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3D Single-Segment Cable Analysis

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ABSTRACT
Cables are invaluable structural elements. They have been used in guyed towers, bridges marine vehicles, offshore structures, transmission lines and tensioning applications etc. Briefly, cables are necessary elements for long spans. As known, cables are tension elements; they cannot carry any compression load due to its unique geometry. This tangential convoluted geometry makes them hard to analyze. Engineers solve cables assuming them as linear elements even now, because cables cannot be solved by classical finite element methods. Having almost zero bending rigidity makes it vulnerable to drastic vertical movements. Hence, engineers either define a bending rigidity or apply a checker for drastic movements in each nonlinear iteration to solve cable by classical FEM. Therefore, engineers propose a different iterative finite element method to solve cables more stabilized way. In this research, a 3D static solution method is presented for a cable supported at its ends. This method first makes the cable determinant by releasing one cusp of the cable. Then playing with the reaction at the other cusp changes the position of the released cusp. Thus, one can determine the correct reactions at the first cusp, which makes the second cusp position same with the released support's. Cable equilibrium equations and stiffness matrix is derived accordingly and some sample cables are solved.

***Keywords**: Single-Segment Cable, Catenary, Rope, Newton-Raphson.*

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I. INTRODUCTION

Cables are materially simple but geometrically complex structural elements. Generally they manufactured by steel, copper or aluminum depending on need of use. They all have the same geometry. This geometry (Figure 1) makes the cable structure having negligibly small shear, flexural and torsional rigidities. This properties gain them a self-shapeability. Under their self-weight, they depict a geometry named as catenary (Figure 2).

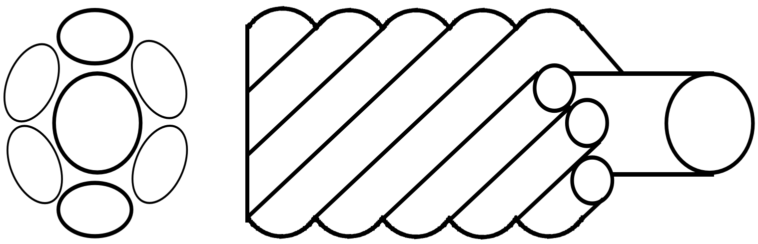

Figure 1 Cross-section and layout of 7 wire strands

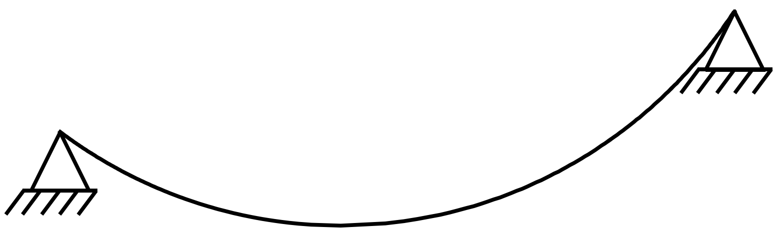

Figure 2 A sample catenary

Solution of a catenary had been made by shape assumptions [1, 2, 3, 4, 5, 6]. Although those assumptions are valid for faster solutions, computers makes it possible to perform very fast nonlinear cable solutions [7, 8, 9, 10]. These iterative nonlinear cable solutions are first used by Michalos and Brinstiel [11], Skop and O'Hara [12, 13]. The method was named as method of imaginary reactions. Polat M.U. [14] introduced Newton Raphson method in 1980's which improves the method of imaginary reactions. This method was defined in 3D in thesis of Demir A. [15]

II. SINGLE-SEGMENT CABLE EQUILIBRIUM EQUATIONS

A cable, having total unstressed length L_U and stressed length L_S, is supported between points A and B. The view of the cable in space is shown in Figure 3.

As illustrated in Figure 3, $\mathbf{P}_A$ and $\mathbf{P}_B$ are the position vectors of cable supports. Let M be any point on the cable defined by the following parameters;

l_u ; unstressed arc length form point A to M .

l_s ; stressed arc length from point A to M .

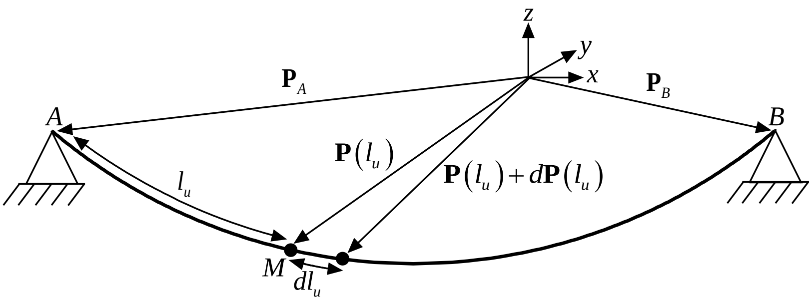

Figure 3 Configuration of single-segment cable in space

The unit tangent along the cable, $\boldsymbol{\tau}(l_u)$, can be defined as;

$$\boldsymbol{\tau}(l_u) = \frac{d\mathbf{P}(l_u)}{dl_s} \tag{1a}$$

Or

$$d\mathbf{P}(l_u) = -\boldsymbol{\tau}(l_u)\,dl_s \tag{1b}$$

The unknowns in Eq. 1b are; $\boldsymbol{\tau}(l_u)$ and the differential stressed arc length of the cable dl_s .

The unit tangent along the cable can also be defined as

$$\boldsymbol{\tau}(l_u) = \frac{\mathbf{R}(l_u)}{T(l_u)} \tag{2}$$

Where $\mathbf{R}(l_u)$ is reaction vector at l_u , $T(l_u)$ is tension at l_u and dl_u is the original differential length of the cable.

So, the elongation of the differential element is

$$\Delta l_u = dl_s - dl_u \tag{3}$$

The strain of this element is the elongation divided by the original length.

$$\varepsilon(l_u) = \frac{dl_s - dl_u}{dl_u} \tag{4}$$

From Eq. 4 the stressed length of the element can be written as

$$dl_s = \left[1+\varepsilon(l_u)\right]dl_u \tag{5}$$

Substituting Eq. 5 into Eq. 1b

$$d\mathbf{P}(l_u) = -\boldsymbol{\tau}(l_u)\left[1+\varepsilon(l_u)\right]dl_u \tag{6a}$$

$$\frac{d\mathbf{P}(l_u)}{dl_u} = -\boldsymbol{\tau}(l_u)\left[1+\varepsilon(l_u)\right] \tag{6b}$$

Finally, writing Eq. 6b in integral form

$$\mathbf{P}(l_u) = \mathbf{P}(0) - \int_0^{l_u} \frac{\mathbf{R}(x)}{T(x)}\left[1+\varepsilon(l_u)\right]dx \tag{7a}$$

Since $\mathbf{P}(0) = \mathbf{P}_A$,

$$\mathbf{P}(l_u) = \mathbf{P}_A - \int_0^{l_u} \frac{\mathbf{R}(x)}{T(x)}\left[1+\varepsilon(l_u)\right]dx \tag{7b}$$

Consequently, $\mathbf{R}_A$, which is equal to $\mathbf{R}(l_0)$, is the only unknown in this equation and it can be regarded as the initial condition of the problem.

III. STIFFNESS MATRIX

If a virtual displacement, $\Delta\mathbf{P}_B$, is given to support B, there will be a change in the reactions at the other support, $\Delta\mathbf{R}_A$. The relation between these parameters are explained by the stiffness matrix, $[S]$.

$$\Delta\mathbf{R}_A = [S]\Delta\mathbf{P}_B \tag{8}$$

The stiffness matrix is determined by using the variational approach as follows:

From variation of Eq. 7b, $\Delta\vec{P}_B$ is determined.

$$\Delta\mathbf{P}_B = -\int_0^{L_U} \Delta\left\{\left[1+\varepsilon(l_u)\right]\frac{\mathbf{R}(l_u)}{T(l_u)}\right\}dl_u$$

$$= -\int_0^{L_U}\left\{\frac{1+\varepsilon(l_u)}{T(l_u)}\Delta\mathbf{R}(l_u) + \mathbf{R}(l_u)\Delta\frac{1+\varepsilon(l_u)}{T(l_u)}\right\}dl_u \tag{9}$$

Unknowns are $T(l_u)$, $\Delta\mathbf{R}(l_u)$ and $\Delta\frac{1+\varepsilon(l_u)}{T(l_u)}$ in Eq. 9.

$$\Delta\frac{1+\varepsilon(l_u)}{T(l_u)} = \frac{\Delta\left[1+\varepsilon(l_u)\right]T(l_u) - \left[1+\varepsilon(l_u)\right]\Delta T(l_u)}{T^2(l_u)}$$

$$= \frac{\Delta\varepsilon(l_u)T(l_u) - \left[1+\varepsilon(l_u)\right]\Delta T(l_u)}{T^2(l_u)} \tag{10}$$

Thus, unknowns are $T(l_u)$, $\Delta T(l_u)$, $\Delta\mathbf{R}(l_u)$, $\Delta\varepsilon(l_u)$ in Eq. 9.

Tension in cable is

$$T(l_u) = \left[\mathbf{R}(l_u)\cdot\mathbf{R}(l_u)\right]^{1/2} \tag{11}$$

In variational form, $\Delta T(l_u)$

$$\Delta T(l_u) = \frac{1}{2}\frac{\left[\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u) + \Delta\mathbf{R}(l_u)\right]}{\left[\mathbf{R}(l_u)\cdot\mathbf{R}(l_u)\right]^{1/2}} \tag{12}$$

Or

$$\Delta T(l_u) = \frac{\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u)}{T(l_u)} \tag{13}$$

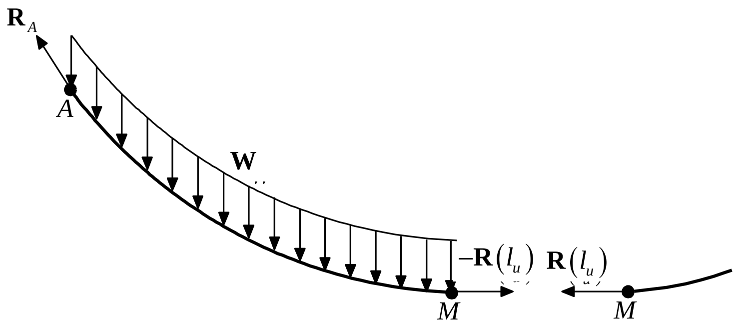

Figure 4 Reactions on cable

Many external forces e.g. wind force, could be applied to the cable. If no external load is applied, there will be only self-weight of the cable.

$$\mathbf{F}_{ext}(l_u) = \mathbf{W}\, l_u \tag{14}$$

From the free body diagram of cable element shown in Figure 4, reaction at point M is;

$$\mathbf{R}(l_u) = \mathbf{R}_A + \mathbf{F}_{ext}(l_u) \tag{15a}$$

For the whole cable

$$\mathbf{R}(L_U) = \mathbf{R}_A + \mathbf{F}_{ext}(L_U) \tag{15b}$$

$$\mathbf{R}_B = \mathbf{R}\left(L_U\right) \tag{16}$$

Substituting Eq. 16 into Eq. 15b

$$\mathbf{R}_B = \mathbf{R}_A + \mathbf{F}_{ext}\left(L_u\right) \tag{17}$$

From variation of Eq. 17

$$\Delta\mathbf{R}(l_u) = \Delta\mathbf{R}_A = \Delta\mathbf{R}_B \tag{18}$$

The strain can also be expressed by the stress-strain relationship as

$$\varepsilon(l_u) = \frac{T(l_u)}{EA}\upsilon \tag{19}$$

$T(l_u)$ is the tension at M and E , A and υ are material properties of cable.

The variational form of strain, $\Delta\varepsilon(l_u)$, from Eq. 4

$$\Delta\varepsilon(l_u) = \upsilon\left[\frac{T(l_u)}{EA}\right]^{\upsilon-1}\frac{\Delta T(l_u)}{EA} \tag{20a}$$

$$= \upsilon\varepsilon(l_u)\frac{\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u)}{T^2(l_u)} \tag{20b}$$

So, substituting Eq. 11 Eq. 13 and Eq. 20b into Eq. 10

$$\Delta\left[\frac{1+\varepsilon(l_u)}{T(l_u)}\right] = \frac{\upsilon\varepsilon(l_u)\dfrac{\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u)}{T^2(l_u)}T(l_u) - \left[1+\varepsilon(l_u)\right]\dfrac{\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u)}{T(l_u)}}{T^2(l_u)}$$

$$= -\frac{1+\left(1-\upsilon\right)\varepsilon(l_u)}{T^3(l_u)}\left[\mathbf{R}(l_u)\cdot\Delta\mathbf{R}(l_u)\right] \tag{21}$$

Finally, substituting Eq. 18 and Eq. 21 into Eq. 9

$$\Delta\mathbf{P}_B = -\int_0^{L_U}\left\{\left|\frac{1+\varepsilon(l_u)}{T(l_u)}\right|\Delta\mathbf{R}_A - \left|\frac{1+(1-\upsilon)\varepsilon(l_u)}{T^3(l_u)}\right|\left[\mathbf{R}(l_u)\cdot\Delta\mathbf{R}_A\right]\mathbf{R}(l_u)\right\}dl_u \tag{22}$$

In global coordinate directions Eq. 22 will be

$$\Delta P_{BX}\,\mathbf{i} = -\int_0^{L_U}\left[C_1\Delta R_{AX}\,\mathbf{i} - C_2C_3C_4\right]dl_u \tag{23a}$$

$$\Delta P_{BY}\,\mathbf{j} = -\int_0^{L_U}\left[C_1\Delta R_{AY}\,\mathbf{j} - C_2C_3C_4\right]dl_u \tag{23b}$$

$$\Delta P_{BZ}\,\mathbf{k} = -\int_0^{L_U}\left[C_1\Delta R_{AZ}\,\mathbf{k} - C_2C_3C_4\right]dl_u \tag{23c}$$

Where

$$C_1 = \left[\frac{1+\varepsilon(l_u)}{T(l_u)}\right]$$

$$C_2 = \left[\frac{1 + (1-\upsilon)\varepsilon(l_u)}{T^3(l_u)} \right]$$

$$C_3 = \left[R_X(l_u)\Delta R_{AX} + R_Y(l_u)\Delta R_{AY} + R_Z(l_u)\Delta R_{AZ} \right]$$

$$C_4 = \left[R_X(l_u)\mathbf{i} + R_Y(l_u)\mathbf{j} + R_Z(l_u)\mathbf{k} \right]$$

Writing Eq. 23a,b,c in the form of Eq. 8

$$\begin{Bmatrix} \Delta P_{BX} \\ \Delta P_{BY} \\ \Delta P_{BZ} \end{Bmatrix} = [S]^{-1} \begin{Bmatrix} \Delta R_{AX} \\ \Delta R_{AY} \\ \Delta R_{AZ} \end{Bmatrix} \tag{24}$$

Where the stiffness matrix is

$$[S] = \begin{bmatrix} -\int_0^{L_e} \left[C_1 - C_2 R_X^2(l_u) \right] dl_u & -\int_0^{L_e} \left[C_2 R_X(l_u) R_Y(l_u) \right] dl_u & -\int_0^{L_e} \left[C_2 R_X(l_u) R_Z(l_u) \right] dl_u \\ -\int_0^{L_e} \left[C_2 R_Y(l_u) R_X(l_u) \right] dl_u & -\int_0^{L_e} \left[C_1 - C_2 R_Y^2(l_u) \right] dl_u & -\int_0^{L_e} \left[C_2 R_Y(l_u) R_Z(l_u) \right] dl_u \\ -\int_0^{L_e} \left[C_2 R_Z(l_u) R_X(l_u) \right] dl_u & -\int_0^{L_e} \left[C_2 R_Z(l_u) R_Y(l_u) \right] dl_u & -\int_0^{L_e} \left[C_1 - C_2 R_Z^2(l_u) \right] dl_u \end{bmatrix}$$

Inverse of stiffness matrix is the flexibility matrix, $[F] = [S]^{-1}$.

So, Eq. 8 can be rewritten as

$$\Delta P_B = [S]^{-1} \Delta R_A \tag{25a}$$

or

$$\Delta P_B = [F] \Delta R_A \tag{25b}$$

IV. NEWTON-RAPHSON METHOD

The Newton-Raphson method is an iterative technique for solving equations numerically. The solution procedure of this method is based on making linear approximations to find a solution for nonlinear systems or equations in each step. It is aimed to achieve the target linearly. Nevertheless, solutions are always approximate. Newton-Raphson method is an appropriate method to find a solution for cable positioning due to its nonlinear behavior.

It will be seen that the position of cable is a function of $\mathbf{R}_A$ from Eq. 7b. However, the reaction at support A for the solution case, $\mathbf{R}_{A,sol}$, is not known. Newton-Raphson method is used to find that reaction. A linear approximation is made for each iteration to reach the solution.

The step-by-step procedure to find the unknown support reactions of cable is explained below and described schematically in Figure 5.

Make an initial approximation for the reactions at support A, $\mathbf{R}_A^{[i]}$,where [i] shows the iteration number which is 0 for the initial guess.

Determine the cable configuration by Eq. 7b. The end of the cable position is $\mathbf{P}^{[i]}(L_U)$. Also calculate the stiffness matrix $\left[S^{[i]} \right]$.

Determine the misclose vector and the error as

$$\mathbf{M}^{[i]} = \mathbf{P}_B - \mathbf{P}^{[i]}(L_U) \tag{27}$$

$$E^{[i]} = \left| \mathbf{M}^{[i]} \right| \tag{28}$$

Calculate a better approximation for the support reactions at A.

$$\mathbf{R}_A^{[i+1]} = \mathbf{R}_A^{[i]} + \left[S^{[i]} \right]^{-1} \mathbf{M}^{[i]} \tag{29}$$

Go to step 2 and continue iterations until $E^{[i]} \leq ERR$. where ERR is the target error for approximate result.
It can be easily comprehended that initial guess for the support reactions is an important step. A convenient initial support reaction will decrease the iteration number considerably.

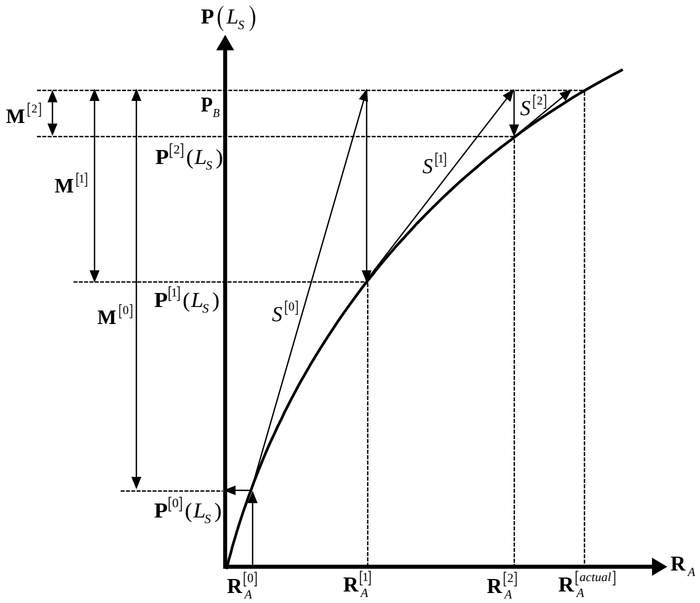

Figure 5 Newton Raphson method in schematic form for single-segment cable

V. VERIFICATION

A numerical iterative algorithm is proposed for the solution of cable systems. Consequently, the predicted solution will only be an approximation to the exact equilibrium state. Therefore, the convergence characteristic of the algorithm as well as its ability to produce the final equilibrium state accurately is of concern. The convergence characteristic of the algorithm is verified by changing the cable configuration in space and its degree of slackness. Accordingly, a verification model presented by Peyrot A.H. and Goulois A.M. [16] is used.
A weightless cable is supported at its ends with a defined load. Load per unit length of cable is 1 N/m. Modulus of elasticity times cross-section area is 3e7 N/m and thermal expansion coefficient is 0.65e-5 / ºC. Initial total length of the cable is 100 m. A temperature change +100 ºC is applied to see the action. Being an iterative method precision is taken as 1e-6 m for proposed method which was named as ERR .

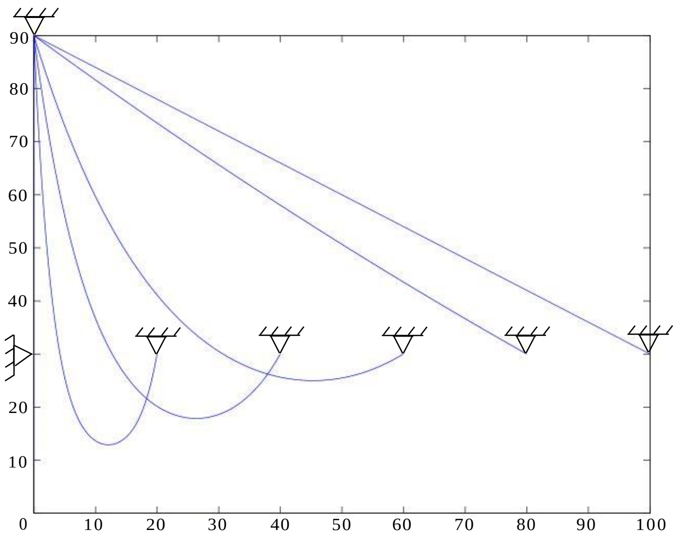

Figure 6 Cable configurations

There are 6 different cable configurations (Figure 6). Coordinates of supports of each configuration is shown in table above. Besides, results of previous study, achieved by commercial computer program ANSYS and proposed method are also tabulated below. In **Error! Reference source not found.** error of percentages are given.

Table 1 Support reactions

Supports	Coordinates		PEYROT & GOULOIS		ANSYS		SSC Solution	
	X (m)	Y (m)	Rx	Ry	Rx	Ry	Rx	Ry
S1	0	30	0	20.02	-0.043	19.091	0	20.02
S2	20	30	3.061	19.93	3.061	19.932	3.061	19.942
S3	40	30	9.172	19.24	9.172	19.242	9.172	19.252
S4	60	30	22.15	15.73	22.146	15.734	22.146	15.744
S5	80	30	504	-328.8	504.1	-328.87	504.102	-328.859
S6	100	30	4170000	-2511000	4258500	-2555000	4258489	-2555043

Table 2 Error of percentages

Supports	PEYROT & GOULOIS		ANSYS	
	Rx	Ry	Rx	Ry
S1	0.000%	0.000%	-	4.640%
S2	0.000%	0.060%	0.000%	0.050%
S3	0.000%	0.062%	0.000%	0.052%
S4	-0.018%	0.089%	0.000%	0.064%
S5	0.020%	0.018%	0.000%	-0.003%
S6	2.078%	1.724%	0.000%	0.002%

VI. CONCLUSION

Cables are highly nonlinear structural elements. This nonlinearity is due to its unique geometry. Scientist proposed iterative methods to capture the nonlinearity of the cable. These methods gain value with common usage of computer programs. Presented method is also a 3D iterative method. Method based on determination of one support reaction giving the right position for second support coordinates. Therefore, a better solution can be achieved by a better position of second end (support) coordinates of cable. A case solution was given by using a known case in literature.

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