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Effects of Variable Viscosity and Thermal Conductivity on MHD free Convection and Mass transfer Flow over an Inclined Vertical Surface in a Porous Medium with Heat Generation

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ABSTRACT
A steady two-dimensional MHD free convection and mass transfer flow past an inclined vertical surface in the presence of heat generation and a porous medium have been studied numerically when the fluid viscosity and thermal conductivity are assumed to be vary as inverse linear function of temperature. The governing partial differential equations are reduced to a system of ordinary differential equations by introducing similarity transformations. The non-linear similarity equations are solved numerically by applying the Runge-Kutta method of fourth order with shooting technique. The numerical results are presented graphically to illustrate influence of different values of the parameters on the velocity, temperature and concentration profiles. Skin friction, Nusselt number and Sherwood number are also completed and presented in tabular form.

***Keywords** - Heat generation, inclined vertical surface, mass transfer, shooting method, variable viscosity and thermal conductivity.*

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I. INTRODUCTION

Under the influence of magnetic field, the study of free convection and mass transfer flow of an electrically conducting fluid over an inclined vertical surface has considerable interest in geophysics, astrophysics and many engineering problems, such as cooling of nuclear reactors, the boundary layer control in aerodynamics. Alam *et.al* [1] studied the heat and mass transfer in MHD free convection flow over an inclined plate with hall current. Anghel *et.al* [2] investigated a numerical solution of free convection flow past an inclined surface. Subhakar *et. al* [3] studied the Soret and Dufour effects on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink.

The study of the flow through porous medium has some important applications in engineering and geophysics such as chemical engineering for filtration and purifications process, agriculture engineering to study the underground water resources, and petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. Chandra *et. al* [4] studied heat and mass transfer along an isothermal vertical porous plate in the presence of heat sink. Sharma *et. al.* [5] studied the effects of dusty viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate

The study of the heat generation or absorption in moving fluids is important problems dealing with chemical reactions and those concerned with dissociating fluids. Reddy [6] studied the effects of heat generation and radiation on steady MHD free convective flow of micro polar fluid past a moving surface and they [7] also observed the MHD free convection heat and mass transfer flow past an inclined vertical surface in a porous medium with heat generation. Islam *et. al* [8] discussed the MHD free convection and mass transfer flow with heat generation through an inclined plate.

Most of these studies are based on constant physical properties. More accurate prediction for the flow and heat transfer can be determined by variation of these properties with temperature. Several investigators have studied the effects of temperature dependent viscosity and thermal conductivity properties of fluid. But temperature dependent physical properties like viscosity of the fluid and thermal conductivity plays a significant role in fluid mechanics. It plays a great role in underground storage system and geothermal energy extraction. Khound *et. al.* [9] observed that a significant variation of velocity distribution and temperature distribution take place with the variation of viscosity parameter and thermal conductivity parameter.

Mahanti *et.al.* [10] studied the effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. They assumed the viscosity and thermal conductivity to vary linearly with temperature. Sarma *et. al* [11] studied the effect of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence magnetic field and they also [12] studied on free convective heat and mass transfer flow with constant heat flux through a porous medium.

The purpose of this present paper is to investigate the effects of variable viscosity and thermal conductivity on MHD free convection and mass transfer flow past an inclined vertical plate in a porous medium with heat generation. The flow governing equations are transformed into ordinary differential equations, before numerically solve by Runge-Kutta shooting method by using similarity transformation. The fluid viscosity and thermal conductivity are taken as inverse linear functions of temperature. The effects of variable viscosity parameter, variable thermal conductivity parameter, Prandtl number, magnetic parameter and Schmidt number have been discussed and shown graphically.

II. MATHEMATICAL FORMULATION

Consider a two dimensional steady laminar boundary layer flow of an incompressible viscous electrically conducting fluid over a semi-infinite inclined vertical plate in a porous medium with an acute angle α. The flow is assumed to be in the x -direction, which is taken along the semi-infinite inclined porous plate and y - axis normal to it. A uniform magnetic field B_0 is imposed along the y-axis. The fluid properties are assumed to be isotropic and constant except for the fluid viscosity and thermal conductivity.

The flow governing equations for the present problem are:

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Equation of momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+g\beta^*(T-T_\infty)\cos(\alpha)+g\beta^{**}(C-C_\infty)\cos(\alpha)-\frac{\sigma B_0^2}{\rho_\infty}u-\frac{\nu_\infty}{k_1}u \qquad (2)$$

Equation of energy (2)

$$\rho_\infty c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q_1(T-T_\infty) \qquad (3)$$

Equation of concentration (3)

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \qquad (4)$$

(4)

where (u,v) is the fluid velocity, μ,ρ_∞,k are the coefficient of viscosity, density and thermal conductivity of fluid. g, β^* and β^{**} are the acceleration due to gravity, thermal expansion coefficient and concentration expansion coefficient. k_1, T and T_∞ are the permeability of porous medium, fluid temperature within the boundary layer and fluid temperature in the free stream. C and D are the concentration and the coefficient of mass diffusivity of the fluid. Q_1, c_p are the heat generation constant and specific heat at constant pressure.

The boundary conditions for the problem are:

$$\left.\begin{array}{l} u=0, v=0, T=T_w, C=C_w \text{ at } y=0, \\ u\to 0, T\to T_\infty, C_p\to C_\infty \text{ at } y\to\infty \end{array}\right\} \qquad (5)$$

Lai and Kulacki [13] has assumed the fluid viscosity as

$$\left.\begin{aligned}&\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right]\\ \text{or,}\quad &\frac{1}{\mu}=a\left(T-T_r\right),\\ \text{here,}\quad &a=\frac{\gamma}{\mu_\infty},\ T_r=T_\infty-\frac{1}{\gamma}\end{aligned}\right\}\tag{6}$$

Similarly,

$$\left.\begin{aligned}&\frac{1}{k}=\frac{1}{k_\infty}[1+\xi(T-T_\infty)]\\ \text{or,}\quad &\frac{1}{k}=c(T-T_k)\\ \text{here,}\quad &c=\frac{\xi}{k_\infty},T_k=T_\infty-\frac{1}{\xi}\end{aligned}\right\}\tag{7}$$

where μ_∞, k_∞ and T_∞ are the viscosity, thermal conductivity and temperature at free stream. a, c, Tr and T_k are constants and their values depend on the reference state and thermal property of the fluid. In general a > 0 for liquids and a < 0 for gases. γ and ξ are constant based on thermal property of the fluid.

Let us introduce the following transformations:

$$\left.\begin{aligned}&u=U_\infty f'(\eta), v=\frac{1}{2}\sqrt{\frac{U_\infty \upsilon_\infty}{x}}\left(\eta f'-f\right), \eta=\sqrt{\frac{U_\infty}{\upsilon_\infty x}}y,\\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\end{aligned}\right\}\tag{8}$$

Using the transformations (5-8), in equations (1-4), it is seen that the equation of continuity satisfies identically and rest of the equations becomes:

$$f'''-\frac{\theta' f''}{\theta-\theta_r}-\frac{\theta-\theta_r}{\theta_r}\left\{\frac{1}{2}ff''+Gr\theta\cos\alpha+Gm\phi\cos\alpha-\left(M+K\right)f'\right\}=0\tag{9}$$

$$\theta''-\frac{\theta'^2}{\theta-\theta_k}-\frac{(\theta-\theta_k)\Pr}{\theta_k}\left(\frac{1}{2}f\theta'+Q\theta\right)=0\tag{10}$$

$$\phi''+\frac{1}{2}Scf\phi'=0\tag{11}$$

H

Here,

Here,

$M=\dfrac{\sigma B_0^{\ 2}x}{\rho_\infty U_\infty}$ is the magnetic field parameter.

$Gr=\dfrac{g\beta^*\left(T_w-T_\infty\right)x}{U_\infty^{\ 2}}$ is the local Grashof number.

$Gm=\dfrac{g\beta^{**}\left(C_w-C_\infty\right)x}{U_\infty^{\ 2}}$ is the mass Grashof number.

$P_r=\dfrac{\mu c_p}{k}$ is the Prandtl number.

$Sc = \dfrac{D}{\upsilon_\infty}$ is the Schmidt number of fluid.

$\theta_r = \dfrac{T_r - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\gamma(T_w - T_\infty)}$ is the viscosity variation parameter.

$\theta_k = \dfrac{T_k - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\xi(T_w - T_\infty)}$ is the thermal conductivity variation parameter.

$K = \dfrac{\upsilon_\infty x}{k_1 U_\infty}$ is the permeability parameter.

$Q = \dfrac{Q_1 x}{\rho_\infty c_p U_\infty}$ is the heat generation parameter.

$\mathrm{Re} = \dfrac{U_\infty x}{\upsilon_\infty}$ is the local Reynolds number.

The transform boundary conditions are

$$\left.\begin{array}{l} f = 0,\ \ f' = 0,\ \theta = 1,\ \phi = 1 \text{ at } \eta = 0 \\ f' = 0,\ \ \theta = 0,\ \phi = 0 \text{ as } \eta \to \infty \end{array}\right\} \qquad (12)$$

. The physical quantities of interest are the skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh for the surface which are define by

$$\left.\begin{array}{l} C_f = \dfrac{\tau_w}{\rho_\infty U_\infty^{\ 2}} = -\dfrac{\theta_r}{(\theta - \theta_r)} \mathrm{Re}^{-\frac{1}{2}} f''(0) \\ Nu = \dfrac{x q_w}{k_\infty (T_w - T_\infty)} = -\dfrac{\theta_k}{\theta - \theta_k} \mathrm{Re}^{\frac{1}{2}} \theta'(0) \\ Sh = \dfrac{x h_m}{D(C - C_w)} = -\mathrm{Re}^{\frac{1}{2}} \phi'(0) \end{array}\right\} \qquad (13)$$

where $\tau_w = \mu\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$, $q_w = k\left(\dfrac{\partial T}{\partial y}\right)_{y=0}$, $h_m = -D\left(\dfrac{\partial C}{\partial y}\right)_{y=0}$

III. RESULTS AND DISCUSSION

The boundary value problems (9)- (13) are solved numerically using fourth order Runge-Kutta shooting methods. The numerical values of different parameters are taken as $\mathrm{Pr} = 0.7,\ \theta_r = 4, \theta_k = 4, M = 0.5,$ $Gr = 0.5, Gm = 0.5, \alpha = 30$ unless otherwise stated. Both velocity, temperature and concentration profiles are displayed graphically.

Velocity profile for various combination of parameters Pr=.7, Ec=.05, $N = 0.5, k = 0.2$ are illustrated in figure 1 to figure 8. It is observed from these figures that the velocity distribution decreases with increase of viscosity variation parameter θ_r, thermal conductivity parameter θ_k, magnetic parameter M, permeability parameter K and Schmidt number Sc. On the other hand velocity increases with increasing values of heat generation parameter Q, local Grashof number Gr and mass Grashof number Gm.

From figure 9 to figure 16, it can be concluded that the temperature increases with the increase of viscosity variation parameter θ_r, magnetic field parameter M, heat generation parameter Q and Schmidt number Sc. Also temperature decreases with the increase of thermal conductivity parameter θ_k, local Grashof number Gr and mass Grashof number Gm.

Concentration profiles for various values of Sc and M are shown in figure 17 to 18. It is clear from these figures that the concentration decreases with increase of Schmidt number Sc. But it is increases with increase of magnetic parameter M.

The missing values $f''(0)$, $\theta'(0)$, $\phi'(0)$ and the coefficient of skin friction Cf, Nusselt number Nu and Sherwood number Sh are estimated for various combinations of parameters and presented in Table 1 to Table 3. From these Tables we have observed that Cf and Nusselt number Nu decrease and Sh increases for the increasing values of viscosity parameter θ_r while $f''(0)$ and Sh increase and Nu decreases with the increase of thermal conductivity parameter θ_k. Also Sh increases with increase mass Grashof number Gm and decreases with Schmidt number Sc.

In the Table 4 we have compared of missing values $f''(0)$, $\theta'(0)$ and $\phi'(0)$ with those of values obtained by Reddy *et. al* [7] for various values of Gr. From this Table we have observed that the accuracy of missing values $f''(0)$, $\theta'(0)$ and $\phi'(0)$ of present study are less than that of Reddy *et. al* [7].

IV. FIGURES AND TABLES

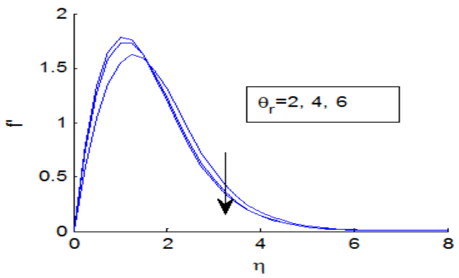

Fig 1: Velocity profile for different θ_r.

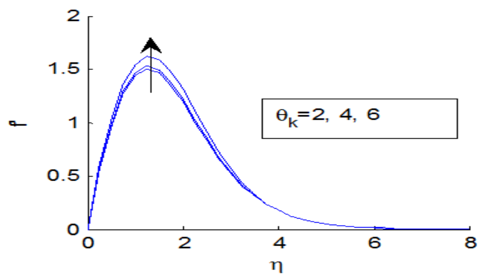

Fig 2: Velocity profile different θ_k.

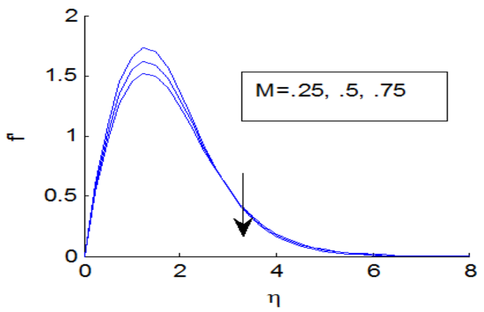

Fig 3: Velocity profile for different M.

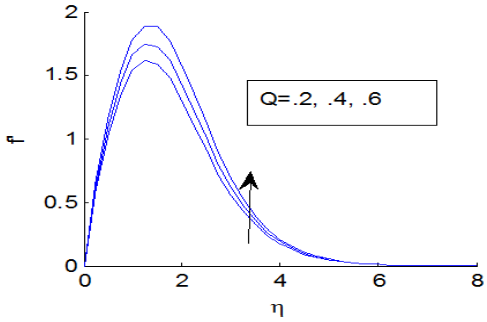

Fig 4: Velocity profile for different Q.

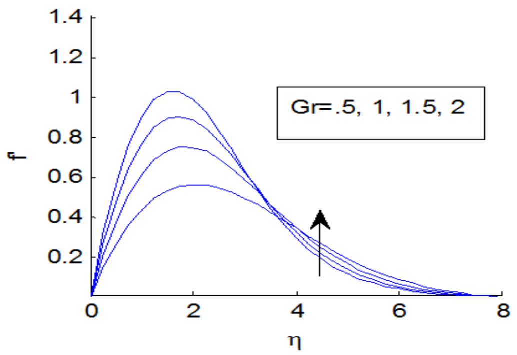

Fig 5: Velocity profile for different Gr.

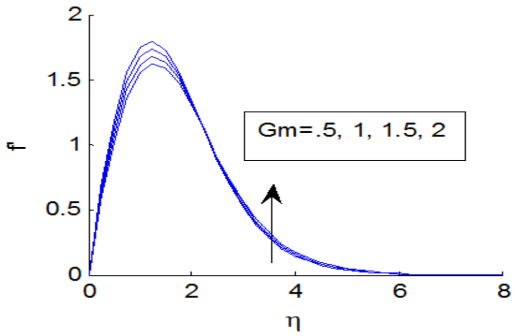

Fig 6: Velocity profile for different Gm.

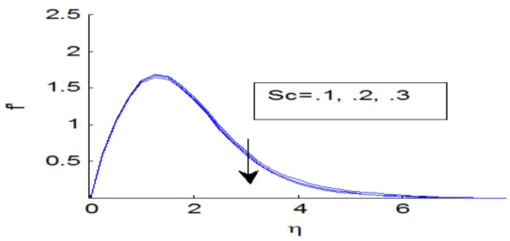

Fig 7: Velocity profile for different Sc.

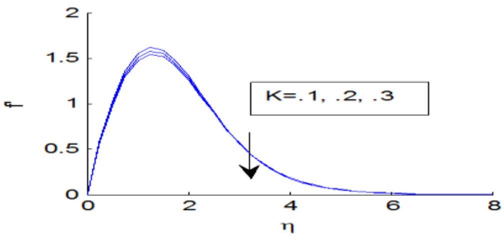

Fig 8: Velocity profile for different K.

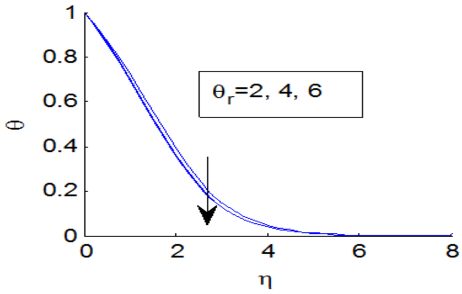

Fig 9: Temperature profile for different θ_r.

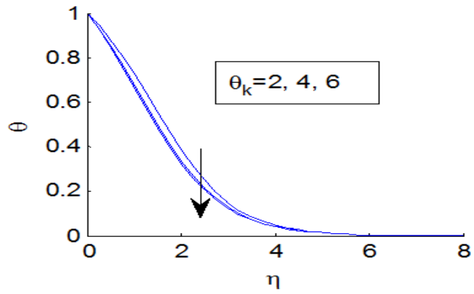

Fig 10: Temperature profile for different θ_k.

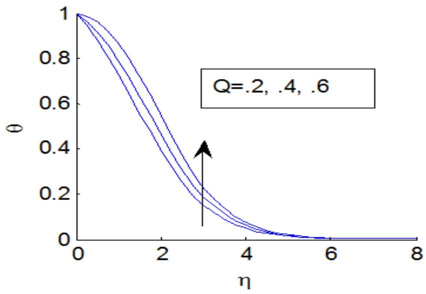

Fig 11: Temperature profile for different Q.

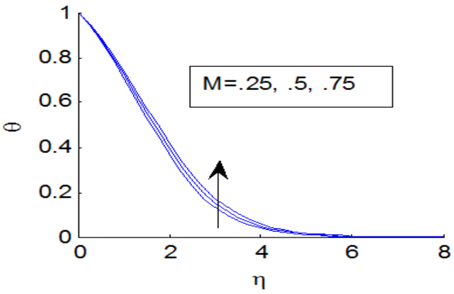

Fig 12: Temperature profile for different M.

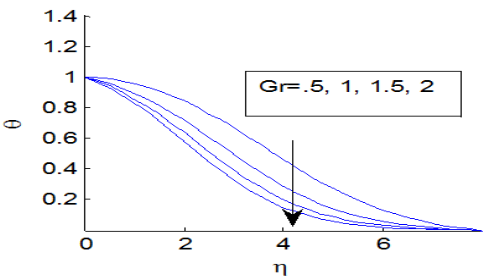

Fig 13: Temperature profile for different Gr.

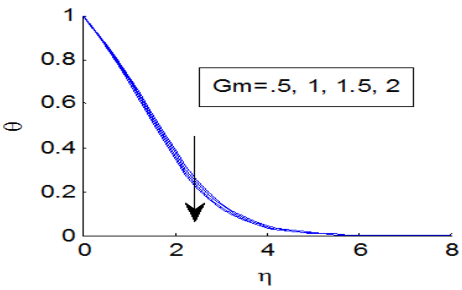

Fig 14: Temperature profile for different Gm.

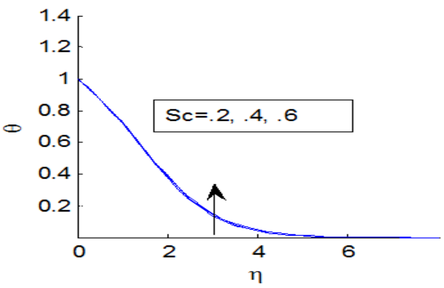

Fig 15: Temperature profile for different Sc.

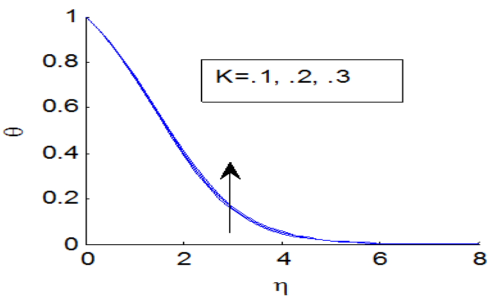

Fig 16: Temperature profile for different K.

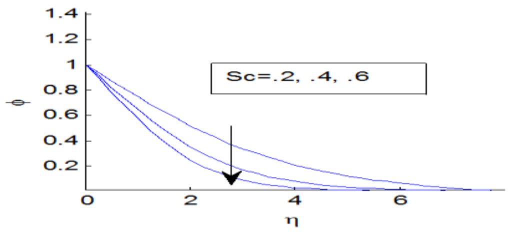

Fig 17: Concentration profile for different Sc.

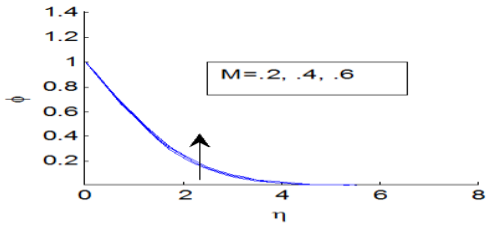

Fig 18: Concentration profile for different M.

Table: 1

M	θ_r	$f''(0)$	$\theta'(0)$	$\phi'(0)$	Cf	Nu	Sh
0.25	2	3.862029	-0.45512	-0.22722	438.6968	-0.14488	0.071854
	4	3.595297	-0.4799	-0.24603	181.3049	-0.15277	0.077801
	6	2.713935	-0.48647	-0.25091	34.10165	-0.15485	0.079345
0.5	2	3.653211	-0.44259	-0.21555	415.1085	-0.14076	0.068162
	4	3.405082	-0.46589	-0.23333	171.7936	-0.14817	0.073787
	6	2.582761	-0.47205	-0.23795	32.48405	-0.15012	0.075245
0.75	2	3.469272	-0.43109	-0.20469	394.3685	-0.13694	0.064728
	4	3.237415	-0.453	-0.2215	163.4325	-0.1439	0.070043
	6	2.466796	-0.45879	-0.22584	31.06169	-0.14572	0.071418

Table 2: Table: 2

Q	θ_k	$f''(0)$	$\theta'(0)$	$\phi'(0)$	Cf	Nu	Sh
0.1	2	2.355011	-0.43614	-0.25368	29.63691	-0.13337	0.08022
	4	2.388106	-0.42422	-0.32137	30.036	-0.13453	0.101625
	6	2.513231	-0.42103	-0.3409	31.51908	-0.13911	0.107802
0.2	2	2.432502	-0.44259	-0.21555	30.67494	-0.13548	0.068162
	4	2.463873	-0.43107	-0.2688	31.05703	-0.13656	0.085002
	6	2.582761	-0.42799	-0.2838	32.48405	-0.14076	0.089746
0.3	2	2.518574	-0.44936	-0.17475	31.8122	-0.13774	0.05526
	4	2.547707	-0.43832	-0.21228	32.16999	-0.13873	0.067129
	6	2.658662	-0.43536	-0.22232	33.51574	-0.14258	0.070302

Table: 3

Gm	Sc	$f''(0)$	$\theta'(0)$	$\phi'(0)$	Cf	Nu	Sh
0.25	0.2	2.514299	-0.28718	-0.21369	31.624	-0.09133	0.067576
	0.4	2.506848	-0.36825	-0.21264	31.53248	-0.11711	0.067241
	0.6	2.500946	-0.43836	-0.21187	31.45971	-0.13939	0.067001
0.5	0.2	2.609755	-0.28969	-0.21915	32.81628	-0.09215	0.069302
	0.4	2.594611	-0.37199	-0.21705	32.63021	-0.11832	0.068637
	0.6	2.582761	-0.44259	-0.21555	32.48405	-0.14076	0.068162
0.75	0.2	2.704833	-0.29217	-0.22448	34.00385	-0.09296	0.070987
	0.4	2.681772	-0.37565	-0.22135	33.7204	-0.1195	0.069996
	0.6	2.663932	-0.44671	-0.21913	33.50032	-0.14209	0.069293

Table: 4

Gr	$f''(0)$	$\theta'(0)$	$\phi'(0)$	$f''(0)$	$\theta'(0)$	$\phi'(0)$
	Ready *et.al* [7]			Present study		
1	1.355485	-0.31921	-0.12004	1.048356	-0.21489	-0.07232
2	2.058302	-0.36851	-0.21732	1.64646	-0.23438	-0.12823
3	2.695334	-0.40412	-0.27765	2.192476	-0.2491	-0.16583

V. CONCLUSION

From the above analysis, it is clear that the viscosity and thermal conductivity variation parameter, heat generation parameter Q, magnetic parameter M, permeability parameter K, local Grashof number Gr, mass Grashof number Gm and Schmidt number etc. have substantial effects on velocity, temperature and concentration profiles within the boundary layer. We can conclude from this analysis as:

1. The increasing values of viscosity retard the velocity but enhance the temperature.
2. Both velocity and temperature increases as heat generation parameter increases.
3. Velocity decreases and temperature increases with increase magnetic parameter.
4. Velocity increases and temperature decreases with increase of free convection parameter.
5. Skin friction and Nusselt number decrease with increase of viscosity.
6. Increasing values of Schmidt number retards the Sherwood number.

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REFERENCES

[1]. Alam M. S., Ali M., Hossain M. D., Heat and mass transfer in MHD free convection flow over an inclined plate with hall current**,** *Int. J. Eng. Science, Vol. 2*(7), 2013, pp. 81-88.

[2]. Anghel M., Hossain M. A, Zeb S., Combined heat and mass transfer by free convection past an inclined flat plate. *Int. J. Appl. Mech. and Eng. Vol. 2*, 2001, pp. 473-497.

[3]. Subhakar M. J., Gangadhar K., Soret and Dufour effects on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink, *Int. J. Modern Engineering Research, Vol. 2*(5), Sep-Oct. 2012 pp-3458-3468.

[4]. Chandra B., Kumer M., Heat and mass transfer along an isothermal vertical porous plate in the presence of heat sink, *J. Comp. & Math. Sci. Vol. 2*(2), 2012, pp. 392-398.

[5]. Sharma, V.K., Sharma, G., Varshney, N. K., Effects of dusty viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate, *Appl. Math. Sci., Vol. 5*(77), 2011, pp. 3827-3836.

[6]. Reddy M. G., Heat generation and radiation effects on steady MHD free convection flow of micropolar fluid past a moving surface, J. *comp. And Apply. Res. In Math. Eng., Vol. 2*(2), March 2013, pp 1-10.

[7]. Reddy M. G., Reddy N. B., Mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium, ***J**. **A**ppl. **F**luid **M**echanics, Vol. 4* (2), 2011, pp. 7-11.

[8]. Islam M. S., Samsuzzoha M., Ara S., Dey P., MHD free convection and mass transfer flow with heat generation through an inclined plate**,** *Annals of Pure and Appl. Math***.** *Vol. 3*(2), 2013, pp. 129-141

[9]. Khaund P.K., Hazarika G.C., The effect of variable viscosity and thermal conductivity on liquid film on an unsteady stretching surface, *Proc.of 46th Annual Tech. Session, Ass.Sc.Soc.,* 2000, pp. 47-56.

[10]. Mahanti N.C., Gaur P., Effect of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink, *J. Appl. Fluid Mechanics, Vol. 2*(1), 2009, pp. 23-28.

[11]. Sarma U., Hazarika G.C., Effect of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence magnetic field, *Int. J. Phy. Education*

[12]. Sarma U., Hazarika G.C., Effects of variable viscosity and thermal conductivity on free convective heat and mass transfer flow with constant heat flux through a porous medium, *J. Com. Math. Sci., Vol. 1*(2), 2010, pp. 163-170.

[13]. Lai F.C., Kulacki F.A., The effect of variable viscosity on conductive heat transfer along a vertical surface in a saturated porous medium, *Int. J. Heat Mass Transfer, Vol. 33*, 1990, pp. 1028-1031.