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An efficient algorithm for ogee spillway discharge with partially-opened radial gates by the method of *Design of Small Dams* and comparison of current and previous methods

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ABSTRACT

Ogee profile flood spillways equipped with radial gates are common, and accurate computation of spilled discharge through partially-opened radial gates is an important problem. A new algorithm is developed for the method given in the latest edition of the book: Design of Small Dams for computation of discharge over ogee spillways equipped with radial gates for the partial opening case. This algorithm is more efficient with less computational load than the one presented in 'Hydraulic Design Criteria, Sheets 311-1 to 311-5' by US Army Corps of Engineers which is the method by 'Design of Small Dams'. For a wide range of partial gate openings on a few existing dams, discharges are computed by this method and are compared with those given by the previous method comprised in the former editions of 'Design of Small Dams'. As both yield close values for small gate openings, the current method gives spillway discharges about 10 % to 30 % greater than the previous method for large gate openings. Next, discharge coefficients are computed using the measured data taken on 1:50 scale laboratory model of the spillway of Kavsak Dam and are compared with those given by the charts in 'Design of Small Dams', which are found to be deviant as much as 10 %.

Keywords: *discharge of radial-gated spillways*

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I. INTRODUCTION

The subsections 194 of the first and 201 of the second editions of the book: *Design of Small Dams* [1, 2] are exactly the same, which both give the equation to compute the discharge over a radial-gated ogee-crested spillway while the gates are partially opened, which is a dimensionally homogeneous equation, as:

$$Q = (2/3)\cdot(\sqrt{2g})\cdot C\cdot L_e\cdot(H_1^{1.5} - H_2^{1.5}) \qquad (1)$$

where, g is the acceleration of gravity, C is a coefficient, L_e is the effective length of crest, H_1 and H_2 are defined as: "*H_1 and H_2 are the total heads (including the velocity head of approach) to the bottom and top of the orifice, respectively*.", and Q is the discharge. A copy of Fig. 197 of the first edition, which is Fig. 257 of the second edition, of *Design of Small Dams*, is given here as Fig. 1. C in equation (1) is given as a function of the ratio (d/H_1) in this figure, where d is the vertical gate opening, which equals $H_1 - H_2$. In two other relevant books by the Bureau of Reclamation, also, which are *Design of Gravity Dams* [3] and *Design of Arch Dams* [4], equation (1) is given along with Fig. 1 for computation of the discharge under partially-opened radial gates.

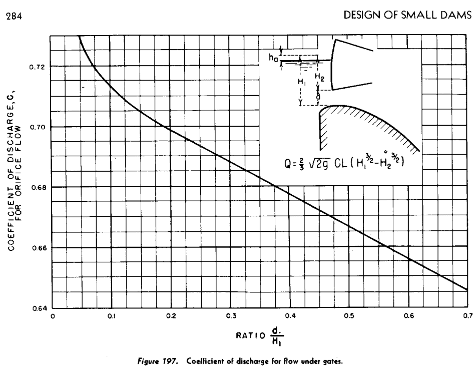

Figure 1. Copy of Fig. 197 in the 'Spillways' chapter of the first edition of *Design of Small Dams* [1], which is the same as Fig. 257 in the 'Spillways' chapter of the second edition of *Design of Small Dams* [2].

The effective length, L_e, is equal to the net length of spillway crest minus a reduction due to the contraction of the flow while passing from the lake first to the approach channel and next into the spillway bays separated by the gate piers, and is expressed as [1, 2]:

$$L_e = L - 2\cdot(N_p\cdot k_p + k_a)\cdot H \quad (1a)$$

where, L is the net length of the spillway crest excluding the sum of widths of the existing piers, N_p is the number of piers on the crest, k_p is the pier contraction coefficient, k_a is the approach abutments contraction coefficient, and H is the actual total head with respect to the spillway apex elevation. Three values for k_p and k_a are suggested as 0, 0.01, 0.02, and 0, 0.1, 0.2, respectively, depending on geometrical shapes of pier noses and of abutment headwalls [1, 2]. In all three *Design of Small Dams* [1, 2, 5], there are numerical examples for the uncontrolled (free flow) ogee spillway discharges, but, there are no examples for the gate-controlled flows.

In Fig. 1, the radial gate looks as if its gate seat is right on the apex in closed position. However, for radial gates whose gate seats are a little further downstream from the apex, which is a common application, the 'bottom of the orifice' should be the point on the spillway crest curve closest to the lip of the partially-opened gate. For computational simplicity, the bottom of the orifice may be taken constantly as the spillway apex elevation. For example, in the final projects of two dams in Turkey [6, 7], the designers used equation (1) and took the spillway apex elevation consistently as the bottom of the orifice, although their gate seats are a little downstream from the apex. The vertical gate opening (d) with respect to the spillway apex is slightly smaller than that with respect to the point closest to the gate lip; but, H_1 with respect to the spillway apex is also just a little smaller than H_1 with respect to the point closest to the gate lip. Therefore, the magnitude of d/H_1 does not change appreciably from both viewpoints, and the magnitudes of the discharge coefficient (C) are close. However, the magnitude of discharge computed with d and H_1 for the more realistic case may be non-negligibly greater than with d and H_1 with respect to the spillway apex.

In subsection 9.16 of the third edition of *Design of Small Dams* [5] however, a different equation is given for discharge over a radial-gated ogee spillway while the gates are partially opened, which is:

$$Q = C\cdot D\cdot L\cdot(2g\cdot H)^{1/2} \quad (2)$$

where, C is a dimensionless coefficient, D is the shortest distance between the gate lip and the spillway crest curve, L is the net length (not the effective length) of the spillway crest, g is the acceleration of gravity, and H is the vertical difference between the total head just upstream of the gate and the center of the gate opening. Equation (2) also is dimensionally homogeneous. Because it is in the updated edition of *Design of Small Dams* [5], equation (2) should be used, and for example, according to [8], the Spanish Committee on Large Dams recommends usage of this equation.

Fig. 2, giving the C coefficient of equation (2) as a function of the angle between the tangent to the crest curve at the point closest to the gate lip and the tangent to the gate at its lip, is a copy of Fig. 9-31 of the third edition of *Design of Small Dams* [5]. This figure is a replica of the figure captioned as: '*Hydraulic Design Chart 311-1*' given in *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9], which was developed by the Waterways Experimentation Station of US Army Corps of Engineers based on measurements on three laboratory models and on three prototype structures; and, it can be downloaded from: *chl.erdc.usace.army.mil/Media/2/8/1/300.pdf*. The angle symbolized by β in *Hydraulic Design Chart 311-1* [9] is denoted by θ in Fig. 9-31 of the third edition of *Design of Small Dams* [5]. And, *Hydraulic Design Chart 311-1* [9] has a high-resolution background for better visual reading of the magnitude of C. Actually, equation (2) and the new method is originally proposed by the Waterways Experimentation Station of US Army Corps of Engineers in the publication: *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9], and, the third edition of *Design of Small Dams* [5] directly refers to this method. Equation (2) and the figure for C as 'Plate 6-1', are given in Chapter 6 of *EM-1110-2-1603* [9], also. Although there are detailed examples for the free-flow (un-gated ogee spillway) case in *Design of Small Dams* [5], the case for the partially-opened gates is given briefly without any examples, where it is stated: "*For additional information and geometric computations see [20]*." (Reference number 20 being the publication: *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9]. Available at the above-mentioned web site, this is a technical report comprising seven pages, which articulately presents the details of a method for computing the C coefficient of equation (2) and finally the discharge for any partial opening. Investigation of this report will reveal that (1) there is a long table having 20 columns which lead to the magnitudes of the angle β and the orifice opening D, (2) there is another table having 15 columns leading to the magnitude of the spilled discharge versus a definite gate opening, and (3) there are two charts for the quantitative relationships of (i) coordinates of point on crest curve closest to the gate lip and of (ii) derivative of crest curve with respect to the x variable. These two charts need to be prepared beforehand in a log-log graph paper for specific values of *K* and *n* coefficients of the analytical expression depicting the downstream crest curve profile of the ogee spillway, which is equation (3) in the next section below. Besides, instead of using the actual analytical expression, a few circular arc replacements are suggested and used. In computing the angle β and the orifice opening *D* in the first table, various steps like using relationships of similar triangles are applied. The method is correct, but it is too winding (This seven-page publication [9] can be downloaded by anyone interested to witness for themselves this cumbersome and too long method.). Instead of using arcs of circles for the crest curve, and instead of such a long procedure involving so many intermediate steps, a new analytical method is developed here in this study, which provides all the terms of equation (2) and finally the discharge as a result of fewer steps with a less computational load. Therefore, the main objective of this study is to present this less complicated method for equation (2).

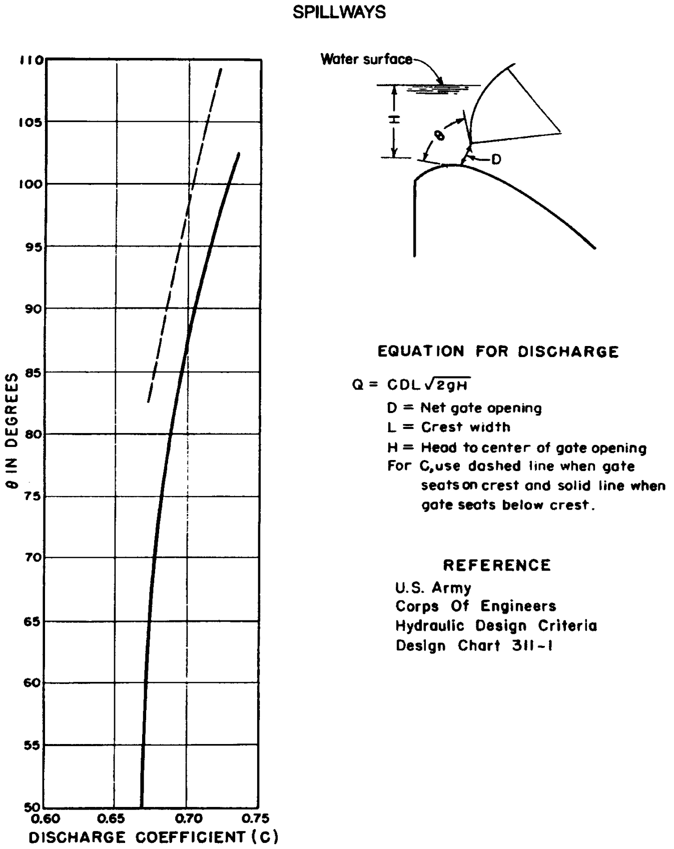

Figure 9-31.—Discharge coefficient for flow under gates. 103-D-1875.

Figure 2. Copy of Fig. 9-31 in the third edition of *Design of Small Dams* [5].

Whether by using either equation (1) or equation (2), computation of discharge passing over a radial-gated ogee flood spillway for the case of partially-opened gates is a common and significant problem, for which the studies by Haktanir et al [10] and Zargar et al [11] are typical examples. Computation of discharge over ogee spillways has always been a significant research topic investigated by various researchers [e.g., 12, 13, 14]. There may be other methods for computing discharge through partially-opened gates. For example, Ansar and Chen [15] presented generalized equations for discharge over ogee spillways with sharp-edged sluice gates using data measured at many prototype canal control structures in South Florida. Bahajantri et al [16] proposed a numerical method based on finite element approach. Saunders et al [17] developed a method using the Smoothed Particle Hydrodynamics model. The objective of the study whose summary is presented here is not to compare the method of *Design of Small Dams* [5] with the others, rather mainly to present an efficient algorithm for the USBR, and hence the USACE, method.

The second objective is to compare the (spillway discharge)↔(lake water surface elevation) relationships over wide ranges of (1) water heads from small magnitudes to the design head and of (2) partial gate openings from as small as 10 cm to close to the full opening given by both equation (1) and equation (2) on radial-gated ogee spillways of a few existing dams.

1. Computation of Spillway Discharge by Equation (2)

The relevant numerical data known from dimensions of the spillway and the radial gates, denoted by the symbols used herein are:

R_g : radius of the gate,
x_{L0} : abscissa of the gate seat (of the gate lip when the gate is at closed position),
y_{L0} : ordinate of the gate seat,
x_T : abscissa of the gate trunnion,
y_T : ordinate of the gate trunnion,
y_L : ordinate of the gate lip when it is partially opened (= vertical gate opening with respect to the spillway apex).

When the origin of a Cartesian coordinates system is located at the apex of an ogee spillway according to the right-hand rule, the profile of its downstream face is expressed analytically as [1, 2, 5]:

$$y = -K \cdot H_d^{(1-n)} \cdot x^n \tag{3}$$

where, y is the ordinate of a point on the crest curve downstream from the apex whose abscissa is x, H_d is the design head in the approach channel with respect to the spillway apex elevation, and K and n are "*constants whose values depend on upstream inclination and on the velocity of approach*" [5, page: 365]. The K and n coefficients are given in Fig. 9-21 of *Design of Small Dams* [5, page: 366] as a function of (1) the ratio: h_a / H_d , where h_a is the velocity head in the approach channel for the design discharge, and of (2) the inclination of the upstream face of the spillway. K varies within the interval: $0.47 < K < 0.54$, and n varies within: $1.74 < n < 1.87$.

Case 1: The gate seat is some distance downstream from the apex of the spillway

Fig. 3 schematically depicts the geometrical configuration of a partially-opened radial gate for this case.

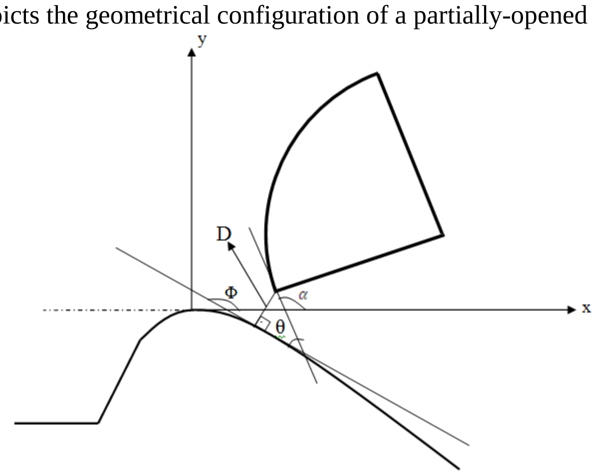

Figure 3. Tangents to the lip of the partially-opened radial gate and to the crest curve of the ogee spillway at the point closest to the gate lip, the angles these tangents make with the x axis, and the angle θ between these tangents.

In order to simplify the analytical expression of the downstream crest curve, equation (3), in Fig. 9-22 of *Design of Small Dams* [5, page: 368] this profile is closely approximated by adjacent arcs of three circles, whose center coordinates and radiuses are indicated in that figure. In this study, equation (3) is used throughout the analyses and circle approximations are not applied. But, the crest curve of an ogee spillway upstream from its apex is necessarily represented by arcs of two adjacent circles [5, page: 368] simply because the range of equation (3) is: $0 < x < +\infty$. As seen in Fig. 3, the shortest distance between the gate lip and the surface of the ogee spillway (D in Fig. 2) is on the line which is perpendicular to the line tangent to the crest curve at the point having the shortest distance to the lip. The line tangent to the radial gate at its lip and the angles these tangents make with the x axis and with each other are also shown in Fig. 3.

According to the adapted Cartesian coordinates, the analytical expression of the circle of which the radial gate is an arc is

$$y^2 - 2 \cdot y_T \cdot y + x^2 - 2 \cdot x_T \cdot x + x_T^2 + y_T^2 - R_g^2 = 0 \tag{4}$$

where, x and y are the coordinates of any point on the circle, x_T and y_T are the abscissa and the ordinate of the trunnion pin of the gate, and R_g is the radius of the gate.

The derivative of y with respect to x in equation (3) gives the slope of the line tangent to the downstream crest curve of the spillway, which is

$$y' = -\, n \cdot K \cdot H_d^{\,(1-n)} \cdot x^{(n-1)} \tag{5}$$

The ordinate of the gate lip (y_L) is equal to the vertical opening of the gate with respect to the spillway apex elevation; and, the abscissa of the lip (x_L) can be computed by inserting y_L for y in equation (4), which can be rewritten as

$$x^2 - 2 \cdot x_T \cdot x + y_L^{\,2} - 2 \cdot y_T \cdot y_L + x_T^{\,2} + y_T^{\,2} - R_g^{\,2} = 0 \tag{6}$$

Out of the two roots of equation (6), a quadratic equation, the one below gives the abscissa of the gate lip ($x_L = r_1$).

$$r_1 \;\; (= x_L) = x_T - (2 \cdot y_T \cdot y_L - y_L^{\,2} - y_T^{\,2} + R_g^{\,2})^{1/2} \tag{7}$$

It can be easily shown that as long as the vertical gate opening with respect to the spillway apex elevation, y_L, is smaller than ($y_T + R_g$), which corresponds to the point on top of the circle of the gate, the term in parentheses is positive-valued, and so equation (7) yields real roots all the time. The second root gives the abscissa of the point at the other side of the circle whose ordinate is the same as that of the gate lip, which has no physical meaning in this problem.

The line of the shortest distance between the gate lip and the point on the surface of the spillway is perpendicular to the line tangent to the spillway surface at that point closest to the lip. Denoting the slope of this line by S_1 and the slope of the line tangent to the spillway crest curve by S_2, the following equation holds.

$$S_1 \cdot S_2 = -1 \tag{8}$$

Symbolizing the ordinates of the point on the surface of the spillway closest to the gate lip by x_c and y_c, S_2 is given by the right-hand side of equation (5) by inserting x_c for x. And, from equation (8) S_1 is depicted as

$$S_1 = 1 \,/\, [n \cdot K \cdot H_d^{(1-n)} \cdot x_c^{(n-1)}] \tag{9}$$

From geometry, the analytical equation of the line passing through the point (x_L, y_L) with the slope S_1 can be written as

$$y = S_1 \cdot x + y_L - S_1 \cdot x_L \tag{10}$$

Because this line passes through the point (x_c, y_c), equation (10) holds at this point also:

$$y_c = S_1 \cdot x_c + y_L - S_1 \cdot x_L \tag{11}$$

The point (x_c, y_c) satisfies equation (3) also because it is on the surface of the spillway. Therefore, when x_c is inserted for x in equation (3), y at the left-hand side becomes y_c, and when the right-hand side of equation (3) in this form is inserted for y_c in equation (11), the below equation results:

$$S_1 \cdot x_c - S_1 \cdot x_L + K \cdot H_d^{(1-n)} \cdot x_c^{\,n} = -\, y_L \tag{12}$$

Now, inserting the right-hand side of equation (9) for S_1 in equation (12), after arranging, the below equation is obtained.

$$x_c^{(2-n)} - x_L \cdot x_c^{(1-n)} + n \cdot [K \cdot H_d^{(1-n)}]^2 \cdot x_c^{\,n} + n \cdot K \cdot H_d^{(1-n)} \cdot y_L = 0 \tag{13}$$

The root of this equation is the abscissa of the point on the spillway crest curve closest to the gate lip (x_c). It is obvious that an analytical solution for the root is not possible. However, it can be easily computed by the iterative Newton-Raphson algorithm because, symbolizing its left-hand side by $f(x_c)$, the analytical derivative of $f(x_c)$ with respect to x_c can be taken easily, which is:

$$f'(x_c) = (2-n)\cdot x_c^{(1-n)} - (1-n)\cdot x_L\cdot x_c^{-n} + n^2\cdot[K\cdot Hd^{(1-n)}]^2\cdot x_c^{(n-1)} \tag{14}$$

The recursion formula of the iterative algorithm is

$$x_{c,1} = x_{c,0} - f(x_{c,0}) / f'(x_{c,0}) \tag{15}$$

where, $f(x_{c,0})$ is the value of the left-hand side of equation (13) with the numerical value $x_{c,0}$ inserted for x_c, and $x_{c,1}$ is a value closer the root than $x_{c,0}$. The absolute relative difference between $x_{c,1}$ and $x_{c,0}$ is computed by: $RD = |[f(x_{c,0}) / f'(x_{c,0})] / x_{c,1}|$. The iterations stop when $RD \leq 10^{-6}$, which means the last $x_{c,1}$ is the root correct to 6 significant digits. Otherwise, $x_{c,1}$ is assigned as the new $x_{c,0}$, and iterations continue.

For case 1, the gate seat being 20 ~30 ~ 40 cm or so below and a couple of meters or so downstream from the spillway apex, the point on the crest curve closest to the gate lip must be between the apex, origin of the x–y coordinates (0, 0), and the gate seat (x_{L0}, y_{L0}). Therefore, initially, the interval (0, x_{L0}) is subdivided into ten equal subintervals, and that one in which $f(x_c)$ changes sign is determined. Next, the bisection algorithm is applied for the root of equation (13) ($x_{1,3} = (x_{1,1} + x_{1,2}) / 2$) until the relative difference between the last two iterations is less than or equal to 1×10^{-3} ($|(x_{1,3} - x_{1,2}) / x_{1,3}| \leq 1\times10^{-3}$). The last value of $x_{1,3}$ is taken as the initial estimate of the Newton-Raphson algorithm ($x_{c,0} = x_{1,3}$) and in just a few iterations the root to 6 significant digits is computed. This scheme yields convergent solutions always.

Next, the ordinate of the point on the spillway crest curve closest to the gate lip, y_c, is computed by equation (3) by inserting the value of x_c for x.

Next, the closest distance between the gate lip and the surface of the spillway, D, is computed by:

$$D = [(x_L - x_c)^2 + (y_L - y_c)^2]^{1/2} \tag{16}$$

Derivative of y in equation (4) with respect to x is

$$y' = (x_T - x) / (y - y_T) \tag{17}$$

which gives the slope of the tangent to the circle of the gate at the point whose ordinates are x and y. By inserting x_L and y_L for x and y, y' equals the tangent of the angle of the line tangent to the gate at its lip with respect to the x axis, denoted by α in Fig. 3. Hence, the angle α in Fig. 3 is computed by:

$$\alpha = \arctan[(x_T - x_L) / (y_L - y_T)] \tag{18}$$

Next, the angle of the line tangent to the crest curve at the point (x_c, y_c) with respect to the x axis, denoted by Φ in Fig. 3, is computed by equation (5), which becomes

$$\Phi = \arctan[-n\cdot K\cdot H_d^{(1-n)}\cdot x_c^{(n-1)}] \tag{19}$$

Next, the angle symbolized by θ in Fig. 9-31 of *Design of Small Dams* [5], and in Fig. 3 here, is computed by:

$$\theta = \Phi - \alpha \tag{20}$$

Next, the magnitude of the C coefficient in equation (2) is taken from either Fig. 9-31 of *Design of Small Dams* [5] or from *Hydraulic Design Chart 311-1* of *Hydraulic Design Criteria*, *Volume 2* [9], preferably from the latter because it has a higher resolution, as a function of the angle θ.

Next, the vertical distance between the water surface elevation just upstream of the partially-opened radial gate and the center of the gate opening is computed by:

$$H = H_2 + (y_L + |y_{L0}|) / 2 = H_2 + (y_L - y_{L0}) / 2 \tag{21}$$

Here, H_2 is the vertical distance between the water surface elevation plus the velocity head in the approach channel just upstream of the gate and the gate lip.

Finally, the discharge through the partially-opened radial gates is computed by equation (2). Obviously, the last three steps are the same as those of the method of *Hydraulic Design Chart 311-1* of *Hydraulic Design Criteria, Volume 2* [9].

Case 2: The gate seat is on the apex of the spillway

The upstream side of the crest curve of the ogee spillway is expressed as arcs of two adjacent circles in Fig. 9-21(A) of *Design of Small Dams* [5, page: 366]. The first circle is upstream of the apex of the spillway with a radius of R_1, and its center is on the ordinate axis. According to Fig. 9-21(f) of *Design of Small Dams* [5, page: 367], R_1 varies within the interval: $(0.34)\cdot H_d \leq R_1 \leq (0.56)\cdot H_d$. Taking R_1 as a positive real number, the analytical expression of this circle is

$$y = (R_1^2 - x^2)^{1/2} - R_1 \qquad (22)$$

The analytical manipulations for the desired quantities in case 2 are made such that for all positions of the partially-opened radial gate, the point on the spillway crest curve closest to the gate lip is on the first circle, because it is geometrically impossible for this point to be far upstream on the second circle. In this case, the profile of the upstream part of the spillway surface is the arc of the circle number 1 of Fig. 9-21(A) of *Design of Small Dams* [5, page: 366]. In Fig. 4, the geometrical configuration of the partially-opened radial gate and of the spillway of the Yellowtail Afterbay Dam is given as an example for definition of the relevant geometrical peculiarities for a spillway whose radial gate sits on the apex in closed form. This figure is drawn to scale for a vertical gate opening of 6 feet with respect to the spillway apex (y_L = 6.0 ft). The Yellowtail Dam is in Montana, USA [18, 19].

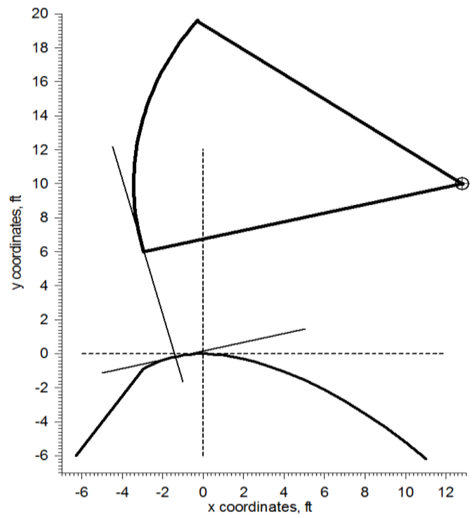

Figure 4. Profile of the gate and the spillway of the Yellowtail Afterbay Dam for a partial opening of d = 6.0 ft and the tangents to the gate lip and to the crest curve at the point closest to the gate lip.

First, the abscissa of the lip of the partially-opened gate (x_L) is computed by equation (7) for this case also. If x_L is negative ($x_L < 0$), which occurs more commonly for this case, then using equation (22) instead of equation (3) and performing the same algebraic analyses of the previous section between equation (8) through equation (12) ultimately results in equation (23) below for the abscissa of the point on the spillway crest curve closest to the gate lip (x_c).

$$x_c = -R_1 / \{[(y_L + R_1) / x_L]^2 + 1\}^{1/2} \qquad (23)$$

Here, R_1 is the radius of the first circle forming the spillway crest upstream of the apex. Notice that for this case x_c is directly computed whereas it is the root of equation (13) for the first case. Rarely, for very large gate openings, the point on the spillway crest curve having the shortest distance to the lip may be somewhere just a little downstream from the apex, for case 2 also. Then, x_c is computed as the root of equation (13), instead, similar to case 1.

If x_L is negative ($x_L < 0$), then the ordinate of the point on the spillway crest curve closest to the gate lip (y_c) is computed by equation (22) by inserting x_c for x. If x_L is positive ($x_L > 0$), then the ordinate of the point on the spillway crest curve closest to the gate lip (y_c) is computed by equation (3) by inserting the value of x_c for x.

Next, the closest distance between the gate lip and the surface of the spillway (D) and the angle of the line tangent to the gate at its lip with respect to the x axis (α) are computed by equations (16) and (18), respectively, for case 2 also.

Next, the angle of the line tangent to the crest curve at the point (x_c, y_c) with respect to the x axis, denoted by Φ here, is computed using the derivative of y in equation (22) with respect to x and inserting the value of x_c for x, which results in

$$\Phi = \arctan[-x_c / (R_1^2 - x_c^2)^{1/2}] \qquad (24)$$

For case 2, from geometrical relationships similar to those in Fig. 4, the angle symbolized by θ in Fig. 9-31 of *Design of Small Dams* [5] is computed by equation (25) below.

$$\theta = \Phi + (\pi - \alpha) \qquad (25)$$

The rest of the procedure for computing the discharge is the same as that for case 1. Notice that neither in *Design of Small Dams* [5] nor in *Hydraulic Design Criteria, Volume 2, Sheets 311-1 to 311-5* [9] there is any method suggested for computation of the angle θ and the orifice gate opening D, for case 2.

II. Application of the Developed Algorithm

The algorithm summarized above is coded as a computer program and applied to a few dams. In order to verify its accuracy, this method is first applied to the example given in *Sheets 311-1 to 311-5 Tainter Gates on Spillway Crests, Discharge Coefficients* [9], which is John Doe Dam in America. The relevant properties of this dam and those of Yellowtail Afterbay Dam in America, Kavsak Dam in Turkey, and Bakhra Dam in India are given in Table 1.

Table 1. Geometrical data of the flood spillways and their radial gates and some pertinent data of the dams analyzed in this study as examples

Peculiarity	**John Doe**(1)	**Yellowtail Afterbay**(2)	**Kavsak**(3)	**Bhakra**(4)
Maximum WSE(*)	: 325.00 ft	3192.0 ft	321.20 m	1685.00 ft
Top of active pool	: 315.00 ft	3189.5 ft	320.00 m	1678.00 ft
Gate trunnion elevation	: 300.00 ft	3189.5 ft	308.23 m	1662.25 ft
Spillway apex elevation	: 288.00 ft	3179.5 ft	300.00 m	1645.00 ft
Gate seat elevation	: 287.46 ft	3179.5 ft	299.86 m	1642.57 ft
Gate top elevation at closed position	: 316.00 ft	3193.0 ft	326.27 m	1680.00 ft
Radius of gate	: 30.75 ft	16.25 ft	18.00 m	38.0 ft
K and n coefficients of	: 0.50	0.47	0.50	0.48
downstream crest curve	:1.85	1.77	1.85	1.83
Radius of upstream circle	: 18.0 ft	5.4 ft	11.24 m	18.0 ft
Number of gates	: 4	5	3	4
Net spillway length	: 42.0 ft	150.0 ft	37.8 m	230.0 ft
Angle of upstream face	: 90º	2h:3v	2h:3v	90º
Design head, H_d	: 37.0 ft	12.5 ft	21.2 m	38.5 ft
Spillway discharge at H_d	: 23330 ft^3/s	25270 ft^3/s	7640 m^3/s	210000 ft^3/s

*: WSE: water surface elevation
(1): [9]
(2): [18, 19]
(3): [20]
(4): [21]

In the John Doe Dam example of the mentioned report [9], the pertinent quantities and spillway discharges are computed for 12 different combinations of lake water surface elevations and partial gate openings. The numerical values of some salient properties for all these 12 cases given in that report and those computed by the method of this study are presented in Table 2.

Next, the spillway discharges have been computed by both equations (1) and (2) for a few partial gate openings from very small values up to the full opening for John Doe and Yellowtail Afterbay Dams in America, Kavsak Dam in Turkey, and Bhakra Dam in India for a lake water surface elevation (WSE) half way between the maximum water surface elevation (Hmax) and top of active pool, and the results are given in Tables 3, 4, 5, and 6. As seen in these tables, the previous (1973) method mostly gives discharges smaller than the present (1987) method, the difference being less than 10 % for gate openings smaller than 40 % of the total head, but being as high as 30 % for large openings. The present method gives greater discharges for gate openings of about 70 ~ 80 % of the total head than the free flow discharge at the same head, which is a contradiction.

Table 2. Comparison of values given in *Hydraulic Design Charts 311-2* and *311-5* (abbreviated as *HDC* below) [9] for the example of John Doe Dam with the values computed by the algorithm developed in this study (lengths are in feet, discharges are in cfs)

Gate opening, dsa	Angle θ		*D* of Eqn.(2)		*C* coefficient		Water surface elevtn	Head on spillway	Discharge, *Q*	
	HDC	this study	*HDC*	this study	*HDC*	this study			*HDC*	this study
(ft)	(degrees)		(ft)				(ft)		(cfs)	
3.70	67.2°	67.3°	3.96	3.96	0.676	0.676	300	10.27	2900	2892
							315	25.27	4500	4538
							325	35.27	5400	5361
7.40	76.1°	76.1°	7.59	7.55	0.683	0.684	310	18.36	7500	7459
							315	23.36	8400	8413
							325	33.36	10100	10054
11.10	84.0°	84.0°	11.25	11.21	0.694	0.693	310	16.49	10700	10638
							315	21.49	12200	12144
							325	31.49	14800	14700
14.80	91.2°	91.1°	14.91	14.91	0.707	0.707	315	19.63	15800	15743
							320	24.63	17600	17634
							325	29.63	19300	19341

Table 3. Results of spillway discharge computations for partial gate openings by the 1973 and 1987 methods of *Design of Small Dams* and for free flow at John Doe Dam in America.

All lengths are in ft and discharges are in cfs			
Gate trunnion coordinates (y, x)	: 12.00	33.55	
Gate seat coordinates (y, x)	: -0.54	5.48	
Lake water surface elevation	: 320.0		
Variations of angle theta, orifice coefficient, and discharges by 1973 & 1987 editions of *Design of Small Dams* versus vertical gate opening:			

dsa(1)	dbo(2)	theta	Corifice	Q-1973	Q-1987	Rel.Diff.
5.0	5.18	71	0.679	5351	6486	-17 %
6.0	6.15	73	0.681	6259	7646	-18 %
7.0	7.13	75	0.683	7157	8801	-18 %
8.0	8.11	77	0.685	8031	9948	-19 %
9.0	9.10	80	0.688	8881	11092	-19 %
10.0	10.09	82	0.690	9707	12226	-20 %
11.0	11.08	84	0.693	10507	13351	-21 %
12.0	12.08	86	0.696	11280	14473	-22 %
13.0	13.07	88	0.700	12027	15598	-22 %
14.0	14.07	90	0.704	12746	16707	-23 %

15.0	15.07	91	0.708	13437	17800	-24 %
16.0	16.07	93	0.712	14099	18886	-25 %
17.0	17.08	95	0.716	14733	19971	-26 %
18.0	18.08	97	0.721	15338	21046	-27 %
19.0	19.09	99	0.725	15913	22104	-28 %
20.0	20.10	100	0.729	16457	23139	-28 %
21.0	21.11	102	0.735	16971	24206	-29 %
22.0	22.12	104	0.735	17454	25068	-30 %
24.0	24.15	107	0.735	18322	26713	-31 %
26.0	26.20	111	0.735	19053	28252	-32 %
28.0	28.25	114	0.735	19634	29687	-33 %
Fully open		---	-----	19279	19279	----

(1): vertical gate opening with respect to spillway apex elevation
(2): vertical gate opening with respect to bottom of orifice

Table 4. Results of spillway discharge computations for partial gate openings by the 1973 and 1987 methods of *Design of Small Dams* and for free flow at Yellowtail Afterbay Dam in America.

All lengths are in ft and discharges are in cfs			
Gate trunnion coordinates (y, x)	: 10.00	12.81	
Gate seat coordinates (y, x)	: 0.0	0.0	
Lake water surface elevation	: 379.5		
Variations of angle theta, orifice coefficient, and discharges by 1973 & 1987 editions of *Design of Small Dams* versus vertical gate opening:			

dsa(1)	dbo(2)	theta	Corifice	Q-1973	Q-1987	Rel.Diff.
1.0	1.03	63	0.674	2854	2769	4 %
2.0	2.09	71	0.674	5521	5515	1 %
3.0	3.13	77	0.674	7960	8136	-2 %
4.0	4.15	82	0.674	10153	10588	-4 %
5.0	5.17	86	0.680	12102	12959	-6 %
6.0	6.17	90	0.686	13809	15185	-9 %
7.0	7.17	94	0.692	15275	17227	-11 %
8.0	8.16	97	0.698	16495	19083	-13 %
9.0	9.15	100	0.704	17454	20744	-15 %
10.0	10.13	103	0.709	18122	22207	-18 %
Fully open		---	-----	21362	21362	----

(1): vertical gate opening w.r.t. spillway apex elevation
(2): vertical gate opening w.r.t. bottom of orifice

Table 5. Results of spillway discharge computations for partial gate openings by the 1973 and 1987 methods of *Design of Small Dams* and for free flow at Kavsak Dam in Turkey.

All lengths are in m and discharges are in m^3/s			
Gate trunnion coordinates (y, x)	: 8.23	17.98	
Gate seat coordinates (y, x)	: -0.14	2.04	
Lake water surface elevation	: 320.6		
Variations of angle theta, orifice coefficient, and discharges by 1973 & 1987 editions of *Design of Small Dams* versus vertical gate opening:			

dsa(1)	dbo(2)	theta	Corifice	Q-1973	Q-1987	Rel.Diff.
0.2	0.31	57	0.672	173	161	8 %
0.5	0.60	59	0.672	322	304	6 %
1.0	1.07	61	0.673	565	543	5 %
1.5	1.55	64	0.675	804	782	3 %
2.0	2.03	66	0.676	1040	1022	2 %
2.5	2.52	68	0.677	1274	1261	2 %
3.0	3.02	70	0.679	1505	1500	1 %

3.5	3.51	73	0.680	1733	1737	0 %
4.0	4.01	75	0.682	1954	1974	-1 %
4.5	4.50	77	0.684	2175	2210	-1 %
5.0	5.00	79	0.686	2389	2445	-2 %
6.0	6.00	82	0.691	2804	2911	-3 %
7.0	7.00	86	0.697	3197	3375	-5 %
8.0	8.00	89	0.704	3569	3834	-6 %
9.0	9.00	92	0.710	3920	4285	-8 %
10.0	10.00	95	0.717	4248	4732	-10 %
12.0	12.00	101	0.733	4838	5607	-13 %
14.0	14.01	107	0.735	5336	6333	-15 %
16.0	16.03	112	0.735	5738	6974	-17 %
18.0	18.06	118	0.735	6035	7550	-20 %
19.0	19.09	121	0.735	6137	7816	-21 %
Fully open		---	-----	7208	7208	----

(1): vertical gate opening with respect to spillway apex elevation
(2): vertical gate opening with respect to bottom of orifice

Table 6. Results of spillway discharge computations for partial gate openings by the 1973 and 1987 methods of *Design of Small Dams* and for free flow at Bhakra Dam in India.

All lengths are in ft and discharges are in cfs			
Gate trunnion coordinates (y, x)	: 17.25	45.21	
Gate seat coordinates (y, x)	: -2.43	12.71	
Lake water surface elevation	: 1680.75		
Variations of angle theta, orifice coefficient, and discharges by 1973 & 1987 editions of *Design of Small Dams* versus vertical gate opening:			

dsa(1)	dbo(2)	theta	Corifice	Q-1973	Q-1987	Rel.Diff.
1.0	2.60	49	0.669	19871	20059	0 %
2.0	3.42	51	0.669	25696	26073	-1 %
3.0	4.26	53	0.670	31569	32155	-1 %
4.0	5.12	56	0.671	37479	38277	-2 %
5.0	6.00	58	0.672	43382	44422	-2 %
6.0	6.90	60	0.673	49196	50577	-2 %
7.0	7.81	62	0.674	55004	56728	-3 %
8.0	8.73	64	0.675	60741	62874	-3 %
9.0	9.66	66	0.676	66392	69017	-3 %
10.0	10.60	68	0.678	71944	75142	-4 %
12.0	12.51	72	0.680	82711	87290	-5 %
14.0	14.44	76	0.683	92971	99311	-6 %
16.0	16.40	79	0.687	102669	111211	-7 %
18.0	18.36	83	0.692	111759	122950	-9 %
20.0	20.35	86	0.697	120203	134566	-10 %
22.0	22.34	89	0.703	127966	146136	-12 %
24.0	24.34	92	0.709	135013	157349	-14 %
26.0	26.35	95	0.716	141305	168463	-16 %
28.0	28.37	98	0.724	146793	179367	-18 %
30.0	30.40	101	0.731	151411	189989	-20 %
32.0	32.44	104	0.735	155052	199111	-22 %
Fully open		---	-----	190195	190195	----

(1): vertical gate opening with respect to spillway apex elevation
(2): vertical gate opening with respect to bottom of orifice

III. Analysis of Laboratory Model Data of Kavsak Dam

Since early 1960s experiments are done on physical models of such appurtenant structures as spillways and intake units of some dams in the hydraulics laboratory facility of the Technical Research and Quality Control Department (known in Turkey as TAKK), which is somewhat similar to the hydraulics laboratory of Bureau of Reclamation at Denver CO, of the General Directorate of State Water Works of Turkey (known in Turkey as DSI) (www.dsi.gov.tr). The final report of such a recent work has been provided to the authors with permission to be used in academic studies by the authorities of TAKK [20]. This is about the experiments on the 1:50 scale model of the radial-gated flood spillway of Kavsak Dam, which is a dam on the Goksu River in Seyhan Basin in Turkey. The relevant peculiarities of this spillway are given in Table 1 along with those of the other four dams. The measured data of the gate openings, spillway discharges, and total heads with respect to the spillway apex elevation are given in Table-3.5 of the report about spillway model of Kavsak Dam [20, pages: 36-38].

The discharge coefficients computed using the relevant quantities observed experimentally for the free-flow (fully open gates) case with the Kavsak Dam laboratory model data turned out to be very close to those given by the pertinent charts in *Design of Small Dams* [1, 2, 5], the difference being mostly less than ±5 %. Although the empirical discharge coefficient are computed by multiplying four coefficients: (1) *Co* for the design head, (2) correction for a head other than the design head, (3) correction for the angle of the upstream spillway face, and (4) correction for the downstream effect, while the experimental discharge coefficient is computed from $Q = C_{net} \cdot L_e \cdot H^{1.5}$ only with the measured values inserted in it, both coefficients turn out to be very close to each other for the Kavsak Dam laboratory model data. The discharge coefficients for the partially-opened cases are computed by equation (1), the previous method [2], and by equation (2), the present method [5], with the experimentally measured values inserted in them. The results are presented in Figs. 5 and 6.

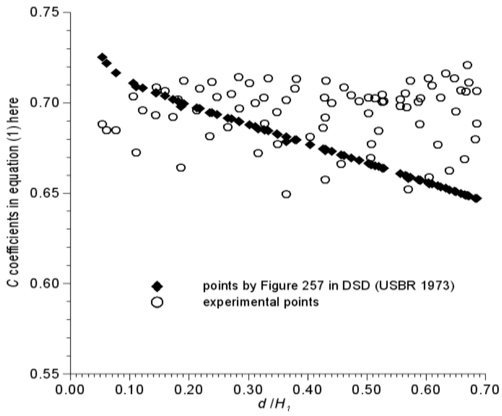

Figure 5. Chart for discharge coefficients given in Figure 257 of the second edition of *Design of Small Dams* [2] plotted together with the discharge coefficients computed by equation (1) with values measured experimentally on the laboratory model of the Kavsak Dam [20].

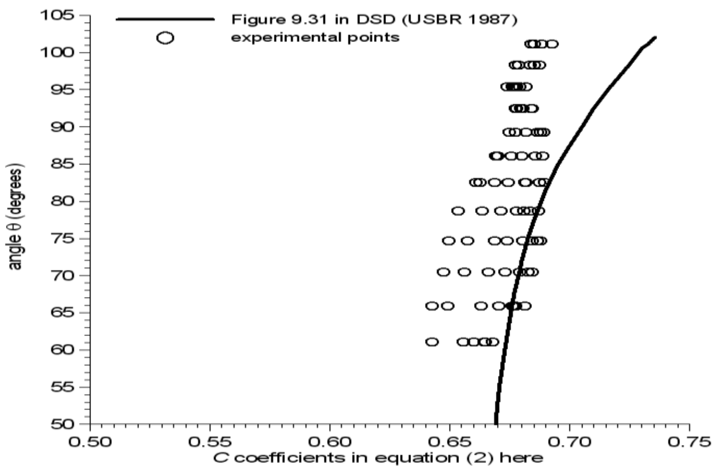

Figure 6. Chart for discharge coefficients given in Figure-9.31 of the third edition of *Design of Small Dams* [5] plotted together with the discharge coefficients computed by equation (2) with values measured experimentally on the laboratory model of the Kavsak Dam [20].

IIII. RESULTS AND DISCUSSIONS

The algorithm developed in this study for computation of spillway discharge for the case of partially-opened radial gates over an ogee spillway is more efficient than the one presented in *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9]. The method of *Hydraulic Design Criteria* [9] necessitates two separate tables comprising sequences of computational steps in 20 columns in the first table followed by 15 columns in the second table. Graphs of y_c / H_d versus x_c / H_d and dy/dx versus x_c / H_d for a wide range of possible values of x_c / H_d pertaining to the geometrical dimensions of that spillway must be prepared initially before getting involved with those tables. As would be appreciated by comparing the procedure outlined in the section: 'Computation of Spillway Discharge by Equation (2)' above with that in *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9], the approach presented in the current study is more straightforward and algorithmically more efficient. It is prepared in form of a Fortran code, which is freely available along with sample input data to anybody interested. For some spillways whose gate seats are directly on the spillway apex, the point on the crest curve closest to the gate lip is on the upstream crest. Although the method suggested in *Hydraulic Design Criteria, Volume 2, Sheets 311-1 to 311-5 Tainter Gates on Spillway Crests, Discharge Coefficients* [9] does not apply to this configuration, the algorithm developed in this study is valid for this case also.

As seen in Tables 3 – 6, discharges over ogee spillways having radial gates when the gates are partially opened given by the method presented in the third edition of *Design of Small Dams* [5] are greater than those computed by the method of the first and second editions of this book [1, 2]. The differences are very small for vertical gate openings (d) up to 20 % of the design head (H_d), they are less than 10 % for 20 % of $H_d < d <$ 50 % of H_d, but they increase as much as 20 ~ 30 % for wider gate openings. The results of John Doe Dam seem to be more pronounced than these values, and the discharge difference of the 1973 method is as much as –35 % from that of the 1987 method. Table 3 of the model study report by USBR [19] presents actually measured discharges from the spillway of Yellowtail Afterbay Dam and those computed by the 1987 method for five different d – WSE combinations for fairly small gate openings between 12 % and 18 % of H_d. According to this table, the average relative difference of the computed values from the observed ones is about +4 % which is affirmative of the result of this study that the 1987 method has a tendency to over-estimate the discharges over partially-opened radial-gated ogee spillways.

Further, the 1987 method gives greater discharges for gate openings of about 70 to 80 % of the total head than the free flow discharge at the same head. On the contrary, the flow through a contracted gate opening should be smaller than the free spilled flow under the same head. Fig. 9-31 of the third edition of *Design of Small Dams* [5], which is a replica of the figure developed by the Waterways Experimentation Station of US Army Corps of Engineers based on measurements on three laboratory models and on three prototype structures and is given as '*Hydraulic Design Chart 311-1*' in *Sheets 311-1 to 311-5* of *Volume 2 of Hydraulic Design Criteria* [9] is a crucial figure as it yields the discharge coefficient (C) of equation (2), the 1987 method. It is believed that there is still room for amendment of this C coefficient. Therefore, this figure, which is Fig. 9-31 of the third edition of *Design of Small Dams* [5] also, needs to be revised based on many more laboratory and prototype data. The laboratory data had better be obtained on models of scales ≤ 1:50.

Yet, as seen in Tables 4 and 5, at two of the four example dams, for very small gate openings of the order of 1 m, the discharges computed by the 1973 method are slightly greater than those of the 1987 method. Such differences are not as significant as the over-estimation of the 1987 method for discharges close to the design flood however, because these are very small flowrates as compared to the real floods.

Although not given here, the 1973 and 1987 methods have been applied to a few more dams, like Bayramhacili, Catalan, Aslantas, Bahcelik, Yamula Dams in Turkey, and Folsom Dam in America, and they all have revealed results similar to those given in Tables 3 – 6.

The total head just upstream of the partially-opened gate (H_1) must be with respect to the bottom of the orifice, which is the point on the spillway crest curve closest to the gate lip, and not with respect to the spillway apex elevation. This matter is not clear either in *Design of Small Dams* [5] or in *HDC-311* [9]. Discharges computed by taking H_1 with respect to the spillway apex for gates whose seats are downstream from the apex will be a little smaller than by taking the geometrically correct H_1.

In the example of the spillway of Yellowtail Afterbay Dam whose gate seat is directly on the spillway apex, the angle θ in Fig. 9-31 of *Design of Small Dams* (USBR 1987), which is symbolized by β in the figure: *Hydraulic Design Chart 311-1* [9] (Fig. 2 here), is found to vary in the interval: $63° \leq \theta \leq 105°$, while the bound values are: $83° \leq \theta \leq 109°$ in this figure. For the other three example dams whose gate seats are a little downstream from the apex, the angle θ is found to vary in the interval: $49° \leq \theta \leq 124°$, while the same interval in Fig. 2 is given as: $50° \leq \theta \leq 103°$. For the first example, the lower bound is a little too short, and for the next four examples the upper bound is a little too short. In this study, the end values of the curves in that figure are taken without extrapolation beyond their bounds.

IV. CONCLUSIONS

The methods to compute discharges over ogee spillways having radial gates when the gates are partially opened presented in the 1973 and 1987 editions of the classical book: *Design of Small Dams* [2, 5] are different from each other. Because the 1987 is a newer date than 1973, the 1987 method should annul the former one. Although *Design of Small Dams* [5] presents the new method, it simply refers to the original source which developed it [9] for the computational procedure. An analytical and numerical scheme for the 1987 method, which is different from and algorithmically more efficient than the approach presented in *Hydraulic Design Criteria, Volume 2, Sheets 311-1 to 311-5 Tainter Gates on Spillway Crests, Discharge Coefficients* [9] is developed in this study.

The spillway discharges over partially-opened radial-gated ogee spillways by both the 1973 and 1987 methods are computed for vertical gate openings of: d = 0.1, 0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, ..., 10, 11, …, *fully open*, in meters, and compared with each other for a few existing dams having radial-gated spillways. And, it is seen that while both methods yield very close discharges for small gate openings, the 1987 method gives discharges greater than the 1973 method, the gap widening with increasing gate openings.

The essential part of the new method [5, 9] is the chart for the discharge coefficient (C). It is believed that this chart should be revised based on more laboratory and prototype data because the present chart has been developed using three laboratory and three prototype data only.

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