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Numerical Solution of Third Order Time-Invariant Linear Differential Equations by Adomian Decomposition Method

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ABSTRACT
In this paper, we state the Adomian Decomposition Method (ADM) for third order time-invariant linear homogeneous differential equations. And we applied it to find solutions to the same class of equations. Three test problems were used as concrete examples to validate the reliability of the method, and the result shows remarkable solutions as those that are obtained by any knows analytical method(s).
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I. INTRODUCTION

Linear time-invariant differential equations plays important role in Physics because it assumes the law of nature which hold now and are identical to those for times in the past or future. This class of equation has wide application in automated theory, digital signal processing, telecommunication engineering, spectroscopy, seismology, circuit and other technical areas. Specifically in telecommunication, the propagation medium for wireless communication systems is often modeled with this class of equations. Also, this class of equation has tremendous application in dynamics.

Many physical systems are either time-invariant or approximately so, and spectral analysis is an efficient tool in investigating linear time-invariant differential equations. In this paper, we apply the powerful ADM to find solution to third order linear time-invariant differential equations. The ADM has been a subject of several studies [1-6]. It seeks to make possible physically realistic solution to complex real life problems without using modeling and mathematical compromises to achieve results. It has been judged to provide the best and sometimes the only realistic simulation phenomena.

In compact form, the general third order time-invariant linear differential equation is given as

$$\dddot{x}(t) = f(t, \dot{x}(t), \ddot{x}(t)) \qquad (1)$$

With initial conditions given as

$$x(\alpha) = A, \quad \dot{x}(t) = B \text{ and } \ddot{x}(t) = C$$

ADM gives a series solution of x which must be truncated for practical application. In addition, the rate and region of convergence of the solutions are potentially short or long coming. The series of x can rapidly converge in small and wide region depending on the problem at hand.

II. THE THEORY OF ADM

By [5] the ADM of equation (1) is given as;

$$\Omega x = \omega \qquad (2)$$

Where Ω is a differential operator, x and ω are functions of t in this case. In operator form, equation (2) is given as;

$$\Gamma x + \Phi x + \Psi x = \omega \qquad (3)$$

Γ, Φ and Ψ are given respectively as; highest order differential operator with respect to t of Ω to be inverted, the linear remainder operator of Ω and the nonlinear operator of Ω which is assumed to be analytic. The choice of the linear operator is designed to yield an easily invertible operator with resulting trivial integration. In this article $\Gamma = \frac{d^3}{dt^3}$. Furthermore, we emphasize that the choice of Γ and concomitantly it inverse Γ^{-1} are determined by a particular equation to be solved. Hence, the choice is non-unique. For equation (1), $\Gamma^{-1} = \iiint (.)\, dtdtdt$. That is, a three-fold definite integral operator from t_0 to t. For a linear form of equation (3), $\Psi x = 0$ and we have;

$$x = \beta + \omega - \Gamma^{-1}[\Phi x] \tag{4}$$

ADM decomposes the solution of equation (1) into a series;

$$x = \sum_{n=0}^{\infty} x_n \tag{5}$$

Where β incorporates all the initial conditions which is considered as x_0. For details of ADM theory see [1-6].

III. ILLUSTRATION

In this section we give examples on how ADM can be used to find solutions to third order linear time-invariant differential equations.

Problem 1

$$\dddot{x} - \ddot{x} + \dot{x} - x = 0 \tag{6}$$

With initial conditions given as

$$x(0) = 1, \quad \dot{x}(0) = 0 \quad \text{and} \quad \ddot{x}(0) = -1$$

The exact solution of (6) is;

$$x = \cos t \tag{7}$$

The series form of (7) is ;

$$x = \frac{1}{0!} - \frac{1}{2!}t^2 + \frac{1}{4!}t^4 - \frac{1}{6!}t^6 + \frac{1}{8!}t^8 - \frac{1}{10!}t^{10} + \ . \ . \ . \tag{8}$$

Applying equations (2) to (5) on (6), we obtain;

$$x_0 = 1 - \frac{1}{2}t^2$$

$$x_1 = \frac{1}{24}t^4 - \frac{1}{120}t^5$$

$$x_2 = \frac{1}{120}t^5 - \frac{1}{360}t^6 + \frac{1}{2520}t^7 - \frac{1}{40320}t^8$$

$$x_3 = \frac{1}{720}t^6 - \frac{1}{1680}t^7 + \frac{1}{8064}t^8 - \frac{1}{72576}t^9 + \frac{1}{1209600}t^{10} - \frac{1}{399168}t^{11}$$

$$x_4 = \frac{1}{5040}t^7 - \frac{1}{10080}t^8 + \frac{1}{40320}t^9 - \frac{13}{3628800}t^{10} + \frac{13}{39916800}t^{11} - \frac{1}{53222400}t^{12} + \frac{1}{1556755200}t^{13} - \frac{1}{87178291200}t^{14}$$

$$x_5 = \frac{1}{40320}t^8 - \frac{1}{72576}t^9 + \frac{1}{259100}t^{10} - \frac{13}{19958400}t^{11} + \frac{1}{13685760}t^{12} - \frac{1}{1779148800}t^{13} + \frac{1}{3353011200}t^{14} - \frac{1}{93405312000}t^{15} + \frac{1}{4184557977600}t^{16} - \frac{1}{355687428096000}t^{17}$$

$$x_6 = \frac{1}{362880}t^9 - \frac{1}{604800}t^{10} + \frac{1}{1995840}t^{11} - \frac{1}{10644480}t^{12} + \frac{1}{83026944}t^{13} - \frac{1}{908107200}t^{14} + \frac{1}{13621608000}t^{15} - \frac{1}{278970531840}t^{16} + \frac{1}{7904165068800}t^{17} - \frac{1}{320118685286400}t^{18} + \frac{1}{20274183401472000}t^{19} - \frac{1}{2432902008176640000}t^{20}$$

$$x_7 = \frac{1}{362880}t^{10} - \frac{1}{5702400}t^{11} + \frac{1}{17740800}t^{12} - \frac{71}{6227020800}t^{13} + \frac{1}{622702080}t^{14} - \frac{1}{6054048000}t^{15} + \frac{89}{6974263296000}t^{16} - \frac{89}{118562476032000}t^{17} + \frac{1}{29640619008000}t^{18} - \frac{1}{868893574348800}t^{19}$$

$$+\frac{71}{2432902008176640000}t^{20}-\frac{1}{18922577117470720000}t^{21}+\frac{1}{16057153253965824000 0}t^{22}$$
$$-\frac{1}{25852016738884976640000}t^{23}$$

Proceeding in this order, we have that;

$$\sum_{n=0}^{10} x_n = 1-\frac{1}{2}t^2+\frac{1}{24}t^4-\frac{1}{720}t^6+\frac{1}{40320}t^8-\frac{1}{3628800}t^{10}+\ldots \tag{9}$$

Comparing equations (8) and (9), we find that they are the same. The similarities between the exact solution and ADM solution of problem 1 is further shown in Fig. 1 and Fig. 2.

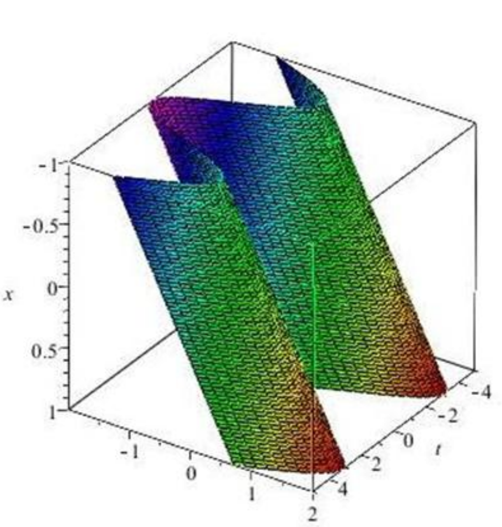

Figure 1: Analytical solution of Problem 1

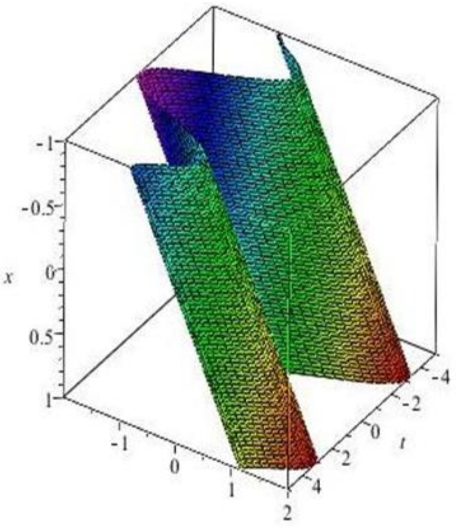

Figure 2: ADM of Problem 1

Problem 2

$$\dddot{x}+4\ddot{x}-5\dot{x}-x=0 \tag{10}$$

With initial conditions given as

$$x(0)=4,\quad \dot{x}(0)=-7 \quad \text{and} \quad \ddot{x}(0)=23$$

The exact solution of (10) is;

$$x=5-2e^{t}+e^{-5t} \tag{11}$$

The series form of (10) is ;

$$x=4-7t+\frac{23}{2}t^2-\frac{127}{6}t^3+\frac{623}{24}t^4-\frac{3127}{120}t^5+\frac{15623}{720}t^6-\frac{11161}{720}t^7+\frac{390623}{40320}t^8-\ldots \tag{12}$$

Similarly, applying equations (2) to (5) on (10), we obtain;

$$x_0=4-7t+\frac{23}{2}t^2$$

$$x_1=-\frac{127}{6}t^3+\frac{115}{24}t^4$$

$$x_2=\frac{127}{6}t^4-\frac{73}{8}t^5+\frac{115}{144}t^6$$

$$x_3=-\frac{254}{15}t^5+\frac{173}{18}t^6-\frac{1555}{1008}t^7+\frac{575}{8064}t^8$$

$$x_4=\frac{508}{45}t^6-\frac{473}{63}t^7+\frac{365}{224}t^8-\frac{10075}{72576}t^9+\frac{575}{145152}t^{10}$$

$$x_5=-\frac{2032}{315}t^7+\frac{100}{21}t^8-\frac{2825}{2268}t^9+\frac{1325}{9072}t^{10}-\frac{125}{16128}t^{11}+\frac{2875}{19160064}t^{12}$$

$$x_6 = \frac{1016}{315}t^8 - \frac{1454}{567}t^9 + \frac{865}{1134}t^{10} - \frac{825}{16632}t^{11} + \frac{38875}{4790016}t^{12} - \frac{73375}{249080832}t^{13} + \frac{14375}{3487131648}t^{14}$$

$$x_7 = -\frac{4064}{2835}t^9 + \frac{488}{405}t^{10} - \frac{13}{33}t^{11} + \frac{175}{2673}t^{12} - \frac{13375}{2223936}t^{13} + \frac{2125}{69118912}t^{14} - \frac{12125}{1494484992}t^{15} + \frac{14375}{167382319104}t^{16}$$

Continuing in this order, we have;

$$\sum_{n=0}^{12} x_n = 4 - 7t + \frac{23}{2}t^2 - \frac{127}{6}t^3 + \frac{623}{24}t^4 - \frac{3127}{120}t^5 + \frac{15623}{720}t^6 - \frac{11161}{720}t^7 + \Im \qquad (13)$$

Where

$$\Im = \frac{390623}{40320}t^8 - \frac{1953127}{362880}t^9 + \frac{1395089}{518400}t^{10} - \frac{48828127}{39916800}t^{11} + \frac{244140623}{479001600}t^{12} - \ldots$$

Equation (13) is the ADM solution of problem 2 i.e. equation (10). The terms of equation (13) are exactly the same as those of equation (12), which is the classical solution of problem 2. The similarity between the exact solution and that of ADM is further depicted in Fig. 3 and Fig. 4.

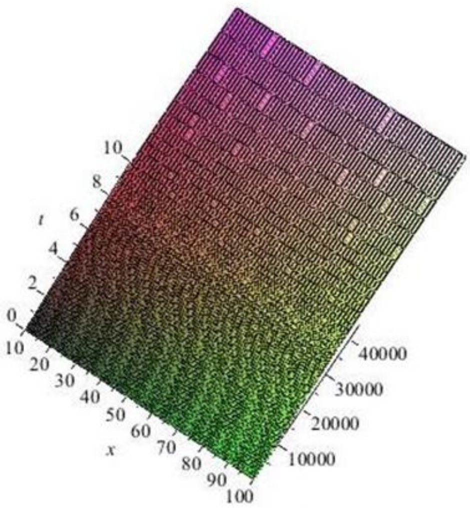

Figure 3: Exact solution of problem 2

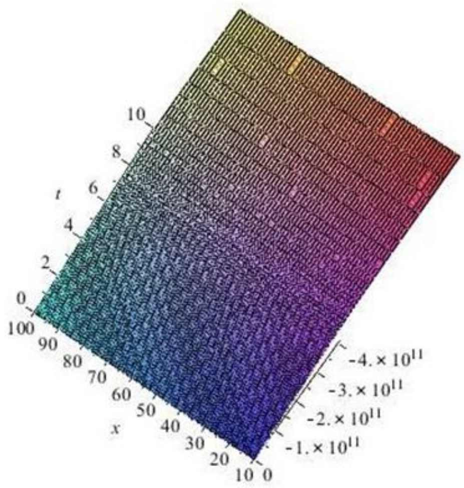

Figure 4: ADM solution of problem 2

Problem 3

$$\dddot{x} + 25\dot{x} = 0 \qquad (14)$$

With initial conditions given as

$$x(0) = 1, \quad \dot{x}(0) = 5 \quad \text{and} \quad \ddot{x}(0) = -5$$

The exact solution of (14) is;

$$x = \frac{4}{5} + \frac{1}{5}\cos 5t + \sin 5t \qquad (15)$$

The series form of (15) is ;

$$x = 1 + 5t - \frac{5}{2}t^2 - \frac{125}{6}t^3 + \frac{125}{24}t^4 + \frac{625}{24}t^5 - \frac{625}{144}t^6 - \frac{15625}{1008}t^7 + \ldots \qquad (16)$$

Similarly, applying equations (2) to (5) on (14), we obtain;

$$x_0 = 1 + 5t - \frac{5}{2}t^2$$

$$x_1 = -\frac{125}{6}t^3 + \frac{125}{24}t^4$$

$$x_2 = \frac{625}{24}t^5 - \frac{625}{144}t^6$$

$$x_3 = -\frac{15625}{1008}t^7 + \frac{15625}{8064}t^8$$

Continuing in this order, we have;

$$\sum_{n=0}^{\infty} x_n = 1 + 5t - \frac{5}{2}t^2 - \frac{125}{6}t^3 + \frac{125}{24}t^4 + \frac{625}{24}t^5 - \frac{625}{144}t^6 - \frac{15625}{1008}t^7 + \frac{15625}{8064}t^8 + \qquad (17)$$

Equation (17) is the ADM solution of problem 3 i.e. equation (14). The terms of equation (17) are exactly the same as those of equation (16), which is the classical solution of problem 3. The similarity between the exact solution and that of ADM is further depicted in Fig. 5 and Fig. 6.

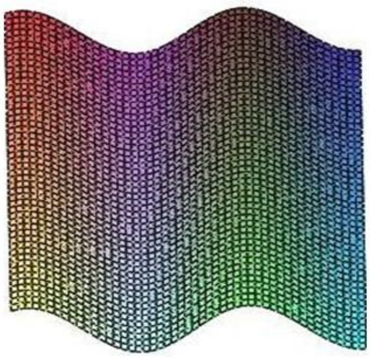

Figure 5: Exact solution of problem 3

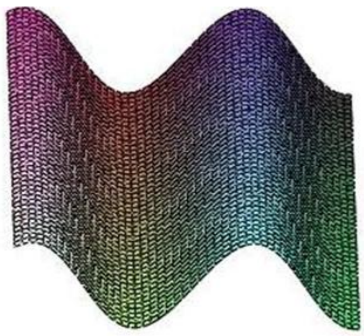

Figure 6: ADM solution of problem 3

IV. CONCLUSION

We have successfully applied ADM to third order linear time-invariant differential equations. Although in the ADM we considered only finite terms of and an infinite series, nonetheless, the result obtained by this method are in total agreement with their exact counterparts. This consideration is some worth obvious in Fig. 2, Fig. 6 and not in any way obvious in Fig. 4. Possible extension of the method to 4^{th} order linear differential equations can be investigated.

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Biography

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