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Modified Adomian Decomposition Method and Padé Approximant for the Numerical Approximation of the Crime Deterrence Model in Society

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ABSTRACT
In this research paper, the Laplace transform method combined with the semi-analytical Adomian decomposition method (LADM) is proposed to solve the mathematical model of crime deterrence in society. The model is solved to obtain analytical solution to the governing parameters of interest in the form of a rapidly convergent series to illustrate its reliability, capability and efficiency of this hybrid method. The practical result obtained reveal, the method is accurate and an efficient tool for solving wide variety of several first and higher order models.
KEYWORD: *Adomian decomposition method, crime model, hybrid, analytical, modified Adomian decomposition method*

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I. INTRODUCTION

We consider the following crime deterrence model in society [1-2], which is an autonomous nonlinear differential equation with four parameters of interests: susceptible, criminals, police force and prisoners.

$$\begin{cases} \frac{dS}{dt} = \left(1-\rho(P)\right)A - \frac{\beta SC}{N} + \theta v R - dS \\ \frac{dC}{dt} = \rho(P)A + \frac{\beta SC}{N} - \frac{\gamma CP}{N} + (1-\theta)vR - (\alpha + d)C \\ \frac{dR}{dt} = \frac{\gamma CP}{N} - vR - dR \\ \frac{dP}{dt} = \phi C - \phi_0(P - P_0) \end{cases} \tag{1}$$

Subject to the initial condition

$$S(0) = S_0 > 0, C(0) = C_0 \geq 0, R(0) = R_0 \geq 0, P(0) = P_0 > 0, \text{and } 0 < \theta < 1 \tag{2}$$

where, $S(t), C(t), R(t), P(t)$ are the people prone to criminality but yet to commit any crime, people who are already involved in criminal activities, prisoners in jail and number of police force at time t. Similarly, parameters, $\frac{\beta SC}{N}, \frac{\gamma CP}{N}, \rho(P), 1-\rho(P)$ and A represent conversion rate of people susceptible to criminality, criminals arrested and imprisoned according to the law, immigrants' criminals as a decreasing function of police force and constant immigration of criminals.

Equally, if the number of police personnel required to manned a given region at time, t is inadequate or decreased due to retirement or mortality, ϕ_0, there will be influx of immigrant which will provide a possibility of high transmission rate of criminals among the immigrants, β. Hence in order to restore sanity and keep crime to a minimum, there is need for additional recruitment to the police force, ϕ, with this more criminal activities can be nipped in the bud, thus will lead to high incarceration rate, γ, more criminals will be residing in jail or rehabilitation centre, R. Consequently, the outflow rate of people due to emigration or mortality, d and crime associated death rate will increase, α and $(1-\theta)$ become the recidivistic fraction that gets involved in criminality.

The semi-analytical Laplace Adomian decomposition method (LADM) which is the fusion between the Adomian decomposition and the Laplace transform methods was first proposed by Khuri [3]. The advantage of this hybrid method is its ability to obtain exact solutions to diverse functional equations ranging from both ordinary and nonlinear partial differential equations in less time with minimal computational time. [4-6]. The method has been applied successfully to various problems in engineering and science. Khuri and Alchikh [7] explored Pade approximation to the solution obtained using LADM to increase its convergence. The approximate solution of a class of nonlinear ordinary differential equation was investigated by Khuri et al [8]. Yusofoglu et al [9] examined the Duffing equation using the Laplace Adomian decomposition method. Nasser [10] successively applied LADM to solve the Falker-Skan equation and obtain a series of rapidly converging solution. Ongun [11] solved the HIV infection model of CD4+T cells using LADM. He constructed an analytical solution in the form of polynomial. Pue-on [12] investigated the Newell-Whitehead-Segel equation using the Laplace Adomian decomposition method. The method was shown to have the capacity to solve both linear and nonlinear functional equation Cherruault [13]. Solution of the systems of ordinary differential equation by combined Laplace Adomian decomposition method was examined by Dogan [14]. Also, the combined Laplace Adomian decomposition method is explored in the following literatures: Linear and nonlinear Volterra Integral equations with weak kernel Hendi [15], Volterra Integro-differential equations wazwaz [16], Approximate solution of nonlinear fractional differential equations Yang et al [43], nth-order Integro-differential equation by Waleed [17] and Manafianheris [18]. Numerical solution of the nonlinear system of partial differential equation has been treated with LADM-Pade by Mohamed et al [19], two-dimensional viscous fluid with shrinking sheet by Kamara et al [20], nonlinear coupled partial differential equation Jafari et al [48], Numerical solution to logistics differential equation Khan et al. [21], convection diffusion-dissipation equations by Youssouf et al. [22]

In this manuscript, our main objective is to explore the Laplace Adomian decomposition method (LADM) to obtain a series solution of the Crime deterrence model. This has not been studied before to the best of our knowledge. The Pade approximation is then applied to the analytical solution obtained for the parameters of interest to get a better approximation that best match its Taylor series expansion.

The paper is organised as follows: In section 2, the fundamentals of the Adomian decomposition method is explained in detail. The basic procedures of the hybrid Laplace Adomian decomposition method are presented in section 3. Section 4 explained the Pade approximation which best approximates the solution obtained using the LADM. Numerical application of LADM as it applies to the model, results as well as discussions in figures and tables are contained in section 5 & 6, whereas, the conclusions are given in the final part, section 7.

II. FUNDAMENTALS OF ADOMIAN DECOMPOSITION METHOD (ADM)

In this section, we review the basics of the standard Adomian decomposition method. See [23-33]

Consider a generalized differential equation of the form

$$L[y(x)] + R[y(x)] + N[y(x)] = g(x) \tag{3}$$

Where L is the highest order derivative that's invertible, R is the remainder of the linear differential operator, N is a nonlinear term and $g(x)$ is called the source term

Suppose the differential operator is invertible, such that $L^{-1} = \int_0^x (.)dx$, then operating both sides of Eq. (3) with the inverse operator, we obtain

$$L^{-1}[Ly(x)] + L^{-1}[Ry(x)] + L^{-1}[Ny(x)] = L^{-1}[g(x)] \tag{4}$$

$$L^{-1}[Ly(x)] = \phi(x) - L^{-1}[Ry(x)] - L^{-1}[Ny(x)]$$

$$L^{-1}[Ly(x)] = \phi(x) - L^{-1}[Ry(x) - Ny(x)] \tag{5}$$

$$y(x) = f(x) - L^{-1}[Ry(x) - Ny(x)] \tag{6}$$

Where $f(x)$ is the term obtained from the integration of the source term, $g(x)$ defined as

$$\phi_0(x) = \begin{cases} u(0) & L = \frac{d}{dt} \\ u(0) + xu'(0) & L = \frac{d^2}{dx^2} \\ u(0) + xu'(0) + \frac{x^2}{2}u''(0) & L = \frac{d^3}{dx^3} \\ u(0) + xu'(0) + \frac{x^2}{2}u''(0) + \frac{x^3}{3!}u'''(0) & L = \frac{d^4}{dx^4} \end{cases} \tag{7}$$

By the ADM hypothesis, the zeroth component of Eq. (6) become

$$y_0 = f(x) \tag{8}$$

The recursive scheme of the problem is then given by

$$\begin{cases} y_{k+1} = -L^{-1}(Ry_n) - L^{-1}(Ny_n), \ \ n \geq 0 \\ y_1 = -L^{-1}(Ry_0) - L^{-1}(Ny_0) \\ y_2 = -L^{-1}(Ry_1) - L^{-1}(Ny_1) \\ y_3 = -L^{-1}(Ry_2) - L^{-1}(Ny_2) \end{cases} \tag{9}$$

Decomposing the unknown solution in the form of an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \qquad 10)$$

The nonlinear term is determined by an infinite series of Adomian polynomials as follows

$$N(y) = \sum_{n=0}^{\infty} A_n(y_0, y_1, y_2, \dots) \tag{11}$$

Where the $A's$ are calculated by the relation

$$A_n(y_0, y_1, y_2, \dots) = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{k=0}^{n} \lambda^k y_k)]_{\lambda=0}, \ n = 0,1,2,3 \tag{12}$$

Using the general formula in Eq. (12), the Adomian polynomials $A_n'^s$ are obtained as follows

$$A_1 = \frac{d}{d\lambda} N(u_0 + u_1\lambda)\Big|_{\lambda=0} = N(u_0)u_1$$

$$A_2 = \frac{1}{2!}\frac{d}{d\lambda}(u_1 + 2u_2\lambda)N''(u_0 + u_1\lambda)\Big|_{\lambda=0} = N'(u_0)u_2 + \frac{1}{2!}N''(u_0)u_1^2$$

$$A_3 = N'(u_0)u_3 + \frac{2}{2!}N''(u_0)u_1u_2 + \frac{1}{3!}N'''(u_0)u_1^3$$

$$A_4 = N'(u_0)u_4 + \frac{1}{2!}N''(u_0)(2u_1u_3 + u_2^2) + \frac{3}{3!}N'''(u_0)u_1^2u_2 + \frac{1}{4!}N^{(iv)}(u_0)u_1^4$$

$$A_5 = N'(u_0)u_5 + \frac{1}{2!}N''(u_0)(2u_1u_4 + 2u_2u_3) + \frac{1}{3!}N'''(u_0)(3u_1^2u_3 + 3u_1u_2^2) + \frac{4}{4!}N^{(iv)}(u_0)u_1^3u_2 + \frac{1}{5!}N^{(v)}(u_0)u_1^5$$

$$A_6 = N'(u_0)u_6 + \frac{1}{2!}N''(u_0)(2u_1u_4 + 2u_2u_3) + \frac{1}{3!}N'''(u_0)(3u_1^2u_3 + 3u_1u_2^2) + \frac{4}{4!}N^{(iv)}(u_0)u_1^3u_2 + \frac{1}{5!}N^{(v)}(u_0)u_1^5$$

$$A_7 = N'(u_0)u_7 + \frac{1}{2!}N''(u_0)(2u_1u_4 + 2u_2u_3) + \frac{1}{3!}N'''(u_0)(3u_1^2u_3 + 3u_1u_2^2) + \frac{4}{4!}N^{(iv)}(u_0)u_1^3u_2 + \frac{1}{5!}N^{(v)}(u_0)u_1^5$$

Hence using Eqs. (9-11), the decomposed equation of the problem become

$$\sum_{n=0}^{\infty} y_n(x) = f(x) - \sum_{n=0}^{\infty} L^{-1}\big(y_n(x)\big) - \sum_{n=0}^{\infty} L^{-1}N(A_n)$$

Replacing n by $n+1$ on both sides of the above, we get

$$\sum_{n=-1}^{\infty} y_{n+1}(x) = f(x) - \sum_{n=0}^{\infty} L^{-1}\big(y_n(x)\big) - \sum_{n=0}^{\infty} L^{-1}N(A_n)$$

$$y_0(x) + \sum_{n=0}^{\infty} y_{n+1}(x) = f(x) - \sum_{n=0}^{\infty}\big[L^{-1}\big(y_n(x)\big) - \sum_{n=0}^{\infty} L^{-1}N(A_n)\big] \quad (13)$$

Comparing the above series on both sides of the equation and taking finite terms, the solution become

$$y(x) \approx \sum_{n=0}^{N} y_n(x) \quad (14)$$

Similarly, if we take infinite number of terms, the solution become

$$y(x) = \lim_{N\to\infty} \sum_{n=0}^{N} y_n(x) \quad (15)$$

III. LAPLACE ADOMIAN DECOMPOSITION METHOD (LADM)

To illustrate the basics of the Laplace Adomian decomposition method denoted LADM, we consider the nonlinear functional differential equation in Eq. (3). See [34-36]

$$L_t u(x) + R\big(u(x)\big) + N\big(u(x)\big) = g(x) \quad (16)$$

Subject to the corresponding initial condition as

$$u(x, 0) = \phi(x) \quad (17)$$

Where L_t, N, R and $g(x)$ denotes the first-order differential operator, nonlinear operator, remainder of the linear operator and source term, respectively

Taking Laplace transform of both sides of Eq. (16), we obtain

$$\mathcal{L}[L_t u(x)] + \mathcal{L}\big[R\big(u(x)\big)\big] + \mathcal{L}\big[N\big(u(x)\big)\big] = \mathcal{L}[g(x)] \quad (18)$$

Suppose L_t is a first order differential operator, then the inverse operator exists such that

$$L_t = \frac{d}{dx}, \qquad L^{-1} = \int_0^x (.)dx$$

Implementing LT on Eq. (18) yield an algebraic system of the form

$$\mathcal{L}[L_t u(x)] = \mathcal{L}[g(x)] - \mathcal{L}\big[R\big(u(x)\big)\big] - \mathcal{L}\big[N\big(u(x)\big)\big] \quad (19)$$

$$s\mathcal{L}[u(x)] - \phi(x) = -\mathcal{L}\big[R\big(u(x)\big)\big] - \mathcal{L}\big[N\big(u(x)\big)\big] + \mathcal{L}[g(x)]$$

Rearranging the above, we obtain the algebraic equation of the form

$$\mathcal{L}[u(x)] = \frac{\phi(x)}{s} + \frac{\mathcal{L}[g(x)]}{s} - \frac{\mathcal{L}\big[R\big(u(x)\big)\big]}{s} - \frac{\mathcal{L}\big[N\big(u(x)\big)\big]}{s} \quad (20)$$

Representing the unknown as a decomposition series in Eq. (18), we get

$$\mathcal{L}\big[\sum_{n=0}^{\infty} u_n(x)\big] = \frac{\phi(x)}{s} + \frac{\mathcal{L}[g(x)]}{s} - \frac{\mathcal{L}\big[R\big(u(x)\big)\big]}{s} - \frac{\mathcal{L}\big[\sum_{n=0}^{\infty} A_n\big]}{s} \quad (21)$$

Using Eq. (20), the recursive formula for the above become

$$\mathcal{L}[u_0] = \frac{\phi(x)}{s} + \frac{\mathcal{L}[g(x)]}{s} \quad (22)$$

$$\mathcal{L}[u_{k+1}] = -\frac{\mathcal{L}[R(u(x))]}{s} - \frac{\mathcal{L}[A_n]}{s} \quad (23)$$

Taking the inverse Laplace transform of Eq. (19) yield the required result of the problem in the spatial domain.

IV. THE PADE APPROXIMANT

Frequently in science and engineering, the solution of several mathematical functions is expressed using Taylor series expansion. Among these functions, polynomials are widely used to find the approximation of truncated power series. This is useful, because polynomials never blow up and their singularities are not apparent in a finite region. However, polynomials exhibit oscillations that produce error bound since their radius of convergence cannot contain two boundaries at a time. For this reason, power series may not always be useful to approximate a function. To manipulate the power series for better approximations to the obtained series, a new approximation that best matches the Taylor series as far as possible was proposed by Henri Eugene Pade in 1892 [37-42]. This new approximation is advantageous over the Taylor series in that using a quotient of two polynomials with varying degrees, the inherent errors obtained using Taylor series are improved upon for fast convergence.

Suppose there exists a differentiable function $y(x)$, then the Taylors series expansion of the function near a point $x = a$ is given by the expression

$$y(x) = y(a) + (x-a)\frac{dy}{dx}\bigg|_{x=a} + \frac{(x-a)^2}{2!}\frac{d^2y}{dx^2}\bigg|_{x=a} + \frac{(x-a)^3}{3!}\frac{d^3y}{dx^3}\bigg|_{x=a} + \cdots + \frac{(x-a)^n}{n!}\frac{d^ny}{dx^n}\bigg|_{x=a} + \cdots \quad (24)$$

Thus, a Pade Approximation to a differentiable function, $f(x)$ on a closed interval $[a,b]$ is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of the function. Following Momani [43], the $[M/N]$ Pade approximant to a function is given by

$$[M/N]f(x) = \frac{P_M(x)}{Q_N(x)} = \frac{\sum_{n=0}^{M} a_n x^n}{1+\sum_{n=1}^{N} b_n x^n} = \frac{a_0 + a_1x + a_2x^2 + \cdots a_M x^M}{1 + b_1x + b_2x^2 + \cdots b_N x^N} \quad (25)$$

Where $P_M(x), P_N(x)$ are the polynomials of degree at most M and N

$$Q_N(0) = 1 \quad (26)$$

The central idea behind the Pade Approximation is to replace the power series

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

By a ratio of two polynomials. The error in the approximation is such that

$$y(x) - f(x) = O(X^{M+N+1})$$
$$y(x) - \frac{P_M(x)}{Q_N(x)} = O(X^{M+N+1}), (x \to 0) \quad (27)$$

Multiplying the numerator and denominator by a constant and using the normalization condition in Eq. (27).
Next, $P_M(x)$ and $Q_N(x)$ have common factor such that

$$\left.\begin{aligned} P_M(x) &= a_0 + a_1x + a_2x^2 + \cdots + a_M x^M \\ Q_N(x) &= b_0 + b_1x + b_2x^2 + \cdots + b_N x^N \end{aligned}\right\} \quad (28)$$

To linearize the coefficient equation above, we multiply both sides of Eq. (28) by $Q_N(x)$, using Eqs (27) and (24).
Writing Eq. (23) in rearranged form, we obtain

$$\left.\begin{aligned} a_{M+1} + a_M x_1 + \cdots + a_{N-M+1} x_M &= 0 \\ a_{M+2} + a_{M+1} x_1 + \cdots + a_{M-N+2} x_M &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{M+N} + a_{M+N-1} x_1 + \cdots + a_M x_N &= 0 \end{aligned}\right\} \quad (29)$$

$$\left.\begin{aligned} a_0 &= M_0 \\ a_0 + a_0 N_1 &= M_1 \\ a_2 + a_1 N_1 + a_0 N_2 &= M_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_M + a_{M-1} N_1 + \cdots + a_0 N_M &= M_N \end{aligned}\right\} \quad (30)$$

Solving the set of equations in (29) and (30) for Ms and Ns, the coefficients of $P_M(x)$ and $Q_N(x)$ are easily determined. Then the Pade approximant is obtained using the relation

$$[M/N] = \frac{Det\begin{bmatrix} a_{M-N+1} & a_{M-N+2}\cdots & a_{M+1} \\ \vdots & \vdots & \vdots \\ a_M & a_{M+1} & a_{M+N} \\ \sum_{i=N}^{M} a_{i-M}x^i & \sum_{i=N-1}^{M} a_{i-N+1}x^i & \sum_{i=0}^{M} a_i x^i \end{bmatrix}}{Det\begin{bmatrix} a_{M-N+1} & a_{M-N+2}\cdots & a_{M+1} \\ \vdots & \vdots & \vdots \\ a_M & a_{M+1}\cdots & a_{M+N} \\ x^N & x^{N-1}\ldots & 1 \end{bmatrix}} \quad (31)$$

The diagonal Pade Approximants of different orders such as $[2/2], [3/3], [4/4], [5/5]$ are obtained using symbolic programming software Mathematica.

V. NUMERICAL APPLICATION

In this section, we apply the Laplace Adomian decomposition to the crime deterrence model under investigation in order to seek solutions to the governing parameters subject to the initial conditions. The efficiency of the method is confirmed by the ease with which the solution is obtain. The result obtained when compared to literature is accurate and elegant.

Applying Laplace transform to both sides of Eq. (1), yield the following

$$\begin{aligned}
&\mathcal{L}\left\{\frac{dS}{dt}\right\} = \mathcal{L}\{(1-\rho(P))A\} - \mathcal{L}\left\{\frac{\beta SC}{N}\right\} + \mathcal{L}\{\theta\nu R\} - \mathcal{L}\{dS\}\\
&\mathcal{L}\left\{\frac{dC}{dt}\right\} = \mathcal{L}\{(\rho(P))A\} + \mathcal{L}\left\{\frac{\beta SC}{N}\right\} - \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} + \mathcal{L}\{(1-\theta)\nu R\} - \mathcal{L}\{(\alpha+d)C\}\\
&\mathcal{L}\left\{\frac{dR}{dt}\right\} = \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} - \mathcal{L}\{\nu R\} - \mathcal{L}\{dR\}\\
&\mathcal{L}\left\{\frac{dP}{dt}\right\} = \mathcal{L}\{\phi C\} - \mathcal{L}\{\phi_0(P-P_0)\}
\end{aligned} \tag{32}$$

Applying the formula for Laplace Transforms, we obtain the above equation in the form

$$\begin{aligned}
&w\mathcal{L}\{S\} - S(0) = \mathcal{L}\{(1-\rho(P))A\} - \mathcal{L}\left\{\frac{\beta SC}{N}\right\} + \mathcal{L}\{\theta\nu R\} - \mathcal{L}\{dS\}\\
&w\mathcal{L}\{C\} - C(0) = \mathcal{L}\{(\rho(P))A\} + \mathcal{L}\left\{\frac{\beta SC}{N}\right\} - \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} + \mathcal{L}\{(1-\theta)\nu R\} - \mathcal{L}\{(\alpha+d)C\}\\
&w\mathcal{L}\{R\} - R(0) = \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} - \mathcal{L}\{\nu R\} - \mathcal{L}\{dR\}\\
&w\mathcal{L}\{P\} - P(0) = \mathcal{L}\{\phi C\} - \mathcal{L}\{\phi_0(P-P_0)\}
\end{aligned} \tag{33}$$

Using the initial conditions in Eq. (2), the above equations are reduced to

$$\begin{aligned}
&w\mathcal{L}\{S\} - S(0) = \mathcal{L}\{(1-\rho(P))A\} - \mathcal{L}\left\{\frac{\beta SC}{N}\right\} + \mathcal{L}\{\theta\nu R\} - \mathcal{L}\{dS\}\\
&w\mathcal{L}\{C\} - C(0) = \mathcal{L}\{(\rho(P))A\} + \mathcal{L}\left\{\frac{\beta SC}{N}\right\} - \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} + \mathcal{L}\{(1-\theta)\nu R\} - \mathcal{L}\{(\alpha+d)C\}\\
&w\mathcal{L}\{R\} - R(0) = \mathcal{L}\left\{\frac{\gamma CP}{N}\right\} - \mathcal{L}\{\nu R\} - \mathcal{L}\{dR\}\\
&w\mathcal{L}\{P\} - P(0) = \mathcal{L}\{\phi C\} - \mathcal{L}\{\phi_0(P-P_0)\}
\end{aligned} \tag{34}$$

$$\begin{aligned}
&w\mathcal{L}\{S\} = S_0 + \frac{(1-\rho(P))A}{w} - \frac{\beta}{Nw}\mathcal{L}\{SC\} + \frac{\theta\nu}{w}\mathcal{L}\{R\} - \frac{d}{w}\mathcal{L}\{S\}\\
&w\mathcal{L}\{C\} = C_0 + \frac{\rho(P)A}{w} + \frac{\beta}{Nw}\mathcal{L}\{SC\} - \frac{\gamma}{Nw}\mathcal{L}\{CP\} + \frac{(1-\theta)\nu}{w}\mathcal{L}\{R\} + \frac{(\alpha+d)}{w}\mathcal{L}\{C\}\\
&w\mathcal{L}\{R\} = R_0 + \frac{\gamma}{Nw}\mathcal{L}\{CP\} - \left(\frac{\nu+d}{w}\right)\mathcal{L}\{R\}\\
&w\mathcal{L}\{P\} = P_0 + \frac{\phi}{w}\mathcal{L}\{C\} - \frac{\phi_0}{w}\mathcal{L}\{P\} + \frac{\phi_0 P_0}{w}
\end{aligned} \tag{35}$$

Rearranging the above, we obtain

$$\begin{aligned}
&\mathcal{L}\{S\} = \frac{S_0}{w} + \frac{(1-\rho(P))A}{w^2} - \frac{\beta}{Nw^2}\mathcal{L}\{SC\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R\} - \frac{d}{w^2}\mathcal{L}\{S\}\\
&\mathcal{L}\{C\} = \frac{C_0}{w} + \frac{\rho(P)A}{w^2} + \frac{\beta}{Nw^2}\mathcal{L}\{SC\} - \frac{\gamma}{Nw^2}\mathcal{L}\{CP\} + \frac{(1-\theta)\nu}{w^2}\mathcal{L}\{R\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\{C\}\\
&\mathcal{L}\{R\} = \frac{R_0}{w} + \frac{\gamma}{Nw^2}\mathcal{L}\{CP\} - \left(\frac{\nu+d}{w^2}\right)\mathcal{L}\{R\}\\
&\mathcal{L}\{P\} = \frac{P_0}{w} + \frac{\phi}{w^2}\mathcal{L}\{C\} - \frac{\phi_0}{w^2}\mathcal{L}\{P\} + \frac{\phi_0 P_0}{w^2}
\end{aligned} \tag{36}$$

Where $X = SC,\ Y = CP$ (37)

Next, we decompose the parameters of interest, S, C, R, P as an infinite series of the form

$$S = \sum_{n=0}^{\infty} S_n,\ C = \sum_{n=0}^{\infty} C_n,\ R = \sum_{n=0}^{\infty} R_n,\ P = \sum_{n=0}^{\infty} P_n \tag{38}$$

Where the terms S_n, C_n, R_n, P_n are to be recursively determined

Similarly, the nonlinear terms, X and Y are decomposed as follows

$$X = \sum_{n=0}^{\infty} X_n,\ Y = \sum_{n=0}^{\infty} Y_n \tag{39}$$

Where X_n and Y_n are the so-called Adomian polynomials

The first seven of these polynomials are given below

$$
\begin{aligned}
X_0 &= S_0C_0 \\
X_1 &= S_0C_1 + S_1C_0 \\
X_2 &= S_0C_2 + S_1C_1 + S_2C_0 \\
X_3 &= S_0C_3 + S_1C_2 + S_2C_1 + S_3C_0 \\
X_4 &= S_0C_4 + S_1C_3 + S_2C_2 + S_3C_1 + S_4C_0 \\
X_5 &= S_0C_5 + S_1C_4 + S_2C_3 + S_3C_2 + S_4C_1 + S_5C_0 \\
X_6 &= S_0C_6 + S_1C_5 + S_2C_4 + S_3C_3 + S_4C_2 + S_5C_1 + S_6C_0 \\
X_7 &= S_0C_7 + S_1C_6 + S_2C_5 + S_3C_4 + S_4C_3 + S_5C_2 + S_6C_1 + S_7C_0
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
Y_0 &= C_0P_0 \\
Y_1 &= C_0P_1 + C_1P_0 \\
Y_2 &= C_0P_2 + C_1P_1 + C_2P_0 \\
Y_3 &= C_0P_3 + C_1P_2 + C_2P_1 + C_3P_0 \\
Y_4 &= C_0P_4 + C_1P_3 + C_2P_2 + C_3P_1 + C_4P_0 \\
Y_5 &= C_0P_5 + C_1P_4 + C_2P_3 + C_3P_2 + C_4P_1 + C_5P_0 \\
Y_6 &= C_0P_6 + C_1P_5 + C_2P_4 + C_3P_3 + C_4P_2 + C_5P_1 + C_6P_0 \\
Y_7 &= C_0P_7 + C_1P_6 + C_2P_5 + C_3P_4 + C_4P_3 + C_5P_2 + C_6P_1 + C_7P_0
\end{aligned} \tag{41}
$$

Substituting Eqs. (38) and (39) into Eq. (36), we obtain reduced equation of the form

$$
\begin{aligned}
&\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} = \frac{S_0}{w} + \frac{\big(1-\rho(P)\big)A}{w^2} - \frac{\beta}{Nw^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} X_n\right\} + \frac{\theta\nu}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} R_n\right\} - \frac{d}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} \\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} = \frac{C_0}{w} + \frac{\rho(P)A}{w^2} + \frac{\beta}{Nw^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} X_n\right\} - \frac{\gamma}{Nw^2}\mathcal{L}\{Y_n\} + \frac{(1-\theta)\nu}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} R_n\right\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} \\
&\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} R_n\} = \frac{R_0}{w} + \frac{\gamma}{w^2N}\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} Y_n\} - \left(\frac{\nu+d}{w^2}\right)\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} R_n\} \\
&\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} = \frac{P_0}{w} + \frac{\phi}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} C_n\right\} - \frac{\phi_0}{w^2}\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} + \frac{\phi_0P_0}{w^2}
\end{aligned} \tag{42}
$$

Matching the two sides of Eq. (42) yield the following iterative algorithm

$$
\begin{aligned}
&\mathcal{L}\{S_0\} = \frac{S_0}{w} + \frac{\big(1-\rho(P)\big)A}{w^2} \\
&\mathcal{L}\{S_1\} = -\frac{\beta}{w^2N}\mathcal{L}\{X_0\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R_0\} - \frac{d}{w^2}\mathcal{L}\{S_0\} \\
&\mathcal{L}\{S_2\} = -\frac{\beta}{w^2N}\mathcal{L}\{X_1\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R_1\} - \frac{d}{w^2}\mathcal{L}\{S_1\} \\
&\mathcal{L}\{S_3\} = -\frac{\beta}{w^2N}\mathcal{L}\{X_2\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R_2\} - \frac{d}{w^2}\mathcal{L}\{S_2\} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\mathcal{L}\{S_{n+1}\} = -\frac{\beta}{w^2N}\mathcal{L}\{X_n\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R_n\} - \frac{d}{w^2}\mathcal{L}\{S_n\}
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
&\mathcal{L}\{C_0\} = \frac{C_0}{w} + \frac{\rho(P)A}{w^2} \\
&\mathcal{L}\{C_1\} = \frac{\beta}{w^2N}\mathcal{L}\{X_0\} - \frac{\gamma}{w^2N}\mathcal{L}\{Y_0\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\{C_0\} + \frac{(1-\theta)}{w^2}\,\mathcal{L}\{R_0\} \\
&\mathcal{L}\{C_2\} = \frac{\beta}{w^2N}\mathcal{L}\{X_1\} - \frac{\gamma}{w^2N}\mathcal{L}\{Y_1\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\{C_1\} \\
&\mathcal{L}\{C_3\} = \frac{\beta}{w^2N}\mathcal{L}\{X_2\} - \frac{\gamma}{w^2N}\mathcal{L}\{Y_2\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\{C_2\} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\mathcal{L}\{C_{n+1}\} = \frac{\beta}{w^2N}\mathcal{L}\{X_n\} - \frac{\gamma}{w^2N}\mathcal{L}\{Y_n\} + \frac{(\alpha+d)}{w^2}\mathcal{L}\{C_n\} + \frac{(1-\theta)}{w^2}\mathcal{L}\{R_n\}
\end{aligned} \tag{44}
$$

$$\mathcal{L}\{R_0\} = \frac{R_0}{w}$$
$$\mathcal{L}\{R_1\} = \frac{\gamma}{w^2 N}\mathcal{L}\{Y_0\} - \left(\frac{\nu + d}{w^2}\right)\mathcal{L}\{R_0\}$$
$$\mathcal{L}\{R_2\} = \frac{\gamma}{w^2 N}\mathcal{L}\{Y_1\} - \left(\frac{\nu+d}{w^2}\right)\mathcal{L}\{R_1\} \tag{45}$$
$$\mathcal{L}\{R_3\} = \frac{\gamma}{w^2 N}\mathcal{L}\{Y_2\} - \left(\frac{\nu + d}{w^2}\right)\mathcal{L}\{R_2\}$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$\mathcal{L}\{R_{n+1}\} = \frac{\gamma}{w^2 N}\ \mathcal{L}\{Y_n\} - \left(\frac{\nu+d}{w^2}\right)\mathcal{L}\{R_n\}$$

$$\mathcal{L}\{P_0\} = \frac{P_0}{w} + \frac{\phi_0 P_0}{w^2}$$
$$\mathcal{L}\{P_1\} = \frac{\phi}{w^2}\ \mathcal{L}\{C_0\} - \frac{\phi_0}{w^2}\ \mathcal{L}\{P_0\}$$
$$\mathcal{L}\{P_2\} = \frac{\phi}{w^2}\ \mathcal{L}\{C_1\} - \frac{\phi_0}{w^2}\ \mathcal{L}\{P_1\} \tag{46}$$
$$\mathcal{L}\{P_3\} = \frac{\phi}{w^2}\ \mathcal{L}\{C_2\} - \frac{\phi_0}{w^2}\ \mathcal{L}\{P_2\}$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$\mathcal{L}\{P_{n+1}\} = \frac{\phi}{w^2}\ \mathcal{L}\{C_n\} - \frac{\phi_0}{w^2}\ \mathcal{L}\{P_n\}$$

Taking the inverse Laplace transform to the first equations in Eqs. (43)-(46), we get

$$\mathcal{L}\{S_0\} = \frac{S_0}{w} + \frac{(1-\rho(P))A}{w^2},$$
$$\mathcal{L}\{C_0\} = \frac{C_0}{w} + \frac{\rho(P)A}{w^2} \tag{47}$$
$$\mathcal{L}\{R_0\} = \frac{R_0}{w}$$
$$\mathcal{L}\{P_0\} = \frac{P_0}{w} + \frac{\phi_0 P_0}{w^2}$$

Substitution the values of S_0, C_0, R_0 and P_0 into the first iterates below give the solutions as

$$\mathcal{L}\{S_1\} = -\frac{\beta}{w^2 N}\mathcal{L}\{X_0\} + \frac{\theta\nu}{w^2}\mathcal{L}\{R_0\} - \frac{d}{w^2}\mathcal{L}\{S_0\}$$
$$\mathcal{L}\{C_1\} = \frac{\beta}{w^2 N}\mathcal{L}\{X_0\} - \frac{\gamma}{w^2 N}\mathcal{L}\{Y_0\} + \frac{(\alpha + d)}{w^2}\mathcal{L}\{C_0\} + \frac{(1-\theta)}{w^2}\mathcal{L}\{R_0\}$$
$$\mathcal{L}\{R_1\} = \frac{\gamma}{w^2 N}\mathcal{L}\{Y_0\} - \left(\frac{\nu+d}{w^2}\right)\mathcal{L}\{R_0\} \tag{48}$$
$$\mathcal{L}\{P_1\} = \frac{\phi}{w^2}\ \mathcal{L}\{C_0\} - \frac{\phi_0}{w^2}\ \mathcal{L}\{P_0\}$$

$$\mathcal{L}\{S_1\} = -\frac{\beta}{w^2 N}\left[\left(\frac{S_0+(1-\rho(P))A}{w}\right)\left(\frac{C_0+\rho(P)A}{w}\right)\right] + \frac{1}{w^2}\left[\theta\nu R_0 - d\left(\frac{S_0+(1-\rho(P)A)}{w}\right)\right] \tag{49}$$

$$\mathcal{L}\{C_1\} = \frac{1}{w^2 N}\left(\frac{C_0+\rho(P)A}{w}\right)\left[\frac{\beta}{w}\left(S_0 + (1-\rho(P)A)\right) - \frac{\gamma}{w}\left(C_0 + \rho(P)A\right)\right] + \frac{(1-\theta)\nu R_0}{w^2} + \left(\frac{\alpha+d}{w^2}\right)\left(\frac{C_0+\rho(P)A}{w}\right) \tag{50}$$

$$\mathcal{L}\{R_1\} = \frac{\gamma P_0}{w^2 N}\left(\frac{C_0+\rho(P)A}{w}\right)\left(\frac{1+\phi_0}{w}\right) - \frac{R_0}{w^2}(\nu + d) \tag{51}$$

$$\mathcal{L}\{P_1\} = \frac{\phi}{w^2}\left(\frac{C_0+\rho(P)A}{w}\right) - \frac{\phi_0 P_0}{w^2}\left(\frac{1+\phi_0}{w}\right) \tag{52}$$

Evaluating the Laplace transform of Eqs. (49)-(52) and taking their inverse Laplace transform, we obtain the first approximate solutions, $S_1(t), C_1(t), R_1(t)$ and $P_1(t)$. Similarly, the subsequent solutions of the parameters of interest, $S_2(t), S_3(t), \dots, S_n(t), C_2(t), C_3(t), \dots, C_n(t), R_2(t), R_3(t), \dots, R_n(t),$ $P_2(t), P_3(t), \dots, p_n(t)$ are obtained recursively using Eqs. (49)-(52).

Now to obtain the solution of the parameters of interest in explicit form, we apply the LADM to the model by taking the following values via simulation. $S(0) = S_0 = 1, C(0) = C_0 = 1, R(0) = R_0 = 1, P(0) = P_0 = 1$ for the four components in the model. Next, we set $\alpha = 0.2, \beta = 0.8, \gamma = 0.4, \mu = 0.3, \rho = 0.4, \phi = 1.5, \phi_0 = 0.25, \rho(P) = 0.9, A = 100, \theta = 0.5, \nu = 0.05, D = 2, \phi = 1.5, N = 1000.$ We obtain the first few approximations for the parameters, $S(t), C(t), R(t), P(t)$ as follows

$$\mathcal{L}\{S_0\} = \frac{1}{w} + \frac{10}{w^2}, \mathcal{L}\{C_0\} = \frac{1}{w} + \frac{90}{w^2}, \mathcal{L}\{R_0\} = \frac{1}{w}, \mathcal{L}\{P_0\} = \frac{1}{w} + \frac{50}{w^2}$$

$$\begin{aligned} \mathcal{L}\{S_1\} &= -\frac{0.884}{w^4} - \frac{21.975}{w^3} \\ \mathcal{L}\{C_1\} &= \frac{0.0170107}{w^4} + \frac{4.18}{w^3} + \frac{0.025}{w^2} \\ \mathcal{L}\{R_1\} &= \frac{0.0455}{w^4} - \frac{2.05}{w} \\ \mathcal{L}\{P_1\} &= \frac{134.625}{w^3} \end{aligned} \tag{53}$$

Taking the inverse Laplace transform of both sides of the above equations, we obtain the solutions of the parameters as follows

$$\begin{aligned} S(t) &= 1 + 10t - 105.988t^2 - 0.147333t^3 + 0.18865t^4 - 102.764t^5 + 0.92276t^6 \\ C(t) &= 1 + 90.025t + 2.09t^2 + 0.0283512t^3 + +0.32445t^4 + 3.042t^5 + 0.3572t^6 \\ R(t) &= -1.05 + 0.366667t^3 - 0.41225t^5 + 0.77162t^6 - 0.98112t^8 \\ P(t) &= 1 + 50t + 67.3125t^2 + 98.2t^3 + 123.521t^4 + 142.56t^5 + 160.228t^6 \end{aligned} \tag{54}$$

Next, we calculate the $[4/4]$ Pade approximants of the infinite series solution which gives the following rational fraction approximation of the parameters of interest using Mathematica

$$S_{Pade}(t) = \frac{1 - 1.0055603381t - 216.1577035895t^2 + 1165.18757403778t^3 + 24.768449865626t^4}{1 - 11.0055603381t - 0.1141002086t^2 + 0.018580010169t^3 + 10.679264634963112t^4}$$

$$C_{Pade}(t) = \frac{1 + 89.9293681284t - 6.50795721247t^2 + 0.84819208012t^3 + 0.514060251945t^4}{1 - 0.0956318716t + 0.01130203196t^2 + 0.00224606435t^3 - 0.0335016599824t^4}$$

$$R_{Pade}(t) = \frac{-1.05 - 1.7080263275t - 1.18053301770t^2 + 0.655940772916t^3 + 2.863559041949t^4}{1 + 1.6266917405t + 1.12431715971t^2 - 0.275498831348t^3 - 2.15914748718942t^4}$$

$$P_{Pade}(t) = \frac{1 + 50.6861390516t + 95.12200150658t^2 - 181.598502169621t^3 - 292.3326459478966t^4}{1 + 0.6861390516t - 6.49745107297t^2 - 1.1116834311026587t^3 + 9.711346090612944t^4}$$

VI. RESULTS AND DISCUSSION

In this subsection, the results of the problem in Eq. (1) are presented to show the effects of the governing parameters on the model. The effectiveness and accuracy of the numerical methods are displayed in Tables 1-4 and Figures 1-4. The methods give highly accurate results in few steps. The results obtained when compared are consistent with literature

Table 1: Numerical Computations for S(t)

t	LADM	LADM-PADE	4th Order R-K
0	1	1	1
0.2	-1.27322	-1.27369	-1.27310
0.4	-13.01120	-13.1023	-13.1201
0.6	-39.11090	-41.6953	-41.6102
0.8	-92.2623	-134.119	-134.102
1.0	-196.788	-195.23	-194.21
1.2	-392.44	178.808	178.801

Table 2: Numerical computations for C(t)

t	LADM	LADM-PADE	4^{th} Order R-K
0	1	1	1
0.2	19.0903	19.0903	19.0901
0.4	37.3871	37.3872	37.3862
0.6	56.0688	56.0699	56.0700
0.8	75.5955	75.6116	75.6110
1.0	96.867	96.9949	96.9950
1.2	121.397	122.11	122.020

Table 3: Numerical Computations for R(t)

t	LADM	LADM-PADE	4^{th} Order R-K
0	-1.05	-1.05	-1.05
0.2	-1.04715	-1.04715	-1.04715
0.4	-1.02824	-1.02796	-1.02796
0.6	-0.983335	-0.973338	-0.97337
0.8	-0.959682	-0.833489	-.0.833452
1.0	-1.30508	-0.318347	-0.318340
1.2	-3.3568	-5.94349	-5.94340

Table 4: Numerical Computations for P(t)

T	LADM	LADM-PADE	4^{th} Order R-K
0	1	1	1
0.2	14.7316	14.7307	14.8126
0.4	43.333	42.1704	42.1712
0.6	111.013	125.796	125.700
0.8	273.669	272.619	272.669
1.0	642.822	641.814	641.802
1.2	1416.93	1415.86	1415.72

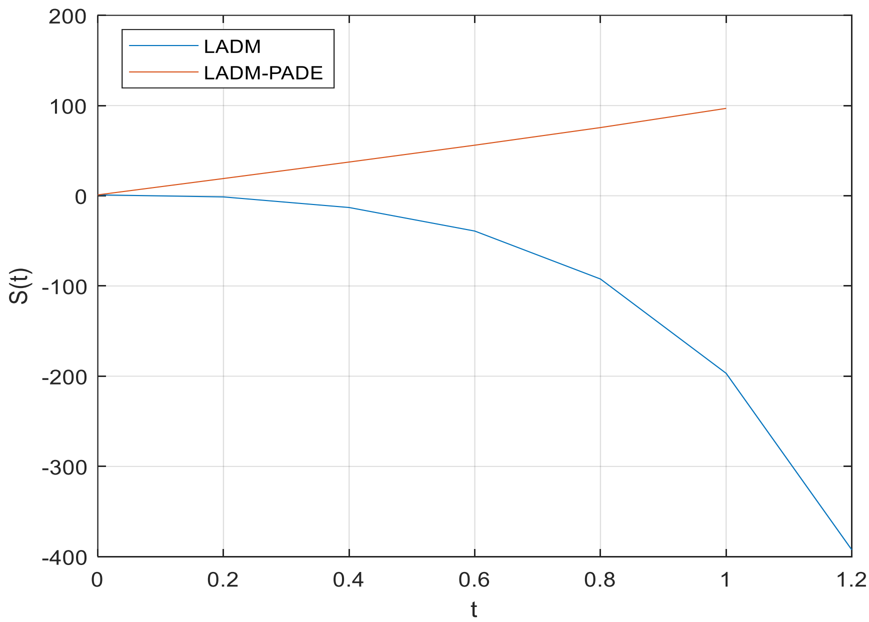

Figure 1. Numerical comparison of LADM and LADM-PADE for $S(t)$

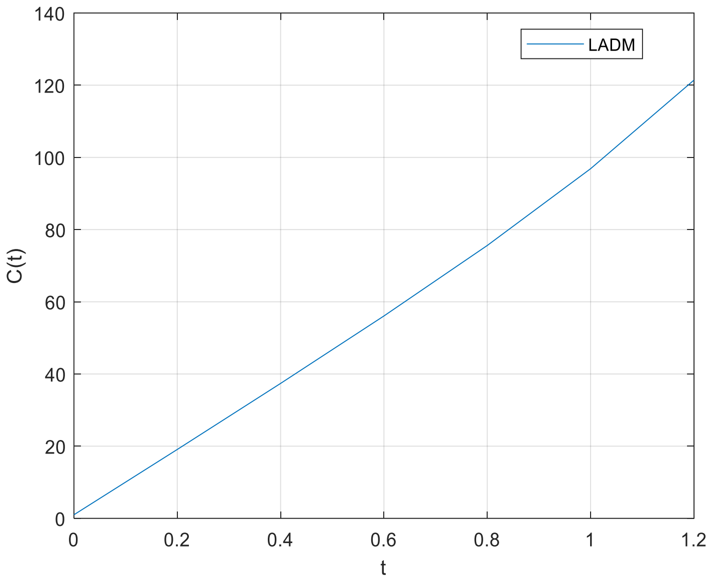

Figure 2. Numerical comparison of LADM and LADM-PADE for $C(t)$

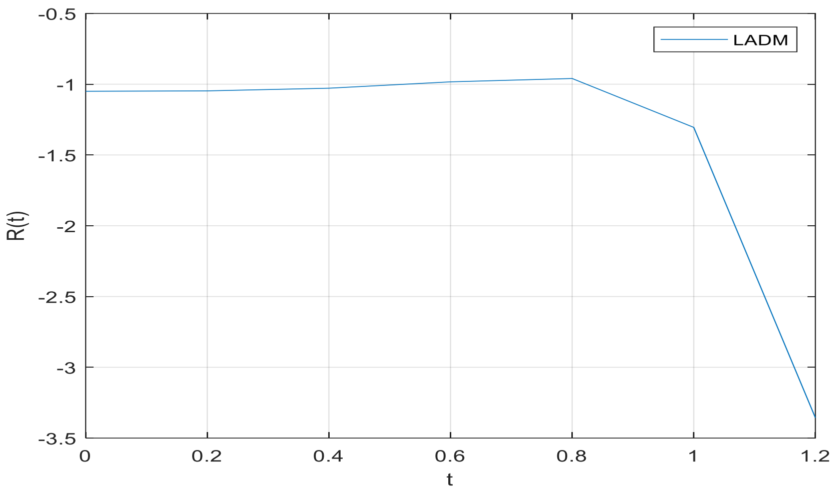

Figure 3. Numerical comparison of LADM and LADM-PADE for $R(t)$

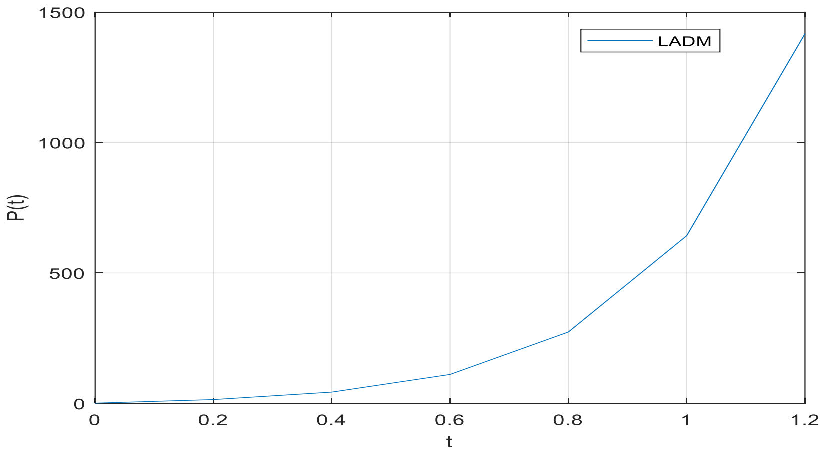

Figure 4. Numerical comparison of LADM and LADM-PADE for $P(t)$

VII. CONCLUSION

In this work, the approximate analytical solution of the mathematical model describing crime deterrence in society is solved using the fusion of Laplace transform and Adomian decomposition method (LADM). The validity, accuracy, flexibility and effectiveness of the method is demonstrated by obtaining the exact solution of the parameters of interest subject to the initial condition. The solution obtained shows the MADM is effective and convenient. Furthermore, MADM is a promising tool to effectively both linear and nonlinear PDEs. The benchmark solution is a ready reference for further works in the crime model.

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