The International Journal Of Engineering And Science (IJES)
|| Volume || 5 || Issue || 9 || Pages || PP 60-63 || 2016 ||
ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805

Problem-Solving Capacity of Students: A Study of Solving Problems in Different Ways

Nguyen Phu Loc[*,1] Duong Huu Tong[2]
School Of Education, Can Tho University, Vietnam
School Of Education, Can Tho University, Vietnam

ABSTRACT

Training towards the development of the capacity of learners has become an inevitable trend of world education. Vietnamese education also emphasizes the comprehensive development of the capacity and the quality of students. In mathematics teaching, there are some notable capacities such as problem-solving capacity, cooperation capacity, capacity for using mathematical language, computing capacity and so on. In particular, the problem-solving capacity is very important to students because it helps them to solve problems not only in mathematics but also in practice. In this paper, we want to investigate the problem-solving capacity of students in primary schools through a problem required to solve in different ways. The results of the study showed that students had enough the problem-solving capacity to find out various solutions to the given problem.

Keywords: *problem-solving capacity, mathematics education, solving a problem in different ways.*

Date of Submission: 20 June 2016 — Date of Accepted: 22 August 2016

I. INTRODUCTION

In his book, Polya (1973) proposed four phases to solve a problem effectively: understanding the problem, devising a plan, carrying out the plan and looking back. If students adhere to this process, their problem-solving capacity will be improved. Indeed, the problem-solving capacity is shown in some aspects. First, students have the ability to analyze data for the problem to find out and discover solutions. Therefore, they need to answer two important questions: “What is the information of the problem given?” and “What is the question of the problem?”. Second, students approach the problem in a systematic way. In particular, they need to find out the plan to solve the problem, present ways to solve the problem and check the results. In addition, to find solutions, students use the thinking operations such as analysis, synthesis, comparison, analogy, generalization and abstraction. Finally, they have the ability to look for and choose solutions. It means that learners can find out how to present a solution, different calculations, multi-solutions.

In our research paper *Different solutions to a mathematical problem: A case study of Calculus 12,* we asked students in Grade 12 to calculate $\int_0^{\frac{\pi}{2}} \sin(\frac{\pi}{4} - x)dx$ in various ways. The result was that they were very active to raise multiple solutions. But in primary schools, how will students solve a problem if placed in a similar situation?

In reality, the current mathematics textbooks in Vietnam provide students many problems about comparing two fractions (Hoan, 2007). However, in our study, there are not many problems requiring solutions in many ways. Sometimes, the request is solving the problem in only two ways. In the study, we put students in an unfamiliar situation with a problem asked to solve in at least five ways. Moreover, we also request them to explain their solutions.

The research question: *In an unfamiliar situation with a problem asked to solve in at least five ways, how is the problem-solving capacity of students expressed?*

II. METHODOLOGY

2.1. Participants

The experiment was conducted in two classes in two primary schools of An Lac Tay and Ke Sach, Soc Trang province, Vietnam. There were all 70 students in two classes.

2.2. Instrument and procedure

Students were requested to solve the following problem in at least five ways: Compare two fractions $\frac{4}{5}$ and $\frac{3}{2}$.

Students had to solve the problem printed on a paper individually (in 30 minutes). More specifically, students could show their problem-solving capacity in dealing with solving the problem in different ways. Besides, we asked them to give explanation to their answers.

2.3. Pre-analyzing the problem

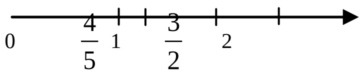
a. The class context of the experimental problem

Students solved the problem after they had finished the decimals topics. In particular, the methods of comparing fractions and decimals were mentioned in mathematics textbooks in primary schools.

b. The solutions expected for the problem

The right solutions to the given problem were raised in Table 1.

Table 1: The solutions expected to the problem

Code	Solution strategy	Solution
S1	Making fractions have a common denominator	$\frac{4}{5}=\frac{4\times 2}{5\times 2}=\frac{8}{10}$; $\frac{3}{2}=\frac{3\times 5}{2\times 5}=\frac{15}{10}$ Because of $\frac{15}{10}>\frac{8}{10}$, we conclude $\frac{3}{2}>\frac{4}{5}$.
S2	Making fractions have a common numerator	$\frac{4}{5}=\frac{4\times 3}{5\times 3}=\frac{12}{15}$; $\frac{3}{2}=\frac{3\times 4}{2\times 4}=\frac{12}{8}$ $\frac{12}{15}<\frac{12}{8}$ (since 8 < 15). Conclusion: $\frac{3}{2}>\frac{4}{5}$
S3	Comparing two fractions with 1	$\frac{4}{5}<1$ and $1<\frac{3}{2}$, we conclude $\frac{4}{5}<\frac{3}{2}$.
S4	Expressing two fractions on a number line	 On the number line, $\frac{3}{2}$ is behind $\frac{4}{5}$, so $\frac{3}{2}$ is greater than $\frac{4}{5}$. Conclusion: $\frac{3}{2}>\frac{4}{5}$
S5	Writing two fractions as mixed numbers	$\frac{4}{5}=0\frac{4}{5}$; $\frac{3}{2}=1\frac{1}{2}$. Conclusion: $\frac{4}{5}<\frac{3}{2}$ (since $0<1$)
S6	Writing two fractions as decimals	$\frac{4}{5}=0.8$; $\frac{3}{2}=1.5$ $0.8<1.5$ (since 0 < 1), therefore $\frac{4}{5}<\frac{3}{2}$
S7	Subtracting fractions	$\frac{3}{2}-\frac{4}{5}=\frac{15}{10}-\frac{8}{10}=\frac{7}{10}>0$, then $\frac{3}{2}>\frac{4}{5}$
S8	Dividing fractions	$\frac{3}{2}:\frac{4}{5}=\frac{3}{2}\times\frac{5}{4}=\frac{15}{8}>1$ (since 15 > 8). Conclusion: $\frac{3}{2}>\frac{4}{5}$.

III. RESULTS AND DISCUSSION

Table 2: The number of students with the right solutions

Solutions	A number of students	A number of students doing correctly
S1	70 (0%)	69 (98.6%)
S2	69 (98.6%)	65 (94.2%)
S3	65 (92.9%)	64 (98.5%)
S4	0 (0%)	0 (0%)
S5	70 (100%)	70 (100%)
S6	67 (95.7%)	67 (100%)
S7	0 (0%)	0 (0%)
S8	0 (0%)	0 (0%)

Table 2 indicated that two dominant solutions belonged to S1 and S5 (70/70 students, at 100% respectively). This could be explained easily since these participants were familiar with comparing fractions and decimals in mathematics textbooks in Vietnam. Nevertheless, a student did not have the right answer because he did incorrect calculation and even this student wrote $\frac{4}{5} < \frac{8}{10}$ on his paper.

S2 solution was chosen by nearly 100% of students, but four students had difficulties in making fractions have a common numerator, and they had the wrong answers. It was remarkable that the rule of making fractions have a common numerator was implicit in mathematics textbook in grade 4. Therefore, all they gave incorrect explanation: *"for two fractions having a common numerator, the one with a greater denominator is greater than the other"*. Meanwhile, the right rule had to be *"for two fractions having a common numerator, the one with a greater denominator is smaller than the other"*.

The number of students choosing S3 and S6 was quite similar. S3 solution was also chosen by 65 students (representing 92.9%) with a wrong answer, while there were 67 students (accounted for 95.7%) solving the problem by writing two fractions as decimals correctly. In their explanation, most students properly stated the rule of comparing two decimals such as "*for two decimals having different integer parts, the one with a greater integer part is greater than the other*". Moreover, to compare the fractions with 1, they also gave a reasonable explanation: *"The numerator of a fraction is greater than the denominator, therefore the fraction is greater than 1"* or *"The numerator of a fraction is smaller than the denominator, therefore the fraction is smaller than 1"*.

There were no students choosing S4, S7 and S8. All students did not consider the number line as a tool to compare two fractions although it was explicitly introduced in the mathematics textbooks in primary schools. It was confirmed that they did not adapt to search for new knowledge in this case. Besides, there was an interesting reason for this. The reason was that teachers did not emphasize the importance of the number line when they introduced it to students. Also, they did not discover two equivalent clauses to suggest a solution to the given problem. They were $a > b \Leftrightarrow a - b > 0$ and $a > b \Leftrightarrow a : b > 1$, but they were not presented in the mathematics textbooks. Because of these difficulties, S7 and S8 absolutely did not appear in their answers. In this situation, they could not afford to discover and apply new knowledge to the given problem. In general, the problem-solving capacity of students was expressed very well due to mastering knowledge on the textbooks.

In addition, there were some errors recorded from students' papers. A student wrote: $\frac{4}{5} = \frac{1}{2} = \frac{3}{2}$ or $\frac{4}{5} = \frac{3}{2} = \frac{4}{5}$ therefore $\frac{4}{5} < \frac{3}{2}$. Another student had the first right answers, but the last solution was comparing $\frac{3}{5}$ and $\frac{3}{2}$ instead of comparing $\frac{4}{5}$ and $\frac{3}{2}$. Also, $\frac{4}{5} = 0.45$ was written by a student. He had difficulties in writing a fraction as a decimal. Furthermore, students also had the disadvantages of expressing mathematical language. For example, one student presented a cryptic explanation: *"a fraction with the smaller numerator than the denominator is smaller"*.

Table 3: The number of students according to their right solutions

Numbers of right solutions	0	1	2	3	4	5	6	7	8
Numbers of students	0 (0%)	1 (1.4%)	2 (2.8%)	0 (0%)	3 (4.2%)	64 (91.4%)	0 (0%)	0 (0%)	0 (0%)

Table 3 showed that students did not have 6 right answers or more. This was obvious because none of students did choose S4, S7, and S8. There was only one student having a correct answer with S1 solution. In addition, there were 2 students with 2.8% respectively having 2 correct answers, whereas 3 students with 4 right solutions accounted for 4.2%. Especially, 92% of students had five correct answers, and that was the strength of the survey group. In fact, they all were successful in using S1, S2, S3, S5 and S6 because they completely understood the rule of comparing the fractions introduced in the textbooks. Overall, the experimental attendants represented their problem-solving capacity relatively well. They used the old knowledge in grade 4 to try to satisfy the problem demand.

IV. CONCLUSION

The results of the investigation showed that the majority of students had the capacity to solve the problem with an unfamiliar requirement. Indeed, thanks to the knowledge in their mathematics textbooks, 91.4 of students raised five correct solutions on their papers. Nevertheless, a few students provided the correct answer when making fractions have a common numerator, but they wrote the wrong rule of comparing two fractions in the case of the same numerator. In addition, a few students gave the correct answer when making fractions have a common numerator, yet they wrote the wrong rule of comparing two fractions in the case of the same numerator. Furthermore, the weakness of the investigated group was that they all did not have enough capacity to solve the problem due to the discovery of new knowledge. Also, some children had mistakes in calculations and solution presentation.

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