, viscous dissipation.*

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Nomenclature

a	ratio of yield shear stress to wall shear stress
$b(R)$	solution of Eqs. (19) and (20)
$Bt(X)$	local Brinkman number, $\frac{K u_m^{n+1}}{(2r_0)^n q_{\omega(X)}}$
C	dimensionless constant employed in Eq. (18)
c_p	specific heat at constant pressure
f	function of R employed in Eq. (15)
F	function of R and $b(R)$ employed in Eq. (22)
g	arbitrary function of r and x
K	consistency index (Pa.s)
m	inverse of power-law exponent, $\frac{1}{n}$
n	power-law exponent
Nu	Nusseltnumber, $\frac{2r_0 q_w}{[\lambda(T_w - T_b)]}$
Pe	Peclet number, $\frac{2r_0 u_m \rho c_p}{\lambda}$
q_w	wall heat flux, J/s
r	radial coordinate, m
r_0	radius of the tube, m
R	dimensionless radial coordinate, $\frac{r}{r_o}$

T	temperature, K
T_0	inlet temperature distribution, K
u	velocity component in the axial direction, $m \cdot s^{-1}$
u_m	mean axial velocity, $m \cdot s^{-1}$
U	dimensionlessaxialvelocity, $\dfrac{u}{u_m}$
x	axial coordinate, m
X	dimensionless axial coordinate, $\dfrac{x}{2 r_0 Pe}$

Greeks Symbols

β	dimensionless parameter defined in Eq. (14)
λ	thermal conductivity of fluid, $W \cdot m^{-1} \cdot K^{-1}$
ω	dimensionless parameter, $\omega = 1 - 2\left(\dfrac{a(1-a)}{m+2} + \dfrac{(1-a)^2}{m+3}\right)$
ρ	fluid density, $kg \cdot m^{-3}$
τ_c	yield shear stress, Pa
τ_w	wall shear stress, Pa
θ	dimensionless temperature, $\dfrac{\lambda r_0^{n-1}(T - T_{0b})}{K u_m^{n+1}}$
Θ	dimensionless temperature, $\dfrac{(T_w - T)}{(T_w - T_b)}$

Subscripts

b	bulk quantity
w	wall condition
∞	quantity evaluated for $X \to +\infty$

I. Introduction

Considerable attention has been devoted to convective heat transfer in non-Newtonian fluids during the past few years, mainly because of the increasing importance of these fluids in variouschemical, processing, and nuclear industries.Heat transfer to Herschel-Bulkley fluids in laminar flow through tubes has been investigated to some extent. Nouar et al. [1] presented a theoretical and experimental study, considering a constant wall heat flux boundary condition. In this paper, a correlation of Nusselt number is proposed taking into account the modification of the wall shear rate induced by the rheological properties, and the temperature dependent character of the fluid. In a similar study, Nouar et al. [2] obtained numerical results assuming fully developed flow at the entrance of the heated region. Two boundary conditions have been considered, constant wall heat flux and constant wall temperature. Axial conduction was neglected, and the temperature dependence of the consistency index was considered. Correlations for friction factor and Nusselt number were also proposed. Javaherdeh and Devienne [3] presented experimental and numerical results concerning heat transfer for Herschel-Bulkley fluids, the consistency of which depends on temperature. They have considered the flow through a horizontal cylindrical duct submitted to a wall cooling by an external counter current flow.They developed a simple model predicting the wall temperature distribution.

Sayed-Ahmed [4] introduced a numerical solution for laminar heat transfer of a Herschel-Bulkley fluid in the entrance region of a square duct assuming fully developed velocity profile. He solved the energy equation with dissipation effect using an implicit Crank-Nicolson method. Analytical solutions are obtained by Pinho [5] for heat transfer in concentric annular flows of viscoelastic fluids modeled by the simplified Phan-Thien-Tanner constitutive equation. Solutions for thermal and dynamic fully developed flow are presented for both imposed constant wall heat fluxes and imposed constant wall temperatures, always taking into account viscous dissipation. Khatyr et al. [6] give analytical solutions for fully developed laminar forced convection in circular ducts for a Herschel-Bulkely fluid in a horizontal duct heated uniformly, and with various axial distributions of wall heat flux for which polynomial and logarithmic functions was considered as examples. Heat transfer with the effect of viscous dissipation for steady, laminar, both hydro-dynamically and thermally fullydeveloped pseudo-plastic fluid through a channel of Couette-Poiseuille flow, where both the plates are kept at specified but different constant heat flux ratios being considered as thermal boundary conditions is studied by Sheela-Francisca [7]. Rashidi and Erfani [8] studied analytically the thermal-diffusion (Soret effect) and diffusion transfer of a steady MHD (magnetohydrodynamic) convective and slip flow due to a rotating disk with viscous dissipation and ohmic heating. They presented the influence of the slip parameter and the magnetic field parameter and of Eckert, Schmidt, DuforandSoret numbers on the profiles of the dimensionless velocity, temperature and concentration distributions. Rashidi et al. [9] studied analytically the effect of the buoyancy force and thermal radiation in MHD boundary layer viscoelastic fluid flow over a continuously moving stretching surface in a porousmedium. They concluded that the effect of viscoelastic parameter is to decrease the velocity and increase the temperature in boundary-layer.Abbasbandy et al. [10] presented the numerical and analytical solutions for Falkner-Skan flow of MHD Oldroyd-B fluid. They used homotopy analysis method and numerical Keller-box method. They concluded that the skin friction coefficient in Oldroyd-B fluid is larger when compared with viscous fluid, and that the relaxation and retardation times have opposite effects on the velocity components. Recently, fourth order Runge-Kutta method has been used to investigate the unsteady MHD free convective boundary-layer flow due to a permeable stretching vertical surface in a nano-fluid [11].

To our knowledge, no semi-analytical solution of the forced convection with viscous dissipation in a circular duct of Herschel-Bulkley fluid with no-uniform wall heat flux distribution $q_w(x)$ which tends to infinity for large value of x, is available in the literature. The aim of the present work is to study a fully developed laminar forcedconvection in circular ducts for a Herschel-Bulkely fluid with viscous dissipation and negligible axial heat conduction in the fluid. The effect of the dimensionless radius of the plug core, the power-law exponent and the Brinkman number are presented and compared to those obtained in previous works.

This article is organized as follows: in Section 2, the considered fully developed velocity profile and the energy equations are presented; Section 3 is devoted to the establishment and discussion of the results of the asymptotic behavior of the temperature field and the Nusselt number; the conclusion is summarized in Section 4.

II. Analysis

Let us consider a Herschel-Bulkley fluid of constant physical properties flowing ina circular duct of radius r_0, submitted to a variable axial wall heat flux $q_w(x)$. The flow is supposed to be steady, laminar, fully developed and axisymmetric. Thefully developed velocity profile for a laminar pipe flow of a Herschel-bulkley is given as follows [12]:

$$u(r) = \begin{cases} \frac{u_m}{\omega}\left(1-\left(\frac{\frac{r}{r_0}-a}{1-a}\right)^{m+1}\right) & \text{if } r_c \le r \le r_0 \\ \frac{u_m}{\omega} & \text{if } 0 \le r \le r_c \end{cases} \tag{1}$$

where, $\omega = 1 - 2\left(\frac{q(1-a)}{m+2} + \frac{(1-a)^2}{m+3}\right)$, $m = \frac{1}{n}$ is the inverse of exponent index n, $a = \frac{\tau_c}{\tau_w} = \frac{r_c}{r_0}$ is the dimensionless radius of the plug flow region, τ_c the yield shear stress, τ_w the wall shear stress, r the radial coordinate, r_c the yield radius, and u_m the mean value of velocity.

The energy equation and associated boundary conditions are given by Bejan[13]

$$\rho c_p u \frac{\partial T}{\partial x} = \frac{\lambda}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] + k\left|\frac{du}{dr}\right|^n \frac{du}{dr} - \tau_c \frac{du}{dr} \tag{2}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0, \left.\frac{\partial T}{\partial r}\right|_{r=0} = \frac{q_w(x)}{\lambda} \tag{3}$$

$$T(r, x=0) = T_0(r) \tag{4}$$

where, ρ, λ, k and c_p are the density of fluid, thermal conductivity, the consistency index, and the specific heat at constant pressure, respectively. The condition that leads to an asymptotic thermally developed region in the case of the forced convection problem described above is defined by Bejan [13]

$$\lim_{x \to +\infty} \frac{T_w(x) - T(r,x)}{T_w(x) - T_b(x)} = \lim_{x \to +\infty} \Theta\left(\frac{r}{\frac{r_{0,x}}{2r_0 Pe}} \right) = \Theta_\infty\left(\frac{r}{r_0}\right) \tag{5}$$

where, $T_w(x)$ and $T_b(x)$ are the wall temperature and the bulk temperature, respectively, Pe is the Peclet number, and $\Theta_\infty\left(\frac{r}{r_0}\right)$ is the asymptotic dimensionless temperature which is a continuous and differentiable function of r. The bulk value of an arbitrary function $g(r,x)$ is defined as

$$g_b(x) = \frac{2}{u_m r_0} \int_0^{r_o} g(r,x) u(r) r dr \tag{6}$$

If condition (5) holds, the asymptotic value of the Nusselt number Nu_∞ exists in Bejan[13] and is given by

$$\lim_{x \to \infty} Nu = 2r_0 \lim_{x \to \infty} \frac{\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w(x) - T_b(x)} = -2r_0 \left.\frac{d\Theta_\infty}{dr}\right|_{r=r_0} = Nu_\infty \tag{7}$$

The proof presented by Barletta [14], allows to check that the boundary value problem, expressed by Eqs. (2)-(4), has a unique solution, and that both the asymptotic behaviour of the temperature field and of the Nusselt number are independent of the inlet section temperature distribution.
Introducing the dimensionless quantities

$$X = \frac{x}{2r_0 Pe}, R = \frac{r}{r_0}, U(R) = \frac{u(r)}{u_m}, \theta = \lambda r_0^{n-1} \frac{T - T_{0b}}{K u_m^{n+1}} \tag{8}$$

Eqs. (2) and (3) can be rewritten in the dimensionless form

$$\frac{\partial}{\partial R}\left[R \frac{\partial \theta}{\partial R}\right] = \frac{RU}{4}\frac{\partial \theta}{\partial X} + \frac{a}{(1-a)^{n+1}}\left(\frac{m+1}{\omega}\right)^n R\frac{dU}{dR} - R\left|\frac{dU}{dR}\right|^{n-1}\left(\frac{dU}{dR}\right)^2 \tag{9}$$

$$\left.\frac{\partial \theta}{\partial R}\right|_{R=0} = 0 \qquad \left.\frac{\partial \theta}{\partial R}\right|_{R=1} = \frac{1}{2^n Br(X)} \tag{10}$$

where, $Br(X)$ is a local Brinkman number defined as:

$$Br(X) = \frac{K u_m^{n+1}}{(2r_0)^n q_\omega(X)} \tag{11}$$

Integrating Eq. (9) over the interval $0 \le R \le 1$ and employing Eq. (10) yields

$$\frac{\partial \theta_b}{\partial X} = \frac{2^{3-n}}{Br(X)} + \frac{8}{(1-a)^{n+1}}\left(\frac{m+1}{\omega}\right)^n \tag{12}$$

where, $\theta_b(X)$ is the bulk value of the dimensionless temperature $\theta(R,X)$

III. Asymptotic Behavior of the Temperature Field

In this work, the asymptotic temperature field and the asymptotic Nusselt number are analyzed in the case of axial distributions of wall heat flux which yield a thermally developed region, such as

$$\lim_{X \to +\infty} Br(X) = 0 \tag{13}$$

and

$$\lim_{X \to +\infty} \frac{1}{Br(X)} \frac{dBr(X)}{dX} = -2\beta \tag{14}$$

where, β is a non-vanishing positive real number. Eq. (13) shows that the effect of viscous dissipation is negligible in the thermally developed region. Eqs. (13) and (14) are satisfied by axial wall heat flux distributions which tends to infinity when $X \to +\infty$, and which behave asymptotically as $Q(X)e^{2\beta X}$, $Q(X)$ can be a polynomial function, or rational function where the degree of the numerator is greater than or equal to the degree of the denominator, or any other function satisfying Eq. (13).

Therefore, in these distributions the dimensionless temperature field for large value of X can be expressed by

$$\theta(R, X) = \theta_m(X) + \frac{f(R)}{Br(X)} \tag{15}$$

By substituting Eq. (15) in Eqs. (9) and (10) and taking into account Eqs. (12)(14), one obtains

$$\frac{df}{dR}\left[R\frac{df}{dR}\right] = \frac{RU}{2}\left(2^{2-n} + \beta f\right) \tag{16}$$

$$\left.\frac{df}{dR}\right|_{R=0} \qquad \left.\frac{df}{dR}\right|_{R=1} = \frac{1}{2^n} \tag{17}$$

Eq. (16) can be reduced to a first-order differential equation using the following transformation

$$f(R) = \frac{2^{2-n}}{\beta}\left[C \exp\int_0^R b(R')\,dR' - 1\right] \tag{18}$$

where, C is a constant given by the boundary condition at $R = 1$ and $b(R)$b is a continuous and differentiable function of R.

Substituting Eq. (18) into Eq. (16), gives

$$R\frac{db}{dR} + b + Rb^2 = \frac{R\beta U}{2} \tag{19}$$

Eqs. (17) and (18) become then

$$b(0) = 0 \tag{20}$$

$$C = \frac{\beta}{4b(1)}\exp\left(-\int_0^1 b(R')\,dR'\right) \tag{21}$$

Eq. (19) can also be written as:

$$\frac{db}{dR} = F(R,b) = -\frac{b}{R} - b^2 + \frac{\beta U}{2} \tag{22}$$

This equation with boundary condition (20) is integrated numerically using fourth-order Runge-Kutta method [15, 16]. This method is still one step, but dependent on estimates of the solution at different points, and requires 4 evaluations of function $F(R,b)$ at every time step.

R is the independent variable, $b = b(R)$ is the unknown function of R, $b_0 = b(0)$ is the given condition (Eq. (20)), and F is a given function of R and b which describes the differential Eq. (22)

The variable R is discretized, say R_i for $i = 0,1,2,\ldots,$ then we determine $b_i = b(R_i)$ for $i = 0,1,2,\ldots$ If b_i is calculated, then we construct b_{i+1} asfollow:

$$b_{i+1} = b_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h \tag{23}$$

With

$$k_1 = hF(R_i, b_i)$$

$$k_2 = hF\left(R_i + \frac{h}{2}, b_i + \frac{hk_1}{2}\right)$$

$$k_3 = hF\left(R_i + \frac{h}{2}, b_i + \frac{hk_2}{2}\right)$$

$$k_4 = hF\left(R_i + h, b_i + k_3\right)$$

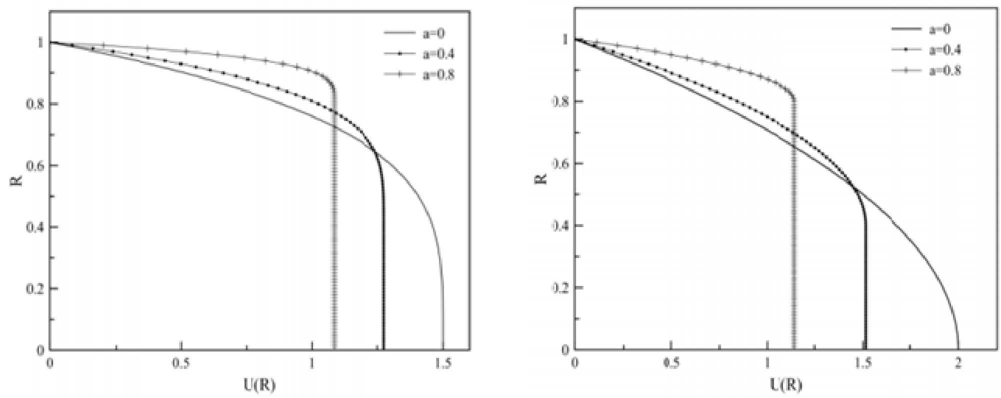
where,

$$h = R_{i+1} - R_i$$

Thereafter, the constant C and the function $f(R)$ are computed by usingSimpson integration method.The asymptotic dimensionless temperature $\Theta_\infty(R)$ isdetermined by the following expression

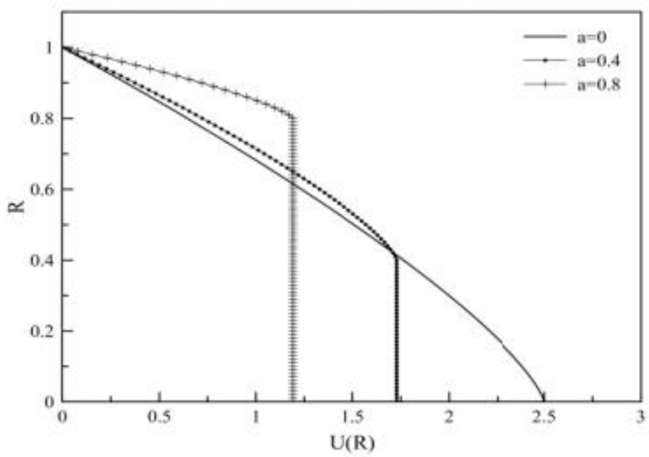
$$\Theta_\infty(R) = \frac{f(1) - f(R)}{f(1)} \tag{24}$$

Taking into account Eqs. (7), (17), (18),(21) and (24), the asymptotic value of Nu is given by



$$Nu_\infty = -2\frac{d\Theta_\infty}{dR}\bigg|_{R=1} = \frac{2\beta b(1)}{\beta - 4b(1)} \tag{25}$$

(a) $n = \frac{1}{3}$ (b) $n = 1$



(c) $n = 3$

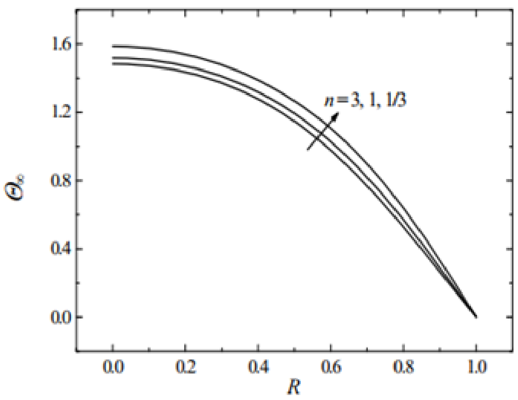
Fig. 1 Velocity profiles used in the simulations.

Figs. 1a-1c represent the variation of dimensionless axial velocity U versus R for different values of a and n .

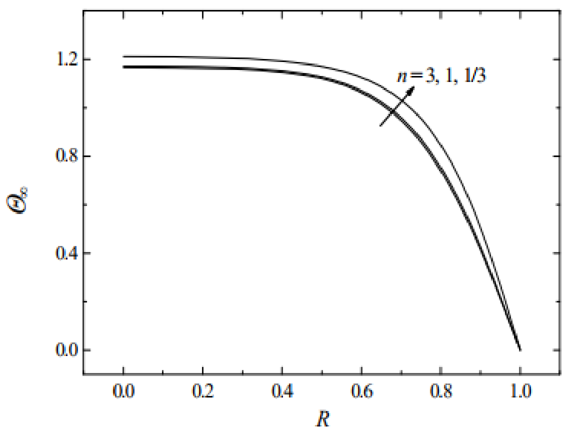
The obtained asymptotic values of Nusselt number arecomparedwiththoseof Barletta[14] inthepower-lawfluidcase $(a = 0)$ (Table1).Wecannote thatthe comparison in the power-law case is verysatisfactory.Figs.2a-2cshow theasymptotictemperatureprofilefordifferentvaluesof β andfor $a = 0.4$. One notes that for large values of β such as $\beta = 1000$ (Fig. 2c), the temperature $\Theta_{\infty}(R)$ does not vary significantly with respect to n . Figs. 3a-3c represent the variation of Nu_{∞} versus β for different values of a and of n . These figures show that for n fixed, Nu_{∞} increaseswith$a$.When β increasesand n decreases,theeffect of the yield stress becomes important(Tables 2 and 3).

Table 1 Values of Nu_{∞} for various values of n compared with those of Barletta [14] in the power-law fluid case $(a = 0)$.

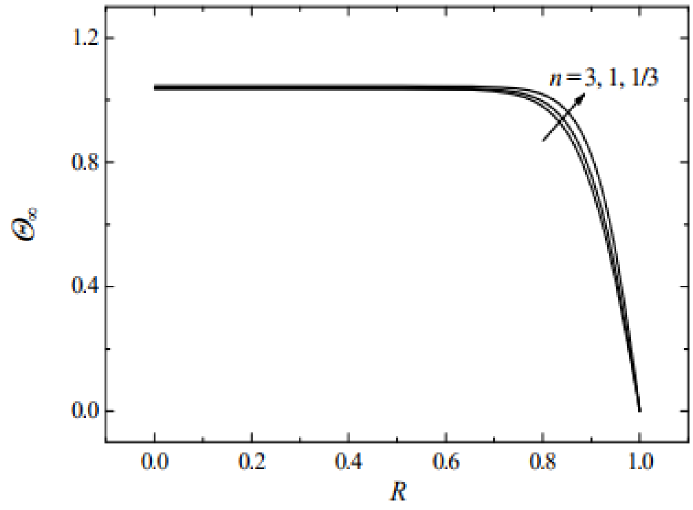
β	$n = \frac{1}{5}$		$n = \frac{1}{3}$		$n = 3$	
	Present work	Barletta [14]	Present work	Barletta [14]	Present work	Barletta [14]
1	5.6141	5.6141	5.1431	5.1431	4.1324	4.1324
5	5.9774	5.9774	5.4818	5.4818	4.4358	4.4358
10	6.3858	6.3858	5.8613	5.8613	4.7715	4.7715
20	7.0896	7.0897	6.5129	6.5129	5.3395	5.3395
30	7.6865	7.6865	7.0634	7.0634	5.8131	5.8132
40	8.2081	8.2082	7.5436	7.5436	6.2230	6.2230
50	8.6740	8.6741	7.9717	7.9717	6.5860	6.5860
60	9.0968	9.0969	8.3597	8.3597	6.9143	6.9144
70	9.4851	9.4852	8.7159	8.7159	7.2145	7.2146
80	9.8452	9.8453	9.0459	9.0459	7.4920	7.4921
90	10.1817	10.1818	9.3542	9.3542	7.7508	7.7509
100	10.4982	10.4983	9.6439	9.6439	7.9937	7.9938
200	12.9731	12.9732	11.9067	11.9067	9.8822	9.8824
500	17.4740	17.4740	16.0133	16.0133	13.2900	13.2902
1,000	22.0460	22.0461	20.1794	20.1794	16.7352	16.7354
10,000	48.3120	48.3125	44.0751	44.0751	36.4280	36.4282



(a) $\beta = 10$

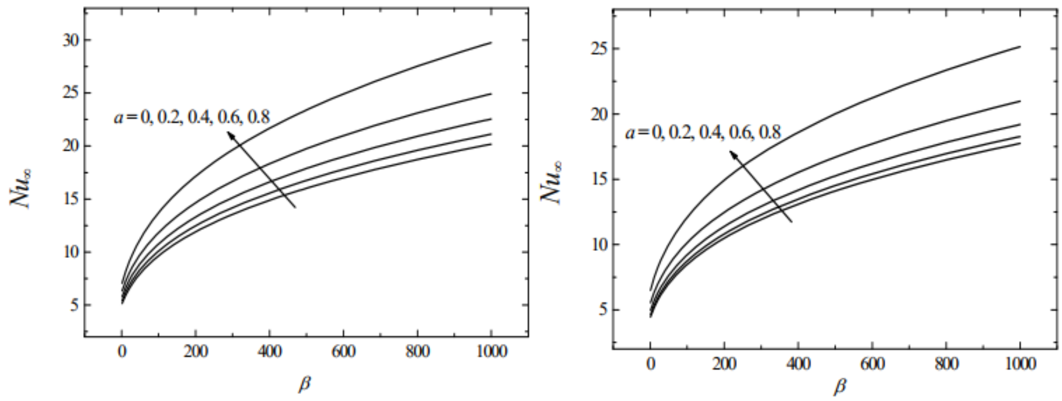


(b) $\beta = 100$

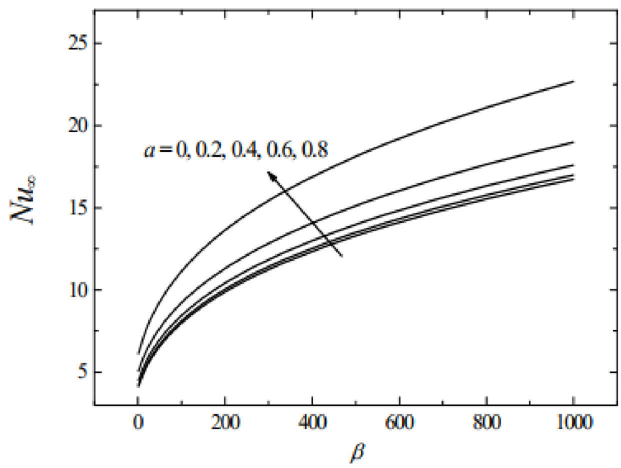


(c) $\beta = 1000$

Fig. 2 Evolution the $\Theta_\infty(R)$ for various values of n and β , and for $a = 0.4$.



(a) $n = \frac{1}{3}$ **(b)** $n = 1$



(c) $n = 3$

Fig. 3 Variation of Nu_∞ versus β for various values of a : (a) $n = \frac{1}{3}$, (b) $n = 1$ and (c) $n = 3$.

Table 2 Asymptotic values of Nu for various values of β and a, and for $n = \frac{1}{3}$.

β	$a = 0$	$a = 0.2$	$a = 0.4$	$a = 0.6$	$a = 0.8$
1	5.1431	5.4157	5.7934	6.3201	7.0691
5	5.4818	5.7652	6.1624	6.7273	7.5491
10	5.8613	6.1576	6.5775	7.1849	8.0916
20	6.5129	6.8333	7.2937	7.9765	9.0367
30	7.0634	7.4054	7.9014	8.6496	9.8454
40	7.5436	7.9051	8.4329	9.2390	10.5569
50	7.9717	8.3511	8.9077	9.7658	11.1949
60	8.3597	8.7557	9.3385	10.2442	11.7756
70	8.7159	9.1272	9.7345	10.6839	12.3101
80	9.0590	9.4717	10.1016	11.0916	12.8070
90	9.3542	9.7935	10.4448	11.4729	13.2717
100	9.6439	10.0960	10.7674	11.8314	13.7093
200	11.9067	12.4610	13.2914	14.6359	17.1407
500	16.0133	16.7582	17.8824	19.7383	23.3965
1,000	20.1794	21.1208	22.5471	24.9243	29.7537
10,000	44.7351	46.1638	49.3499	54.7492	66.2931

Table 3 Asymptotic values of Nu for various values of β and a, and for $n = 3$.

β	$a = 0$	$a = 0.2$	$a = 0.4$	$a = 0.6$	$a = 0.8$
1	4.1324	4.2393	4.5164	5.0840	6.1377
5	4.4358	4.5415	4.8154	5.3939	6.5156
10	4.7715	4.8767	5.1493	5.7413	6.9385
20	5.3395	5.4461	5.7208	6.3386	7.6655
30	5.8331	5.9225	6.2023	6.8438	8.2795
40	6.2230	6.3354	6.6217	7.2849	8.8143
50	6.5863	6.7021	6.9952	7.6786	9.2904
60	6.9143	7.0334	7.3335	8.0358	9.7210
70	7.2145	7.3368	7.6437	8.3638	10.1158
80	7.4920	7.6175	7.9310	8.6680	10.4809
90	7.7508	7.8793	8.1993	8.9522	10.8216
100	7.9937	8.1250	8.4514	9.2195	11.1413
200	9.8822	10.0373	10.4173	11.3105	13.6280
500	13.2900	13.4911	13.9773	15.1144	18.1187
1,000	16.7352	16.9843	17.5825	18.9770	22.6725
10,000	36.4280	36.9584	38.2165	41.1323	48.8644

IV. Conclusions

Laminar and hydrodynamically developed forced convection of aHerschel-Bulkley fluid flowing in a circular tube with a prescribed axial distribution of wall heat flux has been studied. The effect of viscous dissipation has been taken into account, while the axial heat conduction in the fluid has been considered as negligible. It has been supposed that, when $x \to +\infty, q_w(x)$ tends to infinity, while $\left(\frac{1}{q_w(x)}\right)\left(\frac{dq_w(x)}{dx}\right)$ tends to a positive constant. If these conditions are fulfilled, the effect of viscous dissipation becomes negligible in the thermally developed region and the asymptotic value of the Nusselt number is a function of n, a and the dimensionless parameter β. The asymptotic values of the Nusselt number Nu have been evaluated numerically for some values of n, a and β. The comparisons between our theoretical results and those published in the literature for the Newtonian fluid case and the non-Newtonian fluid case (power-law fluid) show very closeagreement.

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