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Design Principle and Realization of Cubic Uniform B-Spline Generator

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ABSTRACT
The article puts forward an approach to compute cubic uniform B-spline by means of combination of digital logic circuit and presents a scheme for the circuit design. By deducing relationship between cubic uniform B-spline basis and the Bernstein basis of Bezier curve, the article discloses that a cubic uniform B-spline can be a linear combination of Bezier curves. Based on such fact, a design for high-speeded B-spline generator is presented in the article. The designed generator is of high speed and can be applied on development of graphics and image process, robotic and CNC systems and other related areas.
Keywords: *B-spline, computation , Embedded system, digital logic circuit,*

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I. INTRODUCTION

Calculation and generation of curves are research topics in many fields such as numerical computation, graphics and CNC, etc. Cubic uniform B-spline has been people's favorite because of its simple computation, good continuity and easy shape-control [1]. Even nowadays, it is still an important tool in graphics process, mechanical computation and system design of numerical control. Literatures [2], [3] and [4] show their achievements of applying cubic uniform B-spline in medical graphics process and camera system; literatures [5] and [6] obtain a more precise results by applying cubic uniform B-spline in mechanical computation; literatures [7],[8] and [9] apply cubic uniform B-spline in development of robots; literature [10] uses uniform B-spline to treat images in GPU; literatures [11] and [12] use cubic uniform B-spline respectively in weapon and building projects and achieve better results; literatures [13] and [14] make more fundamental studies on cubic uniform B-spline. All these studies and applications show that cubic uniform B-spline has permeated into wide fields of science and technology. However it is also seen, as literatures [15] and [16] point out, most algorithms adopted nowadays are base one software. With the development of embedded system, data process on embedded system and FPGA becomes an ordinary routine. Using hardware to compute graphics and to process images is both a demand of real society and a necessity of techniques. Hence the studies have recent bloomed. For example, literatures [15] and [16] make studies on computation of cubic uniform B-spline on FPGA, literatures [17] and [18] design a Bezier interpolator which perform calculation of curve via combination of digital logic circuit, and literatures [19] and [20] study the storage mechanism of the hardware interpolation
This article presents a design for cubic uniform B-spline generator. The generator can compute cubic uniform B-spline via combination of digital logic circuit.

II. PRELIMINARIES

2.1 Bezier Curve and Bernstein Basis

Given points sequence $\{P_i\}$ in 3 dimensional vector space, a Bezier curve Γ_n is constructed by

$$\Gamma_n : r(t) = \sum_{i=0}^{n} P_i B_{i,n}(t), 0 \le t \le 1$$

where $B_{i,n}(t)$ are Bernstein basis functions.

When $n = 3$, the Bezier of degree 3 is determined by four points P_0, P_1, P_2, P_3. For convenience, it is simply denoted by $\boldsymbol{r}(t) =< P_0, P_1, P_2, P_3 >$.

2.2 Subdivision characteristic of Bezier curve

A Bezier curve has subdivision property, which is mathematically described as follows.

Let $\boldsymbol{r}(t) =< P_0, P_1, P_2, P_3 >$ then it holds

$$\boldsymbol{r}(t)=\begin{cases} < P_{0,0}(w), P_{0,1}(w), \ldots\ldots, P_{0,n}(w) >, \dfrac{t}{w} >, 0 \le t \le w \\ < P_{0,n}(w), P_{1,n-1}(w), \ldots\ldots, P_{n,0}(w) >, \dfrac{t-w}{1-w} >, w \le t \le 1 \end{cases}$$

where $P_{i,j}(w)$ satisfies

$$\begin{cases} P_{i,0}(w) = P_i, i = 0,1,\ldots\ldots,n \\ P_{i,j}(w) = (1-w)P_{i,j-1}(w) + wP_{i+1,j-1}(w), i = 0,1,\ldots\ldots,n-j \end{cases}$$

2.3 Cubic Bezier Generator of High Speed and Precision

Subdivision property of Bezier curves makes it possible to be generated by pipeline and parallel computation in term of binary structure. Articles [19] and [20] have made detail studies on the issue. In addition, article [17] and [18] presented detail design of cubic Bezier interpolator. Summarized on literatures [17] to [20], it can know that cubic Bezier generator of high speed and precision can be made by digital logic circuit. Due to limitation of space, this article omits the details of the issue, the detail of which can be seen in literatures [17] to [20].

III. BEZIER REPRESENTATION OF CUBIC UNIFORM B-SPLINE

By knowledge of CAD theory, a cubic uniform B-spline curve $P(t)$ can be constructed by four given points P_0, P_1, P_2, P_3 by

$$P(t) = P_0 N_{0,3}(t) + P_1 N_{1,3}(t) + P_2 N_{2,3}(t) + P_3 N_{3,3}(t), 0 \le t \le 1 \qquad (1)$$

where

$$\begin{cases} N_{0,3}(t) = (1-t)^3 / 3! \\ N_{1,3}(t) = \left(3t^2 - 6t + 4\right) / 3! \\ N_{2,3}(t) = \left(-3t^3 + 3t^2 + 3t + 1\right) / 3! \\ N_{3,3}(t) = \mathrm{t}^3 / 3! \end{cases}$$

A simple mathematical transformation results in

$$\begin{cases} N_{0,3}(t) = B_{0,3}(t) / 3! \\ N_{1,3}(t) = \left(B_{1,3}(t) + 3B_{0,1}(t) + 1\right) / 3! \\ N_{2,3}(t) = \left(B_{2,3}(t) + 3B_{1,1}(t) + 1\right) / 3! \\ N_{3,3}(t) = B_{3,3}(t) / 3! \end{cases} \qquad (2)$$

where $B_{0,3}(t), B_{1,3}(t), B_{2,3}(t), B_{3,3}(t)$ are cubic Bernstein basis, and $B_{0,1}(t), B_{1,1}(t)$ are Bernstein basis of degree 1.

Substituting (2) into (1) yields

$$\begin{aligned} P(t) &= \frac{1}{6}\left(P_0 B_{0,3}(t) + P_1 B_{1,3}(t) + 3P_1 B_{0,1}(t) + P_2 B_{2,3}(t) + 3P_2 B_{1,1}(t) + P_3 B_{3,3}(t) + P_1 + P_2\right) \\ &= \frac{1}{6}\left[\left(P_0 B_{0,3}(t) + P_1 B_{1,3}(t) + P_2 B_{2,3}(t) + P_3 B_{3,3}(t)\right) + 3 \times \left(P_1 B_{0,1}(t) + P_2 B_{1,1}(t)\right) + P_1 + P_2\right] \\ &= \frac{1}{6}\left(BZ_3\left(P_0, P_1, P_2, P_3, t\right) + 3BZ_1\left(P_1, P_2, t\right) + P_1 + P_2\right) \end{aligned} \qquad (3)$$

where $BZ_3(P_0, P_1, P_2, P_3, t)$ and $BZ_1(P_1, P_2, t)$ are respectively cubic Bezier curve and 1-degreed Bezier curve.

Seen from (3) it can know that, a cubic uniform B-spline curve can be obtained by a linear combination of a cubic Bezier curve and a 1-degereed Bezier curve together with a translation. This result shows that, generator of uniform B-spline curve can be designed and obtained by means of Bezier generator.

IV. DESIGNED OF B-SPLINE GENERATOR

Formula (3) shows that, a cubic uniform B-spline generator can be designed by a cubic Bezier generator, a 1-degreed Bezier generator and necessary digital logic circuit. Let CU3 be the cubic Bezier generator, CU1 be the 1-degreed Bezier generator; then the cubic uniform B-spline generator is design by figure 1.

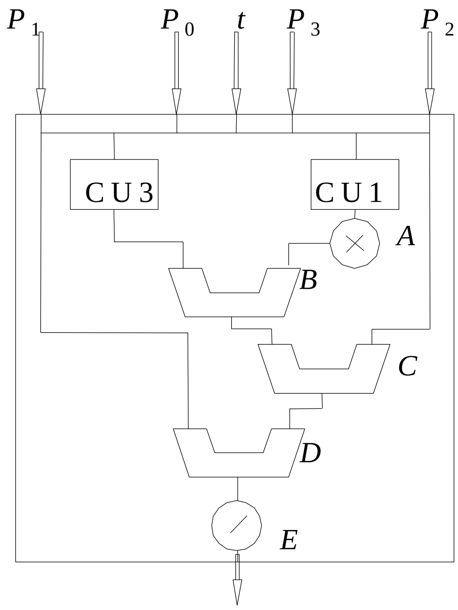

Fig. 1 Designed principle of B-spline generator

In the figure 1, *A* is a multiplier (multiplying unit) which is assigned to a ratio of 3 times to realize multiplication by 3; *B*, *C* and *D* are all adders that perform additions; *E* is also a multiplier which is assigned to a ratio of 1/6 times to realize division by 6. It can see from the figure that, when P_0, P_1, P_2, P_3 and tare input, the 5 parameters first processed by CU1 and CU3, (CU1only needs P_1, P_2 and *t*), then the result of CU1 is amplified 3 times and added with the result from CU3 by adder *B*; the result from *B* is added with P_2 by adder *C* and then added P_1 by adder *D*; finally the result from *D* is divided by 6 in *E* and output. The following Table 1 lists each step and result shown in figure 1.

Table 1 generator's process

No.	Operation	Results
1	$(P_0, P_1, P_2, P_3, t) \Rightarrow CU3, CU1$	$CU3 = P_0B_{0,3}(t) + P_1B_{1,3}(t) + P_2B_{2,3}(t) + P_3B_{3,3}(t)$ $CU1 = P_1B_{0,1}(t) + P_2B_{1,1}(t)$
2	$CU1 \times 3 \Rightarrow A$	$A = 3 \times CU1 = 3(P_1B_{0,1}(t) + P_2B_{1,1}(t))$
3	$A + CU3 \Rightarrow B$	$B = P_0B_{0,3}(t) + P_1B_{1,3}(t) + P_2B_{2,3}(t) + P_3B_{3,3}(t)$ $+3(P_1B_{0,1}(t) + P_2B_{1,1}(t))$
4	$B + P_2 \Rightarrow C$	$C = P_0B_{0,3}(t) + P_1B_{1,3}(t) + P_2B_{2,3}(t) + P_3B_{3,3}(t)$ $+3(P_1B_{0,1}(t) + P_2B_{1,1}(t)) + P_2$
5	$C + P_1 \Rightarrow D$	$D = P_0B_{0,3}(t) + P_1B_{1,3}(t) + P_2B_{2,3}(t) + P_3B_{3,3}(t)$ $+3(P_1B_{0,1}(t) + P_2B_{1,1}(t)) + P_2 + P_1$
6	$D/6 \Rightarrow E$	$E = \frac{1}{6}\begin{bmatrix} P_0B_{0,3}(t) + P_1B_{1,3}(t) + P_2B_{2,3}(t) + P_3B_{3,3}(t) \\ +3(P_1B_{0,1}(t) + P_2B_{1,1}(t)) + P_2 + P_1 \end{bmatrix}$

It is obvious that the generator can match to (3). Since the multiplier and adder are main calculating units, it is reasonable to judge that the precision and speed is the same as that introduced in literature [17] and [18].

V. CONCLUSION

Development of embedded system and reconfigurable computing makes it an ordinary thinking to design hardware to solve some professional problems. The low-valuated trends of hardware make the related applications more and more popular. The motivation of design a B-spline generator relies on the wide applications of B-spline in graphics processing, mechanical computing, robotic controlling and CNC developing. This article starts at analysis of the relationship between basis of cubic uniform B-spline curve and Bernstein basis, derives out relation between B-spline curve and Bezier curve and designs a B-spline generator based on Bezier generator. Such an approach of computing via hardware will greatly enhance the computational capabilities of the embedded system and will be a useful attempt for development of embedded system. The author hopes it will attract more researchers' contribution.

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Author's Biography

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