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A study of unsteady MHD free convection, internal heat generation, and bouncy force for boundary layer flow over a vertical plate in the presence of radiation.

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-----ABSTRACT-----
The present work aims are to study the effects of the magnetic field, internal heat generation, thermal radiation, and bouncy force for the boundary layer on unsteady free convection heat transfer flow of a viscous incompressible electrically conducting fluid along with a vertical plate. The dimensionless partial differential equations of continuity, momentum along energy are analyzed with suitable transformations. For numerical calculation, an explicit finite difference method is applied to solve a set of nonlinear dimensionless partial differential equations. Dimensionless velocity and temperature profile are also investigated due to the effects of the entering parameters in the concerned problem. The analysis of the attained results is presented graphically and the flow field is significantly influenced by incoming parameters. The stability conditions are also examined.

Keywords: *Free convection, heat transfer flow, magnetic field, heat generation, explicit finite difference method.*

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I. INTRODUCTION

Unsteady MHD Free Convection boundary layer flow has a significant effect over vertical plates because of their relevance to a wide variety of technical applications, particularly in the manufacture of fibers in glass and polymers industries. In the presence of radiation,the heat transfer analysis of boundary layer flow is also important in astrophysical flows, electrical power generation, solar power technology space vehicle re-entry, and other industrial areas. Convection heat transfer analysis is very important in involving and controlling high temperatures such as gas turbines, nuclear plants, thermal energy storage. It is created great interest from both theoretical and practical points of view because of its huge applications in several engineering geophysical fields. In 1999, Postelnicu and Pop [1] observed similar solutions of free convection boundary layers over vertical and horizontal surface porous media with internal heat generation. In2000, Sattar *et.al* [2] revealed free convection flow and heat transfer through a porous vertical flat plate immersed in a porous medium with variable suction. In2003, Chamkha *et.al* [3] perceived thermal radiation effects on MHD forced convection flow adjacent to a non-isothermal wedge in the presence of a heat source or sink. In 2004, Abd EL–Naby et.al [4] perceived a finite-difference solution of radiation effects on MHD unsteady free-convection flow on a vertical porous plate. In 2004, Molla *et.al* [5] dissected natural convection flow along a vertical wavy surface with a uniform surface temperature in the presence of heat generation/absorption In 2005, Chamkha and Al-Mudhaf [6] presented unsteady heat and mass transfer from rotating vertical cone with magnetic field and heat generation or absorption effect which are develop our studies nearly to this field. In 2005. Molla *et.al* [7] deliberated magnetohydrodynamic natural convection flow on a sphere in the presence of heat generation. In 2006, Alam *et. al.* [8] proven the numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. In 2006, Rahman and Sattar[9] investigated magnetohydrodynamic convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous flat plate in the presence of heat generation or absorption. In 2006. Alam *et.al* [10] perused a numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. In 2007, Alam et. al. [11] probed the effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation. In 2008, Aydin and Kaya [12] observed radiation effect on MHD mixed convection flow about a permeable vertical plate.In2010, Makinde, and Aziz [13] deliberated a convective boundary condition in an MHD mixed convection flow over an infinite vertical plate. In 2011, Seth *et al.* [14] investigated the impacts of radiation on unsteady hydromagnetic natural convection transient flow near an impulsively moving vertical flat plate with ramped wall temperature in a porous medium. In 2012, Das [15] displayed impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties. In 2012, Khan *et al.* [16] studied the unsteady MHD free convection boundary-layer flow of a Nano-fluid along with a stretching sheet with thermal radiation and viscous dissipation effects. In 2012, Rao *et al.* [17] studied the radiation corporeity of unsteady free convection heat and mass transfer in a walters-B viscoelastic flow past an impulsively started vertical plate. In 2014, Roy et al. [18] deliberated the unsteady MHD free convection flow along with a vertical plate in the presence of radiative heat flux. In 2014, Samad and Saha [19] exposed the two-dimensional heat and mass transfer free convection flow of an MHD Non-Newtonian power-law fluid along with a stretching sheet in the presence of a magnetic field and assuming simplified thermal radiation. In 2014, Fetecau *et al.* [20] premeditated the slip effects on the unsteady radiative MHD free convection flow over a moving plate with mass diffusion and heat source. In 2015, Nagamanemma *et al.* [21] analyzed unsteady MHD free convective heat and mass transfer flow near a moving vertical porous plate with radiation and thermodiffusion effects. In 2015, Tomer and Kumar [22] perused the radiation effects on heat and mass transfer in a steady MHD flow over a porous vertical plate.

Hence the Principal objective of this present paper is to study the above mentioned thermal radiation as well as internal heat generation effects on MHD free convection boundary layer flow over a vertical plate. There is another investigation for the buoyancy effects on the thermal boundary layer over a vertical plate with a convective surface boundary condition.

Mathematical Model and Governing Equations

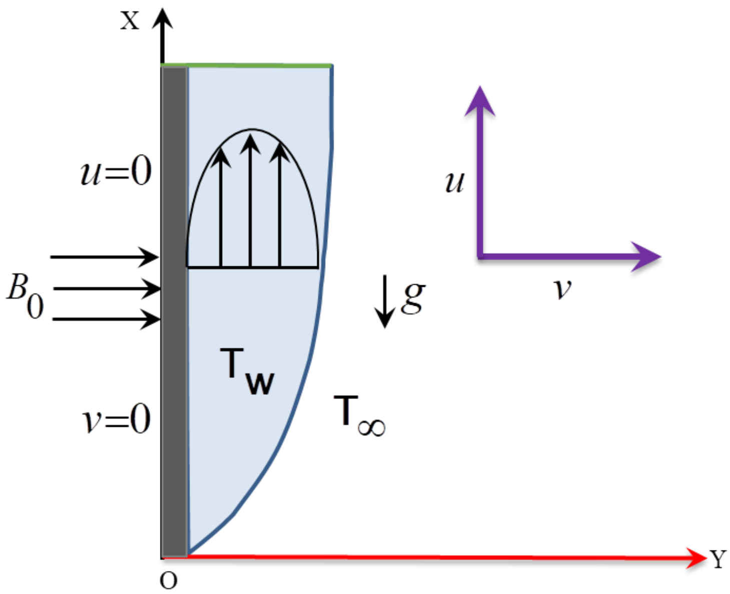


Figure: Physical configuration and coordinates system

Let us consider a two-dimensional unsteady MHD free convection, internal heat generation, and bouncy force for boundary layer flow with electrically conducting incompressible viscous fluid along with a vertical plate in presence of radiation. In the Cartesian coordinate system, the X-axis is taken along the plate in the upward direction and the Y-axis is normal to the plate. T_w is the temperature of the surface plate and T_∞ is the outside of the surface plate separately. A uniform magnetic field $\mathbf{B} = (0, B_0, 0)$ is enacted normal to the plate, and the magnetic field is anticipated to be negligible while B_0 is constant. It is also implicit that the free stream velocity U_∞ parallel to the vertical plate which is constant. Also, a heat source is retained within the flow to support potential heat generation or absorption effects.

The equations for unsteady MHD heat transfer flow, internal heat generation, and thermal radiation over a vertical plate with boundary conditions are given below:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(\mathrm{T}-\mathrm{T}_\infty)-\frac{\sigma \mathrm{B}_0^2}{\rho}u \qquad (2)$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}.\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho C_P}(T-T_\infty)$$

(3)

The relevant boundary condition for velocity and temperature is given by

$$u=U_0,\ v=0,\ T=T_w,\quad at\quad y=0$$
$$u\to 0,\ T\to T_\infty,\quad as\quad y\to\infty \qquad (4)$$

where β is the co-efficient of volumetric expansion, ν is the kinematic viscosity, g is the acceleration due to gravity, T is the temperature of the fluid inside the thermal boundary layer, T_w is the temperature of the plate, T_∞ is the temperature in the free stream, σ is the electric conductivity, B_0 is a constant magnetic field, ρ is the fluid density, κ is the kinematic viscosity, C_p is the specific heat with constant pressure, Q_0 is the heat release per unit mass, U_0 is a constant indicates the uniform velocity of the fluid remaining symbols has their usual meaning. U_0 is a constant measure of the uniform velocity of the fluid.

In the energy equation, the Rosseland approximation is used to define the radiative heat flux. Conferring to Rosseland approximation the radiative heat flux, $q_r=-\frac{4\sigma *}{3\kappa_0}.\frac{\partial T^4}{\partial y}$, where κ_0 is the Rosseland mean absorption coefficient, and σ^* is the Stefan-Boltzman constant. The Rosseland approximation is intended for an optically concentrated medium, so the fluid does not captivate its particular emitted radiation, but it does absorb radiation emitted by the boundaries without self-absorption, It seems reasonable to accept that the temperature differences within the flow are expected to be small and T^4 can be articulated as a linear function of the temperature. Thus T^4 can be extended in Taylor series about T_∞ as:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots,$$

Neglecting the higher-order terms elsewhere the first degree in $(T-T_\infty)$ one can be found as $\mathrm{T}^4\approx -3\mathrm{T}_\infty^4+4\mathrm{T}_\infty^3\mathrm{T}$.

$$\therefore\ q_r=-\frac{16\,\sigma *}{3\kappa_0}\mathrm{T}_\infty^3\frac{\partial \mathrm{T}}{\partial y}$$

By replacing the above expression into energy equation (3) are as follows:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}.\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho C_p}\frac{16\,\sigma *}{3\kappa_0}T_\infty^3\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_P}(T-T_\infty)$$

(5)

Mathematical Formulation

Applying the ensuing usual transformations, the system of partial differential equations with boundary conditions transformed into a non-dimensional equation.

$$u = U_0 U,\ v = VU_0,\ Y = \frac{yU_0}{\nu},\ X = \frac{xU_0}{\nu},\ \eta = \frac{tU_0^2}{\nu},\ T = T_\infty + (T_w - T_\infty)\overline{T}.$$

Applying the above transformation in eq. (1), (2), (5), and with corresponding boundary conditions (4), after simplification we obtain, the following non-linear differential equations in terms of dimensionless variables such as:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

(6)

Momentum equation

$$\frac{\partial U}{\partial \eta} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\,\overline{T} - MU \qquad (7)$$

Energy equation

$$\frac{\partial \overline{T}}{\partial \eta} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \overline{T}}{\partial Y^2} + Ec\left(\frac{\partial U}{\partial Y}\right)^2 + R\frac{\partial^2 \overline{T}}{\partial Y^2} + H\overline{T} \qquad (8)$$

with boundary conditions

$$\begin{aligned} &U = 1,\ V = 0,\ \overline{T} = 1,\ at\ Y = 0 \\ &U = 0,\ \overline{T} = 0,\ as\ Y \to \infty \end{aligned} \qquad (9)$$

where,

Magnetic parameter, $M = \frac{\sigma \nu B_0^2}{\rho U_0^2}$

Grashof number, $Gr = \frac{\nu g \beta (T_w - T_\infty)}{U_0^3}$

Prandtl number, $Pr = \frac{\nu}{\alpha}$

Eckert number, $Ec = \frac{U_0^2}{C_p (T_w - T_\infty)}$

Radiative parameter, $R = \frac{16\sigma^* T_\infty^3}{3\rho C_p \kappa_0 (T_w - T_\infty)}$

Heat Generation parameter, $H = \frac{Q_0 \nu}{\rho C_p U_0^2}$

Numerical Solution

A set of nonlinear partial differential dimensionless governing equations has been solved numerically with related boundary conditions along with an explicit finite difference method which is tentatively stable. The region of the flow is divided into a grid or mesh of lines parallel to X - and Y -axes, where X - axis indicates the plate in the upward direction and Y - axis is normal to the plate. We measured the height of plate X_{max} (=100), i.e., X varies from 0 to 100 and supposed Y_{max} (=25) as taken to $Y \to \infty$, it means that Y varies from 0 to 25.

We have taken $m = 250$ and $n = 250$ grid spacing in the X and Y directions correspondingly and as follows $\Delta x = 0.4(0 \le x \le 100)$ and $\Delta Y = 0.1(0 \le Y \le 25)$ with the minor time step $\Delta \eta = 0.005$. Let U', $\overline{T}'$ indicate the values of U, $\overline{T}$ at the end of a time-step separately.

Applying the explicit finite difference method into the partial equations (6-8) with boundary conditions (9) we get,

$$\frac{U_{i,j} - U_{i,j-1}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0$$

(10)

$$\frac{U'_{i,j} - U_{i,j}}{\Delta \eta} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\,\overline{T}_{i,j} - MU_{i,j}$$

$$\Rightarrow U'_{i,j} = U_{i,j} + \Delta\eta\,(-U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} - V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}$$

$$+ \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\,\overline{T}_{i,j} - MU_{i,j})$$

(11)

$$\frac{\overline{T}'_{i,j} - \overline{T}_{i,j}}{\Delta\eta} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \frac{1}{Pr}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2$$

$$+ R\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + H\overline{T_{ij}}$$

$$\Rightarrow \overline{T}'_{i,j} = \overline{T}_{i,j} + \Delta\eta\left(-U_{I,J}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} - V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} + \frac{1}{Pr}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2\right.$$

$$\left. + R\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + H\overline{T_{ij}}\right) \quad (12)$$

The boundary conditions with the finite difference methods are as follows:

$$U^n_{i,0} = 1,\ V^n_{i,0} = 0,\ \overline{T}^n_{i,0} = 1 \quad (13)$$

$$U^n_{i,L} = 0,\ \overline{T}^n_{i,L} = 0,\ \text{where } L \to \infty.$$

Here, the subscripts i and j indicate the grid points with X and Y coordinates correspondingly and $\overline{T}$ is the temperature.

Stability and Convergence Analysis

Any analysis will remain incomplete without the stability and convergence analysis. It's always supportive of tangible computations. Furthermore, it established numerical solutions (finite difference scheme) that are reliable and consistent. For the constant mesh sizes, the stability criteria of the scheme may be established are as follows.

The general terms of the Fourier expansion for U, θ, φ at time arbitrary say $\eta = 0$ are $e^{i\bar{\alpha}\bar{x}}$ and $e^{i\bar{\beta}\bar{y}}$ apart from a constant, where $i = \sqrt{-1}$.

Then

$$U : \Psi(\eta)e^{i\bar{\alpha}\bar{x}}.e^{i\bar{\beta}\bar{y}}$$

$$\overline{T} : \theta(\eta)e^{i\bar{\alpha}\bar{x}}\,e^{i\bar{\beta}\bar{y}} \quad (14)$$

And after the time step, these terms will converts

$$U : \Psi'(\eta)e^{i\bar{\alpha}\bar{x}}.e^{i\bar{\beta}\bar{y}}$$

$$\bar{T} : \theta'(\eta)e^{i\bar{\alpha}\bar{x}}.e^{i\bar{\beta}\bar{y}} \quad (15)$$

Applying eq. (13) and eq. (14) into equations (10-11), the following equations we found by simplification.

$$\frac{\Psi' - \Psi}{\Delta\eta} + U\frac{\Psi\left(1 - e^{-i\bar{\alpha}\Delta X}\right)}{\Delta X} + V\frac{\Psi\left(e^{i\bar{\beta}\Delta Y} - 1\right)}{\Delta Y} = \frac{2\Psi\left(\Delta Y.\cos\beta - 1\right)}{\left(\Delta Y\right)^2} + Gr\ \theta' - M\Psi$$

$$\Rightarrow \Psi' - \Psi + \frac{\Delta\eta}{\Delta X}U\Psi\left(1 - e^{-i\bar{\alpha}\Delta X}\right) + \frac{\Delta\eta}{\Delta Y}V\Psi\left(e^{i\bar{\beta}\Delta Y} - 1\right) = 2\frac{\Delta\eta}{\left(\Delta Y\right)^2}\Psi\left(\Delta Y\cos\beta - 1\right) + Gr\,\Delta\eta\,\theta' - M\Psi$$

$$\Rightarrow \Psi' = \Psi\left\{1 - \frac{\Delta\eta}{\Delta X}U\left(1 - e^{-\bar{\alpha}i\Delta X}\right) - \frac{\Delta\eta}{\Delta Y}V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{2\Delta\eta}{\left(\Delta Y\right)^2}\left(\Delta Y\cos\beta - 1\right) - M\Delta\eta\right\} + Gr\,\Delta\eta\,\theta'$$

$$\Psi' = A\Psi + B\theta' \qquad (16)$$

where

$$A = 1 - \frac{\Delta\eta}{\Delta x}U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y}V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{2\Delta\eta}{\left(\Delta Y\right)^2}\left(\cos\beta.\Delta Y - 1\right) - M\Delta\eta$$

$$B = Gr\,\Delta\eta$$

and

$$\frac{\theta'(\eta) - \theta(\eta)}{\Delta\eta} + U\theta(\eta)\frac{(1 - e^{-i\bar{\alpha}\Delta X})}{\Delta X} + V\theta(\eta)\frac{(e^{-i\bar{\beta}\Delta Y} - 1)}{\Delta Y}$$

$$= \frac{1}{\Pr}\frac{2\theta(\eta)\left(\cos\beta.\Delta Y - 1\right)}{\left(\Delta Y\right)^2} + Ec\left(\frac{U\Psi(\eta)\left(e^{i\bar{\beta}\Delta Y} - 1\right)}{\left(\Delta Y\right)^2}\right) + R\frac{2\theta(\eta)(\cos\beta.\Delta Y - 1)}{(\Delta Y)^2} + H\theta'$$

$$\Rightarrow \theta'(\eta) = \theta(\eta)\left\{1 - \frac{\Delta\eta}{\Delta x}U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y}V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{1}{\Pr}\frac{2\Delta\eta}{\left(\Delta Y\right)^2}\left(\cos\beta.\Delta Y - 1\right)\right.$$

$$\left. + R\frac{2(\cos\beta.\Delta Y - 1)}{(\Delta Y)^2} + H\theta'\right\} + EcU\,.\frac{\Delta\eta}{\left(\Delta Y\right)^2}\Psi(\eta)\left(e^{i\bar{\beta}\Delta Y} - 1\right)$$

$$\Rightarrow \theta'(\eta) = G\theta + L\Psi$$

(17)

where, $G = 1 - \frac{\Delta\eta}{\Delta X}U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y}V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{1}{\Pr}\frac{2\Delta\eta\left(\cos\beta.\Delta Y - 1\right)}{\left(\Delta Y\right)^2} + R\frac{2(\cos\beta.\Delta Y - 1)}{(\Delta Y)^2}$

and $L = EcU\,.\frac{\Delta\eta}{\left(\Delta Y\right)^2}\left(e^{i\bar{\beta}\Delta Y} - 1\right)$

Equations (15), (16) can be written as:

$$\Psi' = A\Psi.+ B\left(G\theta + L\Psi\right)$$

$$= \left(A + L\right)\Psi + BG\,\theta$$

$$\Rightarrow \Psi' = A_1\Psi + B_1\theta \qquad (18)$$

where, $A_1 = A + L$

$B_1 = BG$

and $\theta' = G\theta + L\Psi$ (19)

Equations (17), (18) can be expressed in matrix form.

$$\begin{pmatrix}\psi' \\ \theta'\end{pmatrix} = \begin{pmatrix}A_1 & B_1 \\ L & G\end{pmatrix}\begin{pmatrix}\psi \\ \theta\end{pmatrix}$$

$i.e.\ \eta' = T\eta$

where, $T = \begin{pmatrix} A_1 & B_1 \\ L & G \end{pmatrix}$ and $\eta = \begin{pmatrix} \psi \\ \theta \end{pmatrix}$

As eigenvalues of the augmentation matrix T are crucial for attaining the stability condition, as a result, let $B_1 \to 0, \quad L \to 0.$

Hence, matrix T is as follows:

$$T = \begin{pmatrix} A_1 & 0 \\ 0 & G \end{pmatrix}$$

Thus, the Eigenvalues of T are

$\lambda_1 = A_1, \quad \lambda_2 = G.$

Here, the values λ_1, λ_2 must not surpass in modulus.

Therefore, the stability conditions are as follows

$|A_1| \le 1, \quad |G| \le 1.$

Let, $a = \frac{u \Delta \eta}{\Delta X}, \quad b = \frac{\Delta \eta}{\Delta Y}.V, \quad c = \frac{\Delta \eta}{(\Delta Y)^2}$

Hence, $A = 1 - a\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - b\left(e^{i\bar{\beta}\Delta Y} - 1\right) + 2c(\cos \beta \Delta Y - 1) - M \Delta \eta$

$L = EcUc \quad \left(C^{i\bar{\beta}\Delta Y} - 1\right).$

The coefficients a, b, c are real and non-negative. Therefore, the maximum modulus of A_1, G arises when $\overline{\alpha}\Delta X = m\pi$ and $\overline{\beta}\Delta Y = n\pi$ where m and n are integers and hence A_1, G are real. The values of $|A_1|, |G|$ are greatest when m and n are odd integers, then

$A_1 = A + L$

$\quad = 1 - 2a - 2b - 4c - 2cEc.$

To satisfy $|A_1| \le 1, \ |G| \le 1$ the most negative permissible values are,

$A_1 = -1, \quad G = -1.$

Thus, the stability conditions of the problem are assumed below.

$1 - 2a - 2b - 4c - 2cEc \le -1.$

$2(a + b + 2c + cEc) \le 2.$

$a + b + 2c + cEc \le 1.$

$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{2\Delta\eta}{(\Delta Y)^2} + \frac{\Delta\eta}{(\Delta Y)^2}Ec \le 1.$$

Analogously, the 2nd conditions are as follows

$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{1}{\Pr}\frac{2\Delta\eta}{(\Delta Y)^2}$$

and convergence criteria of the technique are $\Pr \ge 1.$

II. RESULTS AND DISCUSSION

To scrutinize the problem under deliberation, the results of numerical values of non-dimensional velocity and temperature profiles within the boundary conditions are computed for different values of radiation parameter R, magnetic parameter M, Grashof number Gr, Prandtl number Pr, Eckert number Ec, heat generation parameter H respectively. Furthermore, computed results of velocity and temperature profiles are portrayed and physical explanation is explained here.

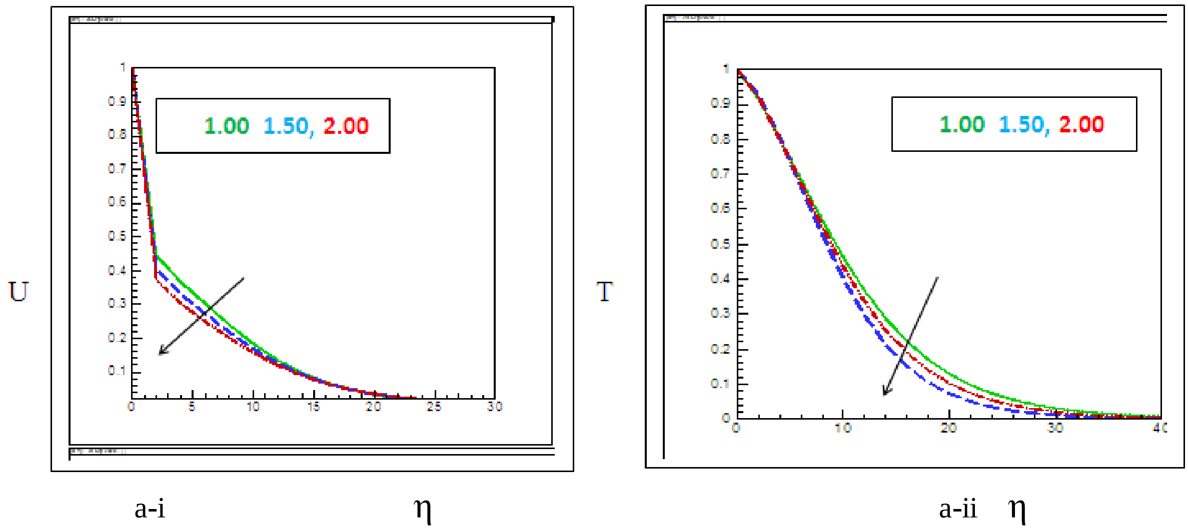


Figure-a: Effect of radiative parameter (R) on (i) velocity and (ii) temperature profile taking Gr = 2.50, Pr=1.00, M = 5.50, Ec = 0.10,H=.3

Figure a-i indicates that velocity profile declines for increasing values of R. Because boundary layer thickness falls for intensifying values of R i.e. retard the flow and reduce fluid velocity.

Figure a-ii demonstrates that temperature profile lessening for increasing values of R. It's the fact that owing to increase values of R leads to an increase in the boundary layer thickness. As a result, the heat transfer rate reduces the present thermal buoyancy force.

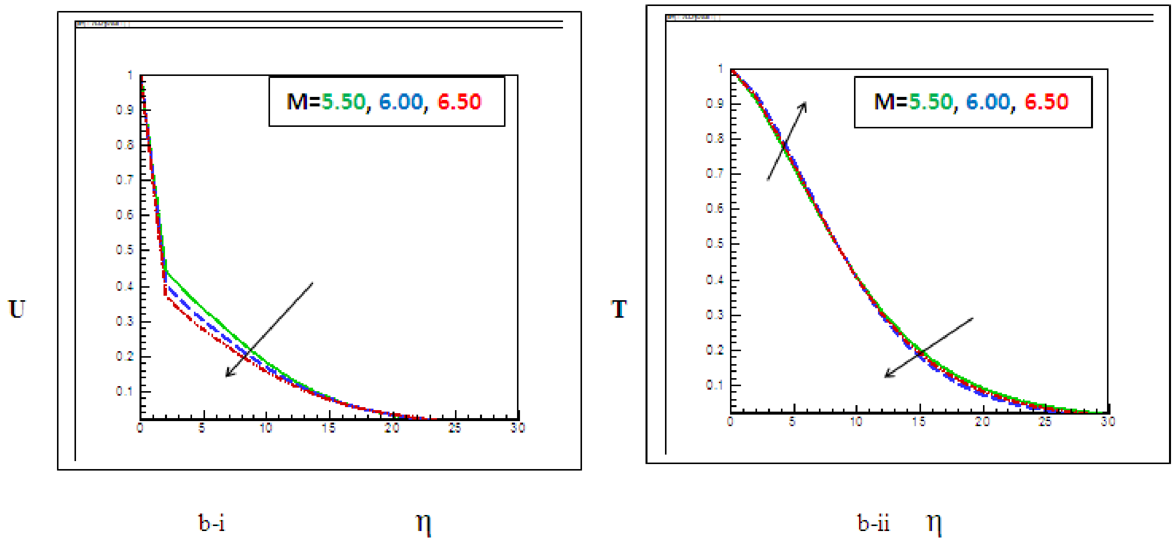


Figure-b: Effect of magnetic parameter M on **(i) velocity and (ii) temperature profile taking Gr = 2.50, Pr=1.00, R = 1.00, Ec = 0.10,H=.3**

Figure b-i displays that for intensifying values of magnetic parameter M , the velocity of the flow field falls. It is because that application of transverse magnetic field to electrically conducting fluids gives to accelerate a body force attributed as Lorentz force. So it is created a tendency to reduce the speed of the motion of the fluid in the boundary layer. Figure b- ii, it is seen that the temperature profiles increase due to the increasing effect of M $(0 < M<7)$ and then decreases. It is also seen that the temperature profiles are increasing near the plate and then gradually decreasing. Due to Lorentz force the fluid motion rigorous and along with the plate simultaneously rises its temperature and decreases far away from the surface for reducing heat generation in the flow with increasing values of the magnetic parameter M . This implies that higher values of magnetic field parameters produce lower heat transfer.

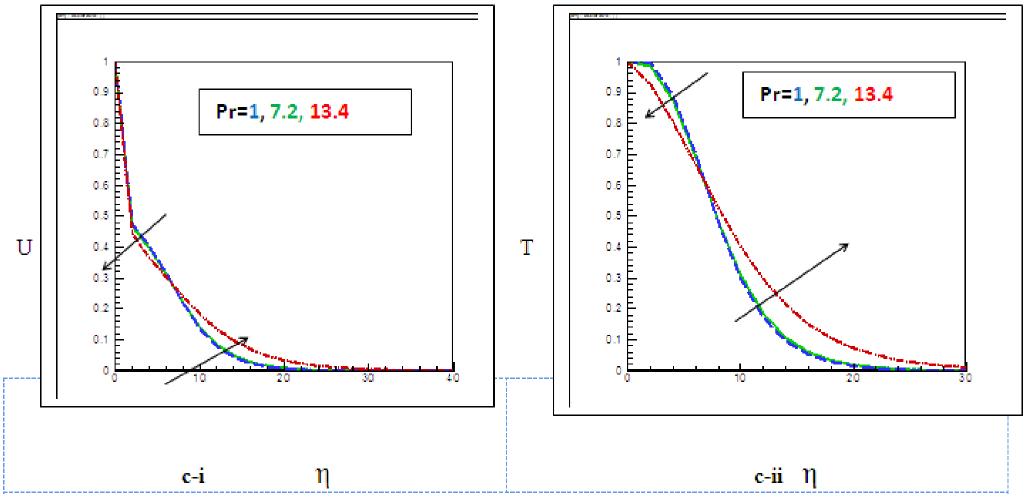


Figure-c: Effect of Prandtl number (Pr) on (i)velocity and (ii) temperature profile taking Gr = 2.50, R=1.00, M = 5.50, Ec = 0.10,H=.3

Figure c-i illuminate velocity profile is a decline for increasing values of Pr ($0 < \mathrm{Pr} < 7$) and then increases. Because a higher Prandtl number creates high viscosity, consequently the velocity profiles move slowly. Figure c-ii, it is comprehended that the temperature profiles decrease due to the increasing effect of Pr ($0 < \mathrm{Pr} < 8$) and then increases. It is due to the fact, increasing values Pr is proportional to reduced thermal conductivity.

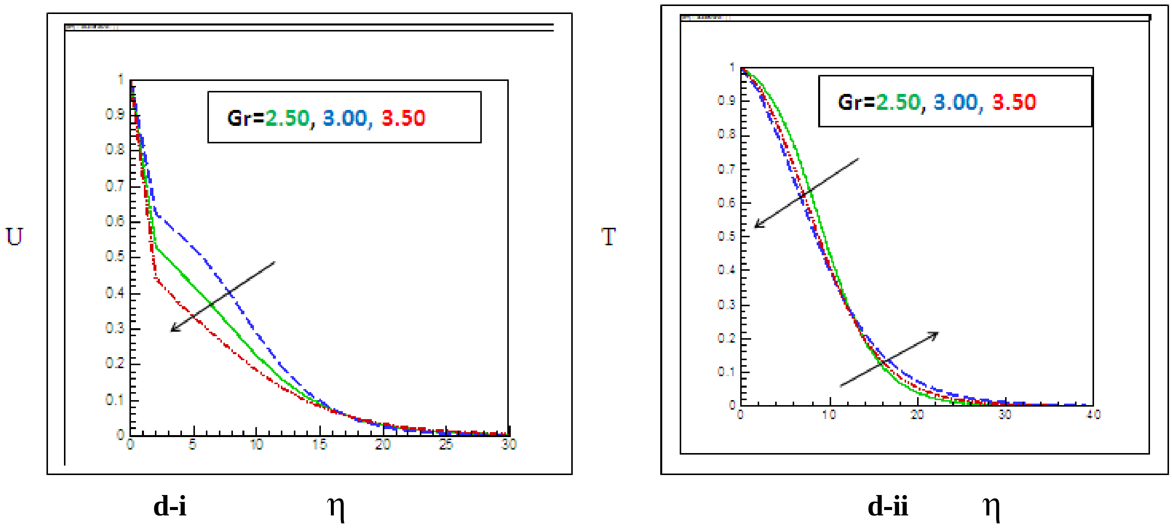
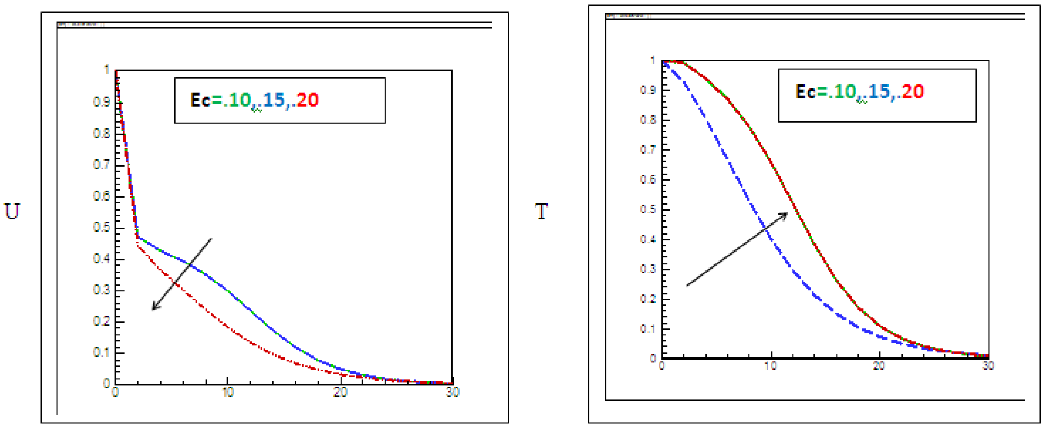


Figure-d: Effect of Grashof number (Gr) on (i) velocity and (ii) temperature profile taking Pr = 1.00, R=1.00, M = 5.50, Ec = 0.10, H=.3

The velocity profile decrease for increasing values of Gr which are shown in figure d-i. Because rising in the buoyancy force hindrances the fluid flow. Figure d-ii elucidates that the temperature profile increases for increasing values of Gr and then decreases. It means that increasing values Gr decreases the rate of flow of temperature within the boundary layer.

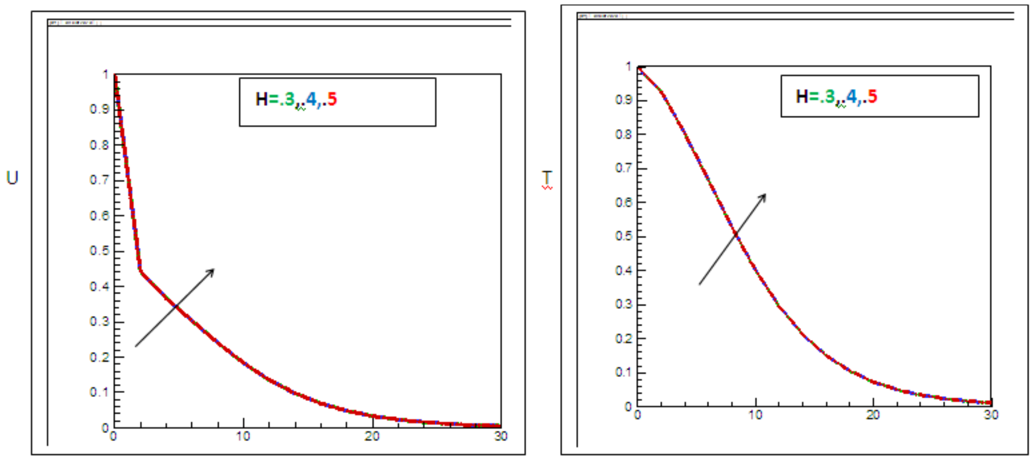


e-i η e-ii η

Figure-e: Effect of Eckert number (Ec) on (i) velocity and (ii) temperature profile taking Gr = 2.50, R=1.00, M = 5.50, Pr=1.00, H=.3

Figure e-i demonstrates that the velocity profile decline because of increasing values of Ec . As a result heat transfer of the flow reduced driving force to the kinetic energy.

Since frictional heating as well as heat energy is stored in liquid, therefore the temperature profile increases for increasing values of Ec which are illustrated in figure e-ii.



f-i η f- I η

Figure-f: Effect of heat generation parameter (H) on (i) velocity and (ii) temperature profile taking Gr = 2.50, R=1.00, M = 5.50, Ec = 0.10, Pr=1.00

Figure f-i appearances that the velocity profile increases for increasing values of H. It is because the heat is produced, the buoyancy force upsurges, which makes the flow rate to increase. Figure f- ii arrivals that with the increasing value of the heat generation parameter, the temperature distribution also increases pointedly.

III. CONCLUSIONS

The numerical solution of the resulting momentum and the thermal equations are reported for representative values of the thermophysical parameters embedded in the fluid convection process. The effects of the Prandtl number Pr, the magnetic parameter *M*, the radiative parameter *R*, the internal heat generation parameter H, Grashof number Gr, Eckert number Ec on velocity and temperature profiles are analyzed and interpreted in physical terms. In this analysis, the following conclusions are drawn here.

- The velocity profiles rise for rising values of Ec and H whereas it decreases with increasing values of R, M, Gr.
- The velocity profiles decrease after then increases for increasing values of Pr.

- The temperature i.e. thermal boundary layer increases owing to the various values of Ec, R, H.
- The temperature profiles increase after then decreases for increasing values M, Gr. whereas the opposite result has been seen in Pr.

Nomenclature

B : magnetic field intensity (JT^{-1}); B_0 : applied uniform magnetic field (JT^{-1})

C_p : specific heat at constant pressure ($kJkg^{-1}.{}^0C$); Ec : Eckert number ;

g : acceleration due to gravity(ms^{-2}); Gr : Grashof number;

M : magnetic parameter; MHD : magnetohydrodynamics ;

q_r : radiative heat flux (wm^{-2}); R : radial parameter;

Pr : Prandtl number ; U_0 : constant velocity (ms^{-1});

$\overline{T}$: dimensionless temperature of the flow fluid ; T_w : the temperature at the fluid (K);

T_∞ : the temperature outside the boundary layers (K):

Greek Symbols

α : thermal diffusivity ($m^2 s^{-1}$); β : co-efficient of volumetric expansion(K^{-1});

β^* : co-efficient of expansion with concentration; κ_0 : Resseland mean absorption co-efficient ;

ρ : density of the fluid (km^{-3}); $\sigma *$: Stefan-Boltzmann constant (5.6697 × 10− 8 W/m2K4; kg m−2 K−4);

θ : dimensionless temperature ; η : dimensionless time ; υ: kinematic viscosity of the fluid (m2s−1);

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