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Analysis of C-C Short Cylindrical Shells under Internal Pressure using Polynomial Series Shape Function.

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ABSTRACT
The traditional approach in the analysis of axisymmetrically loaded short cylindrical shells has been to solve the fourth order differential equation using the Krylov's equation. This involved a transition from exponential functions to krylov's functions using Euler's expressions. This approach is grossly limited by the difficulty in the transition from exponential functions to Krylov's functions. A new approach to static analysis of C-C short cylindrical shell subject to internal liquid pressure is presented in this paper. This involves substituting a polynomial series shape function into the Pasternak's differential equation, by satisfying the boundary conditions for C-C short cylindrical shell, a particular shape function was obtained. This shape function was substituted into the total potential energy functional of the Ritz method and minimized to obtain the unknown coefficient. Stresses and deflections at various points of the shell were determined for different cases of aspect ratio with range $1 \le L/r \ge 4$. For case 1, maximum values of deflection, rotation, bending moment and shear force were $9.856*10^{-3}$metres, $-3.23*10^{-3}$radians, -1366.64KNm and -9566.4639KN respectively. It was observed that as the aspect ratio increases from 1 to 4, the deflections and stresses decreases, and the shell tends to behave like long cylindrical shell.

Keywords *- Axisymmetrically loaded, Boundary condition, C-C Short Cylindrical Shell, Internal liquid pressure, Krylov's function, Polynomial series shape function, Ritz method.*

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I. INTRODUCTION

Large cylindrical shell tanks are widely used in the construction of strategic water or oil reservoir all over the world. In order to lower the cost, and to make the management easier, the volume of such tanks tends to be larger, thus short cylindrical shells. Osadebe and Adamou [1] studied Static Analysis of Cylindrical shell under hydrostatic and Ring forces using initial value method. Ventsel and Krauthammer [2] studied short cylindrical shell using the Krylov's function but did not consider axisymmetrically loaded with internal liquid pressure. Ezeh et al, 2014 [3] studied Static Analysis of C-S short cylindrical shell under internal liquid pressure but did not consider the case of C-C boundary condition.

Thin shells as structural elements occupy a leadership position in engineering and especially in civil engineering, since they can be used in the construction of large liquid storage structures, large span roofs, domes, folded plates and so on.

Vinson [4], and Pasternak [5], by ignoring the effects of longitudinal bending moment, shear forces and torques arrived at the semi-moment theory, which is found to give acceptable results in the analysis of cylindrical shells. Timoshenko and Woinowsky-Krieger [6] experimentally verified on cylindrical shell whose ratio of length to radius (aspect ratio) ranges from 1 to 4. Pasternak [5] showed that when the load on a cylindrical shell is axisymmetric, the stresses and strains are functions of only one variable along the axis of the cylinder. Ventsel and Krauthammer [2] worked on short cylindrical shell using Krylov's function but did not consider the case of axisymmetrically loaded condition. This work is concerned with the analysis of C-C short cylindrical shell subjected to axisymmetric internal liquid pressure.

The objective of this study is to establish an analysis of C-C short cylindrical shell that can be utilized in the design of cylindrical shell reservoir tank and to show the distribution of stress and strain on the C-C shell reservoir under working load. The magnitudes and locations of critical values of stresses and strains along the height of the cylindrical shell tanks were indicated and recorded for the purpose of design.

II. GOVERNING DIFFERENTIAL EQUATION OF A CYLINDRICAL SHELL.

Consider a C-C short cylindrical shell with its dimension L, t and r, as shown in Fig.1 which is subjected to axisymmetric internal liquid pressure.

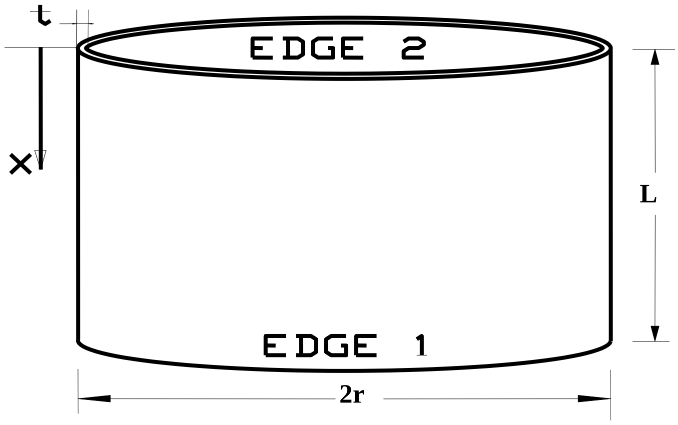

Figure 1: typical C-C (clamped at both edges) short cylindrical shell tank showing the dimensions.

According to Ezeh et al, 2014 [3], the condition for shortness for an unstiffened cylindrical shell is L/r < 5, where L/r is the aspect ratio.
The governing equation of a cylindrical shell according to the semi-moment theory as used by Timoshenko et al [6]; Ugural [7]; Ventsel and Krauthammer [2] is as stated in equation (1).

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{\gamma}{D}x \qquad (1)$$

Where

$$\beta^4 = \frac{3(1-v^2)}{r^2t^2} \qquad (2)$$

γ = Unite weight of the liquid.

Equation (1) is due to Pasternak [5] and is only applicable to cylindrical shell subject to axisymmetric loading.
Ezeh et al, 2014 [3] gave the general polynomial series shape function for short cylindrical shell as:

$$w = k_0 + k_1R + k_2R^2 + k_3R^3 + kR^5 \qquad (3)$$

Where, $k_0 = a_0c;\ k_1 = a_1c;\ k_2 = a_2c;\ k_3 = a_3c;\ k_5 = \Phi b$ (4)

III. THE RITZ METHOD.

According to Vintsel and Krauthammer [2], Timoshenko and Woinowsky-Krieger [6], the Ritz equation derived from the principle of theory of elasticity is given as:

$$\Pi = \frac{1}{2}\int_0^L \left(\frac{d^2w}{dR^2}\right)^2 dR + \frac{\lambda}{2}\int_0^L w^2\, dR - \Phi\int_0^L w\, RdR \qquad (5)$$

Let, $w = Af_i$ (6)

Substituting equation (6) into equation (5) gave:

$$\Pi = \frac{A^2}{2}\int_0^L \left(\frac{d^2f_i}{dR^2}\right)^2 dR + \frac{\lambda A^2}{2}\int_0^L (f_i)^2\, dR - \Phi A\int_0^L f_i\, RdR \qquad (7)$$

IV. SHAPE FUNCTION FOR C-C SHORT CYLINDRICAL SHELL

The C-C short cylindrical shell has the following boundary conditions.

$$w_0 = w(R=0) = 0;\ \theta_0 = \frac{dw}{dR}(R=0) = 0 \qquad (8)$$

$$w_{(1)} = w(R=1) = 0;\ \theta_{(1)} = \frac{dw}{dR}(R=1) = 0 \qquad (9)$$

Applying these boundary conditions in equation (3) gave:

$$w = A(2R^2 - 3R^3 + R^5) = Af_i \qquad (10)$$

That is $f_i = (2R^2 - 3R^3 + R^5)$ (11)

Using equation (11), the following integrations were obtained:

$$\int_0^1 \left(\frac{d^2 f_i}{dR^2}\right)^2 dR = (16 - \frac{144}{2} + \frac{324}{3} + \frac{160}{4} - \frac{720}{5} + \frac{400}{7}) = 5.142857 \quad (12)$$

$$\int_0^1 (f_i)^2 dR = \left(\frac{4}{5} - \frac{12}{6} + \frac{9}{7} + \frac{4}{8} - \frac{6}{9} + \frac{1}{11}\right) = 0.0099567142 \quad (13)$$

$$\int_0^1 (f_i R) dR = \left(\frac{2}{4} - \frac{3}{5} + \frac{1}{7}\right) = 0.042857142 \quad (14)$$

Substituting equations (12), (13) and (14) into equation (7) gave:

$$\Pi = \frac{5.142857 A^2}{2} + \frac{\lambda A^2}{2} (0.0099567142) - \Phi A (0.042857142) \quad (15)$$

Minimizing equation (15) gave:

$$\frac{\partial \Pi}{\partial A} = 5.142857A + 0.0099567142A\lambda - 0.042857142\ \Phi = 0 \quad (16)$$

Minimizing equation (16) gave:

$$A = \frac{0.042857142\ \Phi}{5.142857 + 0.0099567142\lambda} \quad (17)$$

Substituting for Φ and λ, in equation (17) and simplifying further, gave:

$$A = \frac{4.3036346\gamma L^5 r^2}{516.521471 r^2 D + EtL^4} \quad (18)$$

Substituting equation (18) into equation (10) gave:

$$w = \frac{4.3036346\gamma L^5 r^2}{516.521471 r^2 D + EtL^4}(2R^2 - 3R^3 + R^5) \quad (19)$$

Differentiating equation (19) with respect to R gave:

$$\theta = \frac{dw}{dR} = \frac{4.303635\ \gamma L^5 r^2}{516.5215 r^2 D + EtL^4}(4R - 9R^2 + 5R^4) \quad (20)$$

$$M = -D\left(\frac{d^2 w}{dR^2}\right) = -D\left[\frac{4.303635\ \gamma L^5 r^2}{516.5215 r^2 D + EtL^4}\right](4 - 18R + 20R^3) \quad (21)$$

$$Q = -D\left(\frac{d^3 w}{dR^3}\right) = -D\left[\frac{4.303635\ \gamma L^5 r^2}{516.5215 r^2 D + EtL^4}\right](60R^2 - 18) \quad (22)$$

V. NUMERICAL STUDIES

The deformations and stresses at various points of C-C short cylindrical shells were determined for values of aspect ratios ranging from 1 to 4. The equations of the deformations and stresses of C-C short cylindrical shells of various boundary conditions are presented. The numerical values of the following parameters E, D, L, t, r and γ are substituted accordingly into the formulated solutions.

C-C short cylindrical shell water reservoir made of Concrete with real life dimensions was adopted for numerical purposes:

For the four cases considered, the parameters used are as shown in Table 1.

Table 1: Parameters used in the analysis

Case	Aspect ratio(L/r)	Radius (m)	Thicknes s (m)	Height (m)	Unit weight of liquid (KN/m3)	Poisson's ratio, υ	Young Modulus E(KN/m2)
1	1	10.00	0.25	10	9.81	0.25	$26*10^6$
2	2	5.00	0.20	10	9.81	0.25	$26*10^6$
3	3	3.34	0.15	10	9.81	0.25	$26*10^6$
4	4	2.50	0.10	10	9.81	0.25	$26*10^6$

VI. RESULTS AND DISCUSSION

For the cases considered, the graphs of deflections, rotations, bending moments and shear forces were plotted against the height of the shell as shown in figures (2) to (5). The maximum values of deflection, rotation, bending moment and shear force for each case considered were shown in Table 2.

Table 2: The maximum values of deflections, rotations, bending moments and shear forces

Cases	**Maximum Deflection (m)**	**Maximum Rotation (radians)**	**Maximum Bending moment (KNm)**	**Maximum Shear force (KN)**
1	$9.8556 * 10^{-3}$	$-3.22947* 10^{-3}$	-1366.6375	-9566.4631
2	$2.51654 * 10^{-4}$	$-8.2462* 10^{-4}$	-348.9601	-2442.7208
3	$1.12067 * 10^{-4}$	$-3.6722* 10^{-4}$	-155.3993	-1087.7949
4	$6.32514 * 10^{-5}$	$-2.0726 * 10^{-4}$	-87.70857	-613.9600

Deflection**:** From the Graph shown in Fig. 2, it was observed that the maximum deflection for the C-C short cylindrical shell occurs at L/2 of the height from the base.
Rotation**:** From the graphs shown in Fig. 3, it was observed that the maximum slope (rotation) for the C-C short cylindrical shell occurs at the 1/5 of the height L from the base.
Bending moment**:** From the graphs shown in Fig. 4, it was observed that the maximum bending moment occur at the base of the C-C short cylindrical shell, which is at the clamped edge.
Shear force**:** From the graphs of Fig. 5, it was observed that the shear force varied along the height of the shell with the maximum values at the clamped base of the shell.
It was observed that as the aspect ratio increases from 1 to 4, the deflections, rotations, bending moments and shears forces of the C-C short cylindrical shells decreases and tends to behave like long cylindrical shell as shown in Fig. 2 to 5.

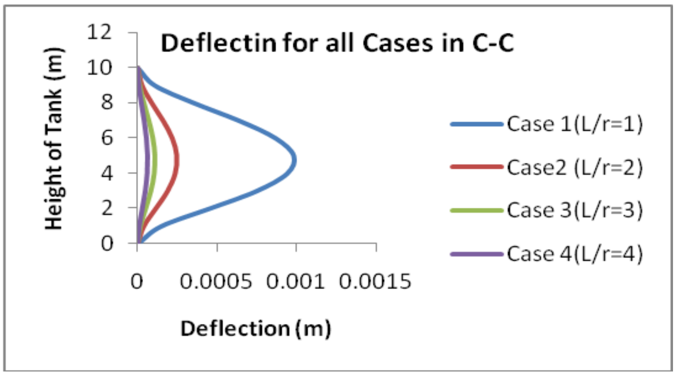

Figure 2: deflection curves for C-C short cylindrical shells of aspect ratios 1to 4.

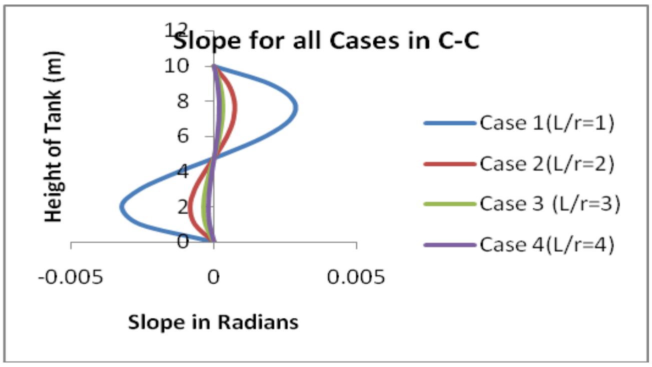

Figure 3: rotation curves for C-C short cylindrical shells with aspect ratios 1 to 4.

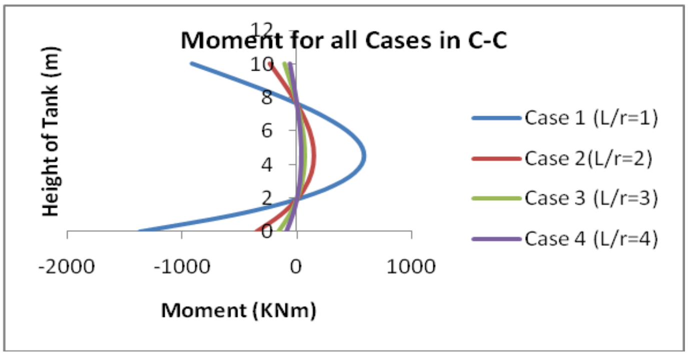

Figure 4: bending moment diagrams for C-C short cylindrical shells of aspect ratios 1to 4.

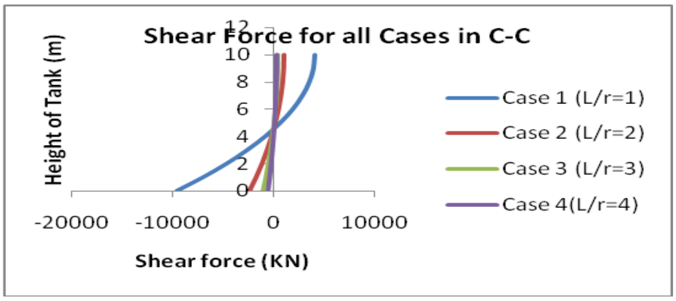

Figure5: shear force diagrams for C-C short cylindrical shells of aspect ratios 1 to 4.

VII. CONCLUSION

Using the polynomial series in the Ritz method is more convenient for analyzing C-C short cylindrical shells than the use of krylov's function. Knowledge of the point of maximum stresses along the height of the shell help for adequate reinforcement to be provided at the appropriate point. In the case of stiffening the shell with rings, this guides in the position of the rings for optimal design.

It is therefore recommended that this approach could be easily applied in solving C-C short cylindrical shell problems during the design of large cylindrical shell water or oil reservoir.

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