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# "Half Power Diameter Of Circularly Symmetric Optical Systems With Higher-Order Parabolic Filters"

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**ABSTRACT**

The Half Power Diameter (HPD) which is also called Full –Width at Half-Maximum (FWHM) is being used in various disciplines where the optical instruments responses are degraded very much due to various factors like, atmospheric turbulence, aberrations, vibrations, etc,-----. In the present paper, we have studied the FWHM or the HPD of an optical system with a set of Higher-Order parabolic filters. These filters are known to be dependence of increasing the resolving power of an optical system. Dependence of FWHM or the HPD on various parameters of the super resolving filters under consideration by us has been found out.

**Key-words:** Resolution, HPD, FWHM, Fourier optics, etc, …

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## I. INTRODUCTION

The spatial resolution of an optical system represents its ability to distinguish between two points after their images are formed. In many practical situations, the measurement of resolving power an optical system will not be correct if the limit of resolution is measured by applying the commonly used Rayleigh criterion[1], Sparrow criterion, etc,-----. This is because of the fact that the individual point spread functions of the two point objects, will be highly complicated and significantly different from the usual Airy patterns [2]. This happens when measurements are done in the presence of image motion, atmospheric turbulence, aberrations, vibrations, etc, -----.

Now, with the advent of new and advance technologies in various fields of science, engineering and medical fields, it is observed that the application of Rayleigh and Sparrow criterion becomes totally incorrect and meaningless. Consequently, a few other criteria of finding the limit of resolution have been proposed by various scientists. Among these newly proposed criteria the most important one is the introduction of the criterion Full-width at Half-Maximum (FWHM). This criterion has been found to be very useful in measuring the unit of resolution of sophisticated optical instruments dealing with compact radioactive sources, biomedical applications, etc-----. This criterion is defined, as the name implies, as the Full-Width at Half- Maximum point source response. Incidentally, FWHM has also been referred to as the Half Power Diameter (HPD) i.e., twice the distance of the point where the PSF drops to 50% of it's peak relative intensity [3].

In the present paper, we have studied the FWHM or the HPD of an optical system with a set of higher-order parabolic filters like second and third order parabolic filters. These filters are known to be capable of increasing the resolving power of an optical system [4]. Dependence of FWHM or the HPD on various parameters of the super resolving filter under consideration by us has been found out. The results obtained have been presented in the form of figures and tables.

## II. MATHEMATICAL FORMULATION:

The Point Spread Function (PSF) of a two- dimensional optical system with circular symmetric will be of the Bessel squared type [4]. In what follows in this section, we shall derive the expression needed to investigate the FWHM or HPD in the diffracted field due to parabolic apodisations filters. This filter can be expressed mathematically as,

$$f(r) = \left(\alpha + \beta r^2\right)^N \quad \ldots\ldots\ldots\ldots(1), \qquad \text{Where } N = 2,3.$$

Where $\beta$ is known as the apodisations parameter and shows the degree of non uniformity of transmission of the pupil. $\beta$ =0 corresponds to Airy type of pupils. $\alpha$ is numerical constant less than one. The Mathematical expression for the point spread function with circular symmetric is given by [5]

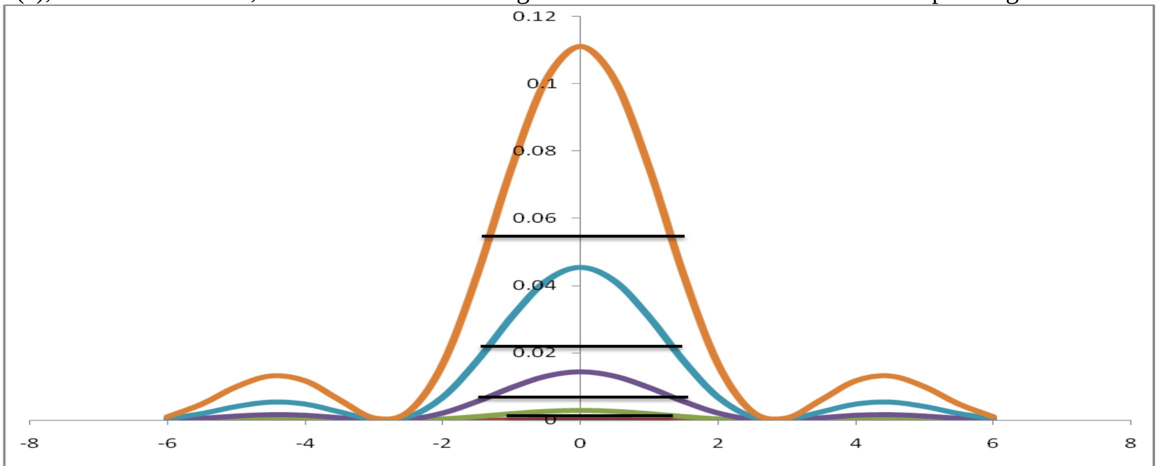
$$I(0,z)=\left[2\int_0^1 f(r)J_0(zr)rdr\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where the various symbols have their usual significances in the scalar theory of diffraction. Substituting for $f(r)$, from (1) into, we get

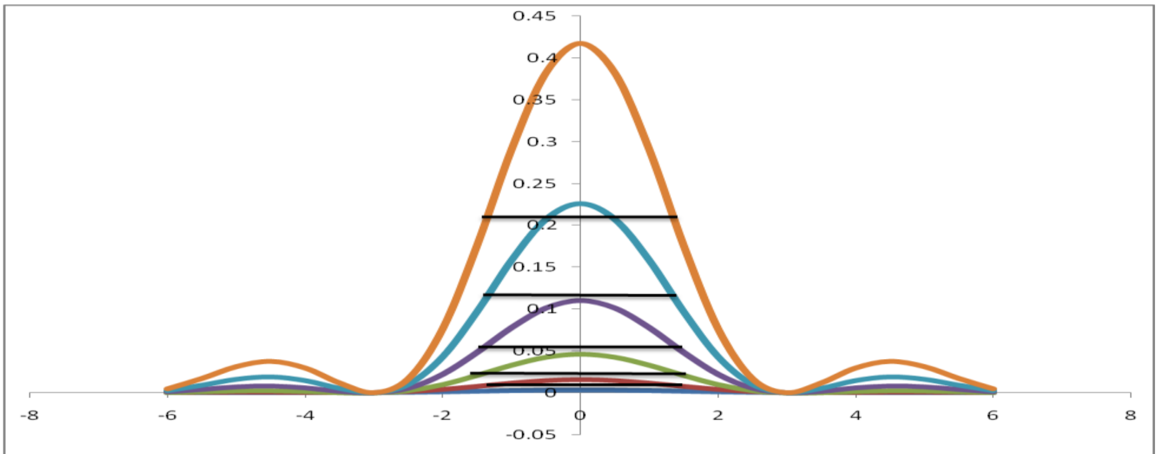
$$I(0,z)=\left[2\int_0^1(\alpha+\beta r^2)^N J_0(zr)rdr\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3), \quad \text{where N=2,3.}$$

## III. RESULTS AND DISCUSSIONS

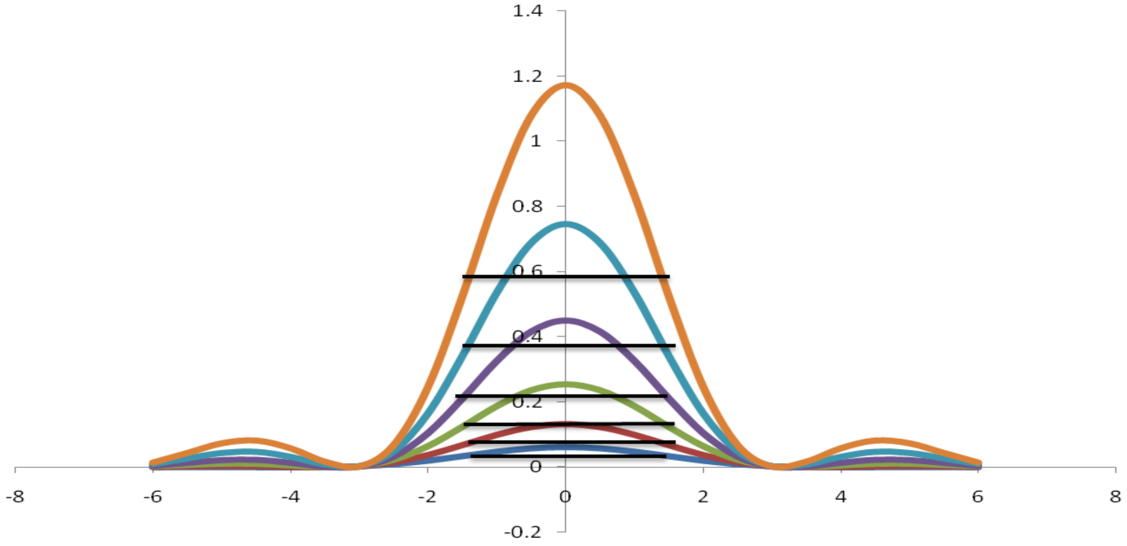
In figures 1 (a) to 1(d), we have plotted expanded IPSF curves for the evaluation of FWHM for $\alpha = 0, 0.25, 0.50 \& 0.75$. For second order. In one of the figures, for example for $\alpha = 0$ i.e. in figure 1(a), we have indicated, the method of evaluating the values of FWHM from the corresponding IPSF curves.



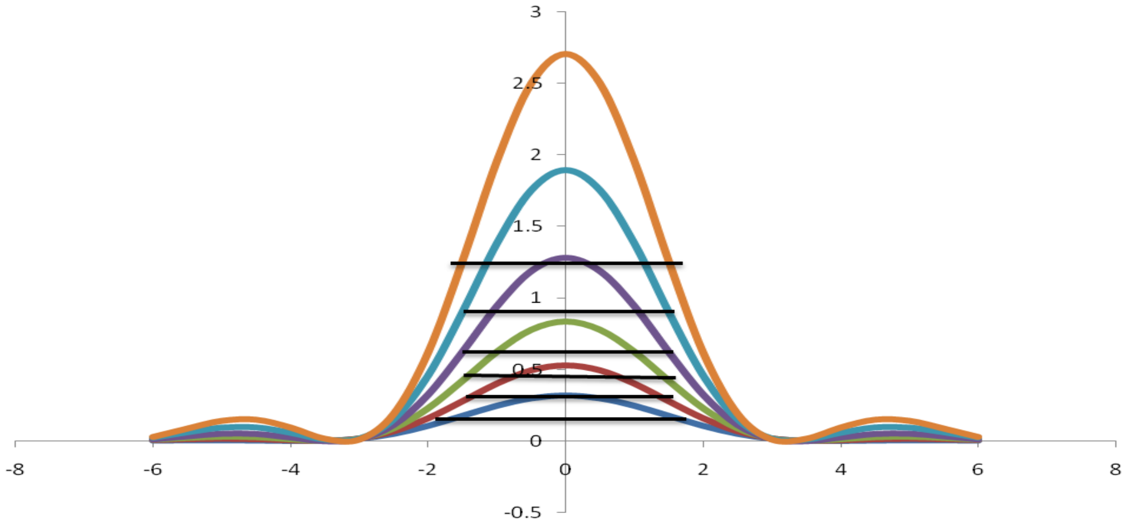

1(a) IPSF Curves for Evaluation of HPD ; for α=0, N=2




1(b) IPSF Curves for Evaluation of HPD ; α=0.25, N=2




1(c) IPSF Curves for Evaluation of HPD ;α=0.5, N=2

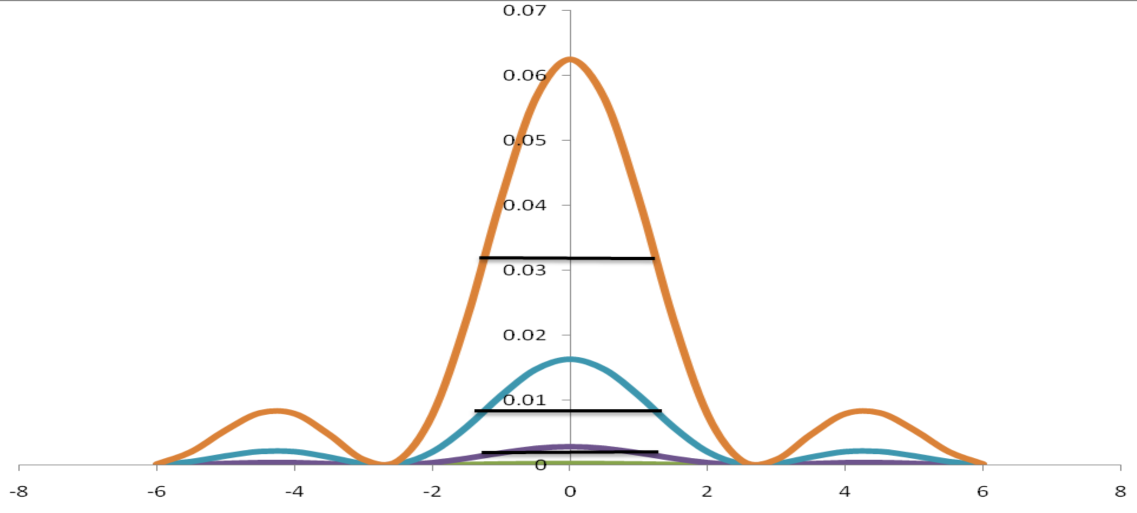



1(d) IPSF Curves for Evaluation of HPD ; α=0.75, N=2

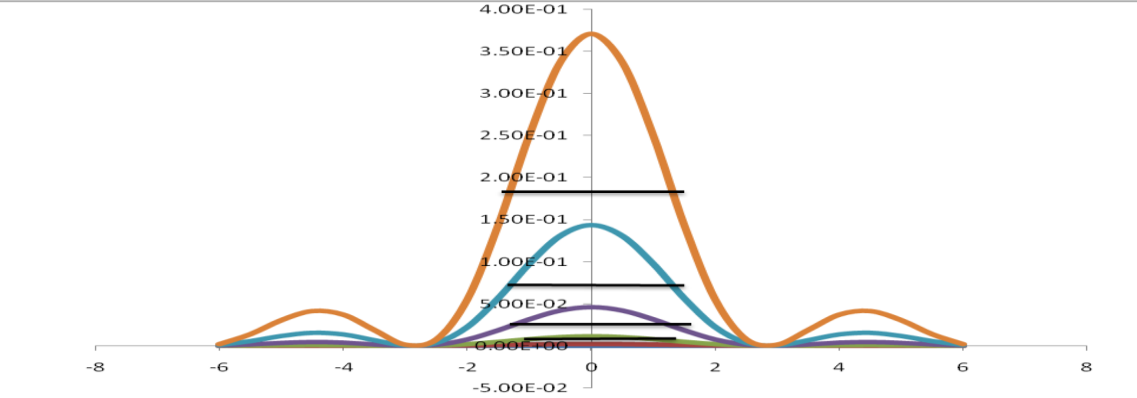
**Table-I**: **Values of FWHM**

| $\beta$ | $\alpha = 0$ | $\alpha = 0.25$ | $\alpha = 0.50$ | $\alpha = 0.75$ |
|---|---|---|---|---|
| **0.2** | **2.8** | **2.9** | **3.2** | **3.0** |
| **0.4** | **2.6** | **3.0** | **3.0** | **3.0** |
| **0.6** | **2.9** | **3.0** | **3.0** | **3.2** |
| **0.8** | **2.7** | **3.0** | **3.1** | **3.0** |
| **1.0** | **2.8** | **3.1** | **3.0** | **3.1** |

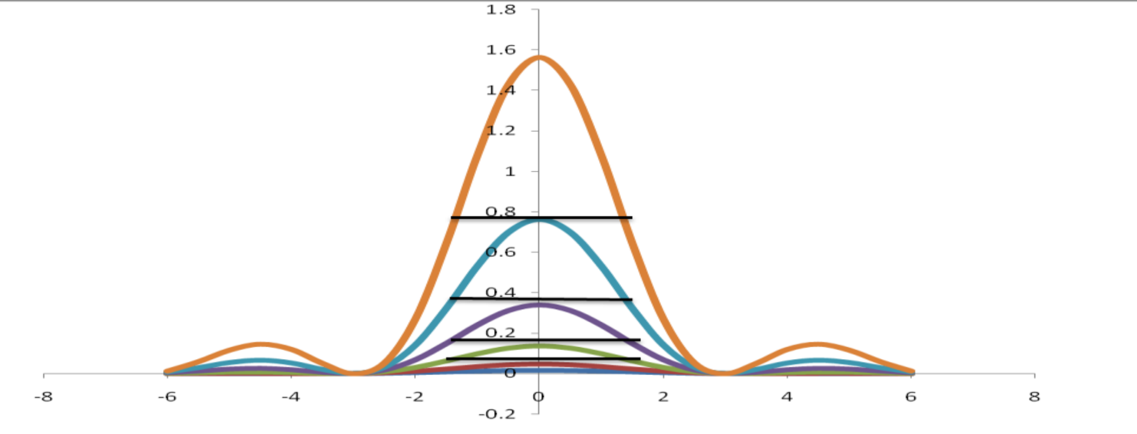
In figures 2 (a) to 2(d), we have plotted expanded IPSF curves for the evaluation of FWHM for $\alpha = 0, 0.25, 0.50 \& 0.75$. For third order. In one of the figures, for example for $\alpha = 0$ i.e. in figure 2(a), we have indicated, the method of evaluating the values of FWHM from the corresponding IPSF curves.



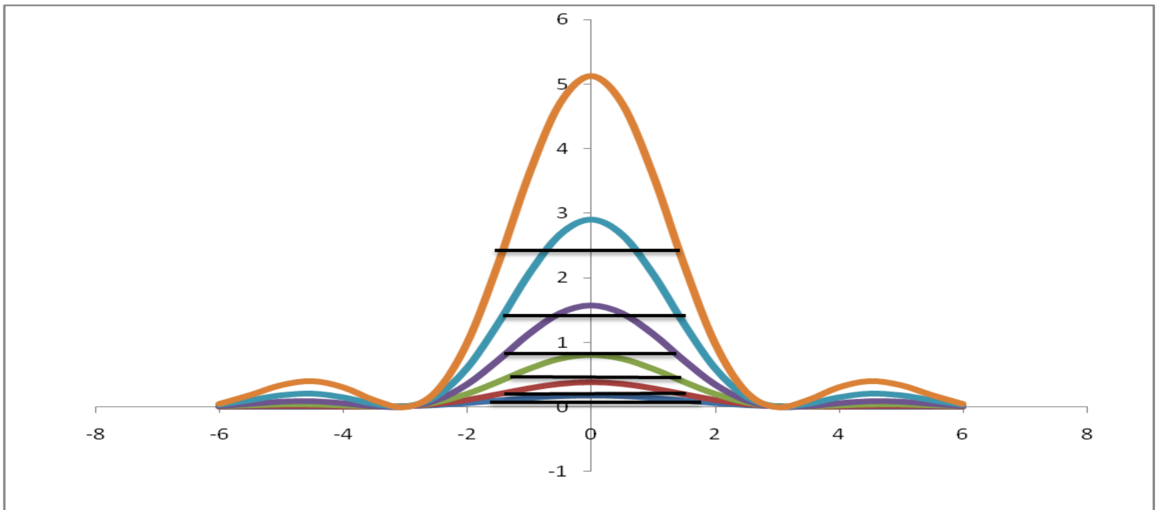

2(a) IPSF Curves for Evaluation of HPD ; α=0, N=3




2(b) IPSF Curves for Evaluation of HPD ; α=0.25, N=3




2(c) IPSF Curves for Evaluation of HPD ; α=0.5, N=3




2(d) IPSF Curves for Evaluation of HPD ; α=0.75, N=3

**Table-II**: **Values of FWHM**

| $\beta$ | $\alpha = 0$ | $\alpha = 0.25$ | $\alpha = 0.50$ | $\alpha = 0.75$ |
|---|---|---|---|---|
| **0.2** | **2.9** | **2.9** | **3.4** | **3.1** |
| **0.4** | **2.6** | **3.3** | **3.2** | **3.1** |
| **0.6** | **2.9** | **3.1** | **3.3** | **3.2** |
| **0.8** | **2.9** | **3.1** | **3.1** | **3.2** |
| **1.0** | **2.9** | **3.2** | **3.1** | **3.3** |

The evaluated values of FWHM have been presented in a tabulated form. In the table I & table II it is observed from the table that FWHM values do not change appreciably with the values of $\alpha \& \beta$ of this particular set of our chosen pupil function. However, the super resolving character of this set of parabolic filters $f(r) = \left(\alpha + \beta r^2\right)^N$ where N=2,3. is clearly evidenced from the fact that the average value of FWHM for second order is 3.2 diffraction units and for third order is 3.2 which are less than Raleigh's classical limit of resolution of 3.83 units for perfect or diffraction limited optical system.

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