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Basian-Markovian Principle in Fitting of Linear Curve

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ABSTRACT

A method of estimation using both of Basian and Markovian principles has been developed in fitting of linear curve to observed data. The method developed has been applied in fitting of linear regression of monthly mean extremum temperature on the monthly average length of day at the five cities namely Guwahati, Dhubri, Dibrugarh, Silchar and Tezpur. It has been found that the application of Basian-Markovian principle in fitting of linear regression of monthly mean extremum temperature on the monthly average length of day is more accurate than the fitting of the same by applying the Basian principle.

KEY WORD: *Linear Regression, Basian – Markovian Principle, Extremum Temperature.*

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I. INTRODUCTION:

In fitting a curve to numerical data by the traditional method of least squares, the parameters involved in the curve are estimated first by solving normal equations of the curve and then the estimated values corresponding to the observed values are obtained from the curve using the estimated values of the parameters and the observed values of the independent variable. The principle of obtaining an estimated value of the dependent variable from the estimates of the parameters (involved) and the corresponding value of the independent variable is nothing but the Basian principle. However, in practice there are situations where a value of the associated variable depends on its just preceding value. In such situations, it seems to be more accurate, if the estimated values of the dependent variable are determined on the basis of the just preceding observed value of it. The principle of obtaining an estimated value of the dependent variable from its just preceding observed value is nothing but the Markovian principle. An attempt has been made to determine estimated value of the dependent variable using the Markovian principle. However, since the values of the parameters involved are unknown, they are to be estimated using the entire observed data (since parameter cannot be estimated from single data). The principle of obtaining estimate of parameter from entire data is the Basian principle. Thus, the fitting of curve where parameters are estimated from entire data and estimate of dependent variable is obtained from its just preceding observed value and estimated values of parameters involved can be termed as Basian-Markovian principle. Rahman and Chakrabarty (2007) have innovated a method of fitting exponential curve by Basian-Markovian principle. In another study, Rahman and Chakrabarty (2008) have innovated similar method of fitting Gompertz curve by the same principle. In the current study, a method of estimation using Basian-Markovian principle has been developed in fitting of linear curve to observed data. The method developed has been applied in fitting of linear regression of monthly mean extremum temperature on the monthly average length of day at the five cities namely Guwahati, Dhubri, Dibrugarh, Silchar and Tezpur.

II. FITTING OF LINEAR CURVE:

The linear curve, more specifically the linear regression of Y on X, considered here is of the form

$$Y = a + bX \qquad (2.1)$$

where a and b are the parameters of the curve which are to be estimated from observed data on the pair

(X , Y) of variables X and Y.

Let the dependent variable 'Y' assume the values

$$Y_1, Y_2, \ldots\ldots\ldots\ldots , Y_n$$

corresponding to the values

$$X_1, X_2, \ldots\ldots\ldots\ldots\ldots , X_n$$

of the independent variable '*X*' respectively.

The objective is to estimate the parameters *a* and *b* of the linear curve described by the equation (2.1) on the basis of the *n* pairs

$$(X_1, Y_1) , (X_2 , Y_2) , \ldots\ldots\ldots\ldots\ldots , (X_n , Y_n)$$

of observations.

Since all pairs of observations may not lie on the linear curve defined above, the equation (2.1) yields

$$Y_i = a + bX_i , \qquad (i = 1, 2 , \ldots\ldots n) \qquad (2.2)$$

Now, from (2.2) we have

$$Y_{i+1} - Y_i = b(X_{i+1} - X_i)$$

which implies

$$Y_{i+1} = Y_i + b\Delta X_i \qquad (2.3)$$

where

$$\Delta X_i = X_{i+1} - X_i \qquad (2.4)$$

This is nothing but the recurrence relationship between Y_{i+1} and Y_i .

This relationship can be suitably applied to determine the estimated value of Y_{i+1} from the observed value of Y_i .

However, this recurrence relationship contains one parameter namely '*b*'. This parameter is to be known for estimating Y_{i+1} from Y_i .

In order to know the parameter '*b*' one can apply the principle of least squares to the linear curve described by the equation (2.1). On the application of the principle of least squares, the following normal equations are obtained:

$$\sum_{i=1}^{n} Y_i = na + b\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n} Y_i X_i = a\sum_{i=1}^{n} X_i + b\sum_{i=1}^{n} X_i^{\,2}$$

Solving these normal equations, one can obtain the estimate of '*b*' as well as of '*a*'.

III. APPLICATION OF THE METHOD TO MAXIMUM AND MINIMUM TEMPERATURE OF FIVE CITIES OF ASSAM.

The method innovated here has been applied to determine the estimated temperatures of five cities in the context of Assam from the data on temperatures collected from the Regional Meteorological Centre at Borjhar, Guwahati.

3.1. Monthly Mean Extremum Temperature at Guwahati:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the Table - 3.1.1.

Table - 3.1.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.536	26.226	29.972	30.883	31.363	31.768	31.995	32.470	31.720	30.383	27.731	24.724

The normal equations of the linear curve described by the equation (2.1) in this case become

$$352.771 = 12a + 144.131b$$

$$\& \; 4271.610108 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of a and b respectively as

$$\widehat{a} = 1.698023869 \quad \text{and} \quad \widehat{b} = 2.306198622.$$

Thus, the linear curve fitted to the data in **Table – 3.1.1** becomes

$$Y_i = 1.698023869 + 2.306198622\, X_i \qquad (3.1)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 2.306198622\, \Delta X_i \qquad (3.2)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.1) and (3.2) have been presented in the **Table- 3.1.2**.

Table- 3.1.2

X_i	Y_i	$\widehat{Y}_{(B)}$ =1.698023869 +2.306198622X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 2.306198622 ΔX_i	$\left\|\widehat{e}_{(B)}\right\| = \left\|Y_i - \widehat{Y}_{(B)}\right\|$	$\left\|\widehat{e}_{(BM)}\right\| = \left\|Y_i - \widehat{Y}_{i+1(BM)}\right\|$
10.553	23.536	26.03533793	-	2.49933793	-
11.105	26.226	27.30835957	24.80902164	1.08235957	1.41697836
11.834	29.972	28.98957836	27.90721880	0.98242164	2.06478120
12.610	30.883	30.77918849	31.76161013	0.10381151	0.87861013
13.266	31.363	32.29205479	32.39586630	0.92905479	1.03286630
13.605	31.768	33.07385612	32.14480133	1.30585612	0.37680133
13.469	31.995	32.76021311	31.45435699	0.76521311	0.54064301
12.921	32.470	31.49641626	30.73120316	0.97358374	1.73879684
12.191	31.720	29.81289127	30.78647501	1.90710873	0.93352499
11.424	30.383	28.04403693	29.95114566	2.33896307	0.43185434
10.755	27.731	26.50119005	28.84015312	1.22980995	1.10915312
10.398	24.724	25.67787714	26.90768709	0.95387714	2.18368709

Mean of the absolute deviations in column 5 = 1.255949775

& Mean of the absolute deviations in column 6 = 1.155245155.

(B) Estimation of Mean Minimum Temperature (Guwahati)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the Table - 3.1.3.

Table - 3.1.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	10.815	14.849	16.087	20.488	22.793	25.045	25.660	25.635	24.650	22.057	16.982	12.226

The normal equations of the linear curve described by the equation (2.1) in this case become

$$237.287 = 12a + 144.131b$$

$$\& \ 2909.964292 = 144.131\,a + 1746.108159\,b$$

which yield the least squares estimates $\hat{a}$ and $\hat{b}$ of a and b respectively as

$$\hat{a} = -28.33315945 \quad \text{and} \quad \hat{b} = 4.005279318$$

Thus, the linear curve fitted to the data in **Table – 3.1.3** becomes

$$Y_i = -28.33315945 + 4.005279318\,X_i \qquad (3.3)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 4.005279318\,\Delta X_i \qquad (3.4)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.3) and (3.4) have been presented in the **Table- 3.1.4**.

Table- 3.1.4

X_i	Y_i	$\hat{Y}_{(B)}$ = -28.33315945 +4.005279318 X_i	$\hat{Y}_{i+1(BM)}$ = Y_i + 4.005279318 ΔX_i	$\lvert\hat{e}_{(B)}\rvert = \lvert Y_i - \hat{Y}_{(B)}\rvert$	$\lvert\hat{e}_{(BM)}\rvert = \lvert Y_i - \hat{Y}_{i+1(BM)}\rvert$
10.553	10.815	13.93455319	-	3.11955319	-
11.105	14.849	16.14546738	13.02591418	1.29646738	1.82308582
11.834	16.087	19.06531600	17.76884862	2.97831600	1.68184862
12.610	20.488	22.17341275	19.19509675	1.68541275	1.29290325
13.266	22.793	24.80087598	23.11546323	2.00787598	0.32246323
13.605	25.045	26.15866567	24.15078969	1.11366567	0.89421031
13.469	25.660	25.61394768	24.50028201	0.04605232	1.15971799
12.921	25.635	23.41905462	23.46510693	2.21594538	2.16989307
12.191	24.650	20.49520072	22.71114610	4.15479928	1.93885390
11.424	22.057	17.42315148	21.57795076	4.63384852	0.47904024
10.755	16.982	14.74361962	19.37746814	2.23838038	2.39546814
10.398	12.226	13.3137349	15.55211528	1.08773490	3.32611528

Mean of the absolute deviations in column 5 = 2.214837646

& Mean of the absolute deviations in column 6 = 1.589418986.

3.2. Monthly Mean Extremum Temperature at Dibrugarh:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the Table - 3.2.1**.**

Table: 3.2.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.089	23.891	26.689	27.206	29.823	31.123	31.057	31.714	30.746	29.820	27.209	24.140

The normal equations of the linear curve described by the equation (2.1) in this case become

$$336.507 = 12a + 144.131b$$

$$\& \ 4074.145399 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of *a* and *b* respectively as

$$\widehat{a} = 2.043788388 \text{ and } \quad \widehat{b} = 2.164569311$$

Thus, the linear curve fitted to the data in **Table – 3.2.1** becomes

$$Y_i = 2.043788388 + 2.164569311X_i \qquad (3.5)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 2.164569311\, \Delta X_i \qquad (3.6)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.5) and (3.6) have been presented in the **Table- 3.2.2.**

Table – 3.2.2

X_i	Y_i	$\widehat{Y}_{(B)}$ =2.043788388 +2.164569311X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 2.164569311 ΔX_i	$\left\|\widehat{e}_{(B)}\right\| = \left\|Y_i - \widehat{Y}_{(B)}\right\|$	$\left\|\widehat{e}_{(BM)}\right\| = \left\|Y_i - \widehat{Y}_{i+1(BM)}\right\|$
10.553	23.089	24.88648833	-	1.79748833	-
11.105	23.891	26.08133059	24.28384226	2.19033059	0.39284226
11.834	26.689	27.65930161	25.46897103	0.97030161	1.22002897
12.610	27.206	29.33900740	28.36870579	2.13300740	1.16270579
13.266	29.823	30.75896487	28.62595747	0.93596487	1.19704253
13.605	31.123	31.49275386	32.32656931	0.36975386	1.20356931
13.469	31.057	31.19837244	30.82861857	0.14137244	0.22838143
12.921	31.714	30.01218846	29.87081602	1.70181154	1.84318398
12.191	30.746	28.43205286	30.13386440	2.31394714	0.61213560
11.424	29.820	26.77182820	29.08577534	3.04817180	0.73422466
10.755	27.209	25.32373133	28.37190313	1.88526867	1.16290313
10.398	24.140	24.55098008	26.43624876	0.41098008	2.29624876

Mean of the absolute deviations in column 5 = 1.463719091

& Mean of the absolute deviations in column 6 = 1.095751493.

(B) Estimation of Mean Minimum Temperature (Dibrugarh)

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.2.3.**

Table: 3.2.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	9.374	12.434	16.089	18.914	21.951	24.277	24.746	25.034	23.963	20.843	14.983	10.229

The normal equations of the linear curve described by the equation (2.1) in this case become

$$222.837 = 12a + 144.131b$$

$$\& \ 2741.911569 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of a and b respectively as

$$\widehat{a} = -33.95633099 \quad \text{and} \quad \widehat{b} = 4.373195023$$

Thus, the linear curve fitted to the data in **Table – 3.2.3** becomes

$$Y_i = -33.95633099 + 4.373195023\, X_i \qquad (3.7)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 4.373195023\, \Delta X_i \qquad (3.8)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.7) and (3.8) have been presented in the **Table- 3.2.4.**

Table: 3.2.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -33.95633099 +4.373195023 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 4.373195023 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	9.374	12.19399609	-	2.81999609	-
11.105	12.434	14.60799974	11.78800365	2.17399974	0.64599635
11.834	16.089	17.79605891	15.62205917	1.70705891	0.46694083
12.610	18.914	21.18965825	19.48259934	2.27565825	0.56859934
13.266	21.951	24.05847419	21.78281594	2.10747419	0.16818406
13.605	24.277	25.54098730	23.43351311	1.26398730	0.84348689
13.469	24.746	24.94623277	23.68224548	0.20023277	1.06375452
12.921	25.034	22.54972190	22.34948913	2.48427810	2.68451087
12.191	23.963	19.35728954	21.84156763	4.60571046	2.12143237
11.424	20.843	16.00304895	20.60875942	4.83995105	0.23424058
10.755	14.983	13.07738148	17.91733253	1.90561852	2.93433253
10.398	10.229	11.51615086	13.42176938	1.28715086	3.19276938

Mean of the absolute deviations in column 5 = 2.305926353

& Mean of the absolute deviations in column 6 = 1.356749793

3.3. Monthly Mean Extremum Temperature at Dhubri:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.3.1.**

Table: 3.3.1.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	22.362	25.748	28.486	30.605	30.900	30.932	31.031	31.696	31.027	29.459	26.486	23.276

The normal equations of the linear curve described by the equation (2.1) in this case become

$$342.008 = 12a + 144.131b$$

$$\& \ 4144.870466 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of *a* and *b* respectively as

$$\widehat{a} = -1.232873987 \quad \text{and} \quad \widehat{b} = 2.475542998$$

Thus, the linear curve fitted to the data in **Table – 3.3.1** becomes

$$Y_i = -1.232873987 + 2.475542998\, X_i \qquad (3.9)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 2.475542998\ \Delta X_i \qquad (3.10)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.9) and (3.10) have been presented in the **Table- 3.3.2**.

Table: 3.3.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = -1.232873987 +2.475542998 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 2.475542998 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	22.362	24.89153127	-	2.52953127	-
11.105	25.748	26.25803101	23.72849973	0.51003101	2.01950027
11.834	28.486	28.06270186	27.55267085	0.42329814	0.93332915
12.610	30.605	29.98372322	30.40702137	0.62127678	0.19797863
13.266	30.900	31.60767943	32.22895621	0.70767943	1.32895621
13.605	30.932	32.44688851	31.73920908	1.51488851	0.80720908
13.469	31.031	32.11021466	30.59532615	1.07921466	0.43567385
12.921	31.696	30.75361710	29.67440244	0.94238290	2.02159756
12.191	31.027	28.94647071	29.88885361	2.08052929	1.13814639
11.424	29.459	27.04772923	29.12825852	2.41127077	0.33074148
10.755	26.486	25.39159096	27.80286173	1.09440904	1.31686173
10.398	23.276	24.50782211	25.60223115	1.23182211	2.32623115

Mean of the absolute deviations in column 5 = 1.262194493

& Mean of the absolute deviations in column 6 = 1.168747773

(B) Estimation of Mean Minimum Temperature (Dhubri).

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.3.3.**

Table: 3.3.3.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	11.938	14.281	18.105	21.600	23.010	24.668	25.568	25.738	24.982	22.645	18.214	13.619

The normal equations of the linear curve described by the equation (2.1) in this case become

$$244.368 = 12a + 144.131b$$

$$\& \; 2989.751653 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of a and b respectively as

$$\widehat{a} = -23.51919938 \quad \text{and} \quad \widehat{b} = 3.653609512$$

Thus, the linear curve fitted to the data in **Table – 3.3.3** becomes

$$Y_i = -23.51919938 + 3.653609512X_i \qquad (3.11)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 3.653609512\, \Delta X_i \qquad (3.12)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.11) and (3.12) have been presented in the **Table- 3.3.4.**

Table – 3.3.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -23.51919938 +3.653609512 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 3.653609512 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	11.938	15.0373418	-	3.09934180	-
11.105	14.281	17.05413425	13.95479245	2.77313425	0.32620755
11.834	18.105	19.71761559	16.94448133	1.61261559	1.16051867
12.610	21.600	22.55281657	20.94020098	0.95281657	0.65979902
13.266	23.010	24.94958441	23.99676784	1.93958441	0.98676784
13.605	24.668	26.18815803	24.24857362	1.52015803	0.41942638
13.469	25.568	25.69126714	24.17110911	0.12326714	1.39689089
12.921	25.738	23.68908912	23.56582199	2.04891088	2.17217801
12.191	24.982	21.02195418	23.07086506	3.96004582	1.91113494
11.424	22.645	18.21963569	22.1796815	4.42536431	0.46531850
10.755	18.214	15.77537092	20.20073524	2.43862908	1.98673524
10.398	13.619	14.47103233	16.9096614	0.85203233	3.29066140

Mean of the absolute deviations in column 5 = 2.145491684

& Mean of the absolute deviations in column 6 = 1.343239858

3.4. Monthly Mean Extremum Temperature at Silchar:

(A) Estimation of Mean Maximum Temperature

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table: 3.4.1.**

Table: 3.4.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	24.548	26.692	29.904	30.607	31.093	31.582	31.629	32.104	31.715	30.965	28.688	25.962

The normal equations of the linear curve described by the equation (2.1) in this case become

$$355.489 = 12a + 144.131b$$

$$\& \; 4297.161584 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of a and b respectively as

$$\widehat{a} = 7.619180167 \quad \text{and} \quad \widehat{b} = 1.832075251$$

Thus, the linear curve fitted to the data in **Table – 3.4.1** becomes

$$Y_i = 7.619180167 \quad + 1.832075251X_i \qquad (3.13)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i \quad + 1.832075251\, \Delta X_i \qquad (3.14)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.13) and (3.14) have been presented in the **Table- 3.4.2.**

Table: 3.4.2

X_i	Y_i	$\widehat{Y}_{(B)}$ = 7.619180167 +1.832075251 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 1.832075251 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	24.548	26.95307029	-	2.40507029	-
11.105	26.692	27.96437583	25.55930554	1.27237583	1.13269446
11.834	29.904	29.29995869	28.02758286	0.60404131	1.87641714
12.610	30.607	30.72164908	31.32569039	0.11464908	0.71869039
13.266	31.093	31.92349045	31.80884136	0.83049045	0.71584136
13.605	31.582	32.54456396	31.71407351	0.96256396	0.13207351
13.469	31.629	32.29540172	31.33283777	0.66640172	0.29616223
12.921	32.104	31.29142449	30.62502276	0.81257551	1.47897724
12.191	31.715	29.95400955	30.76658507	1.76099045	0.94841493
11.424	30.965	28.54880783	30.30979828	2.41619217	0.65520172
10.755	28.688	27.32314949	29.73934166	1.36485051	1.05134166
10.398	25.962	26.66909863	28.03394914	0.70709863	2.07194914

Mean of the absolute deviations in column 5 = 1.159774993

& Mean of the absolute deviations in column 6 = 1.007069435

(B) Estimation of Mean Minimum Temperature (Silchar):

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.4.3.**

Table - 3.4.3

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	12.112	13.980	17.600	21.139	23.164	24.761	25.200	25.267	24.811	23.172	18.152	13.728

The normal equations of the linear curve described by the equation (2.1) in this case become

$$243.086 = 12a + 144.131b$$

$$\& \; 2973.124095 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of a and b respectively as

$$\widehat{a} = -22.63903504 \quad \text{and} \quad \widehat{b} = 3.571434462$$

Thus, the linear curve fitted to the data in **Table – 3.4.3** becomes

$$Y_i = -22.63903504 + 3.571434462\, X_i \qquad (3.15)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 3.571434462\, \Delta X_i \qquad (3.16)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.15) and (3.16) have been presented in the **Table- 3.4.4.**

Table- 3.4.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -22.63903504 +3.571434462 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 3.571434462 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	12.112	15.05031284	-	2.93831284	-
11.105	13.980	17.02174466	14.08343182	3.04174466	0.10343182
11.834	17.600	19.62532038	16.58357572	2.02532038	1.01642428
12.610	21.139	22.39675353	20.37143314	1.25775353	0.76756686
13.266	23.164	24.73961453	23.48186101	1.57561453	0.31786101
13.605	24.761	25.95033082	24.37471628	1.18933082	0.38628372
13.469	25.200	25.46461573	24.27528491	0.26461573	0.92471509
12.921	25.267	23.50746964	23.24285391	1.75953036	2.02414609
12.191	24.811	20.90032249	22.65985284	3.91067751	2.15114716
11.424	23.172	18.16103225	22.07170977	5.01096775	1.10029023
10.755	18.152	15.7717426	20.78271034	2.38025740	2.63071034
10.398	13.728	14.4967405	16.8769979	0.7687405	3.1489979

Mean of the absolute deviations in column 5 = 2.176905501

& Mean of the absolute deviations in column 6 = 1.324688591

3.5. Monthly Mean Extremum Temperature at Tezpur:

(A) Estimation of Mean Maximum Temperature:

The observed values of the mean maximum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table: 3.5.1.**

Table: 3.5.1

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.559	25.976	29.397	30.015	30.895	31.867	31.900	32.197	31.587	30.680	28.195	24.716

The normal equations of the linear curve described by the equation (2.1) in this case become

$$350.984 = 12a + 144.131b$$

$$\& \; 4248.336627 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of *a* and *b* respectively as

$$\widehat{a} = 3.002012944 \quad \text{and} \quad \widehat{b} = 2.185233188$$

Thus, the linear curve fitted to the data in **Table – 3.5.1** becomes

$$Y_i = 3.002012944 + 2.185233188\, X_i \qquad (3.17)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 2.185233188\, \Delta X_i \qquad (3.18)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.17) and (3.18) have been presented in the **Table- 3.5.2**

Table- 3.5.2

X_i	Y_i	$\widehat{Y}_{(B)}$ $=3.002012944 + 2.185233188X_i$	$\widehat{Y}_{i+1(BM)} =$ $Y_i + 2.185233188\, \Delta X_i$	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	23.559	26.06277878	-	2.50377878	-
11.105	25.976	27.26902750	24.76524872	1.29302750	1.21075128
11.834	29.397	28.86206249	27.56903499	0.53493751	1.82796501
12.610	30.015	30.55780344	31.09274095	0.54280344	1.07774095
13.266	30.895	31.99131642	31.44851297	1.09631642	0.55351297
13.605	31.867	32.73211047	31.63579405	0.86511047	0.23120595
13.469	31.900	32.43491875	31.56980829	0.053491875	0.33019171
12.921	32.197	31.23741097	30.70249221	0.95958903	1.49450779
12.191	31.587	29.64219074	30.60177977	1.94480926	0.98522023
11.424	30.680	27.96611688	29.91092614	2.71388312	0.76907386
10.755	28.195	26.50419588	29.21807900	1.69080412	1.02307900
10.398	24.716	25.72406763	27.441487175	1.00806763	2.69887175

Mean of the absolute deviations in column 5 = 1.307337169

& Mean of the absolute deviations in column 6 = 1.109283682

(B) Estimation of Mean Minimum Temperature (Tezpur):

The observed values of the mean minimum temperature (Y_i) corresponding to the average length of day (X_i) for each of the 12 months based on the data from the year 1969 to the year 2010 have been shown in the **Table - 3.5.3.**

Table: 3.5.3.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	11.424	13.651	17.075	19.828	22.377	24.603	25.132	25.264	24.497	21.665	16.808	12.610

The normal equations of the linear curve described by the equation (2.1) in this case become

$$234.934 = 12a + 144.131b$$

$$\& \ 2878.797313 = 144.131\, a + 1746.108159\, b$$

which yield the least squares estimates $\widehat{a}$ and $\widehat{b}$ of *a* and *b* respectively as

$$\widehat{a} = -26.19710002 \quad \text{and} \quad \widehat{b} = 3.811110727$$

Thus, the linear curve fitted to the data in **Table – 3.5.3** becomes

$$Y_i = -26.19710002 + 3.811110727 X_i \qquad (3.19)$$

Also, the recurrence relationship between Y_{i+1} and Y_i becomes

$$Y_{i+1} = Y_i + 3.811110727\, \Delta X_i \qquad (3.20)$$

Estimated values of Y_i obtained by both of the fitted relationship described by the equations (3.19) and (3.20) have been presented in the **Table- 3.5.4**

Table- 3.5.4

X_i	Y_i	$\widehat{Y}_{(B)}$ = -26.19710002 +3.811110727 X_i	$\widehat{Y}_{i+1(BM)}$ = Y_i+ 3.811110727 ΔX_i	$\lvert\widehat{e}_{(B)}\rvert = \lvert Y_i - \widehat{Y}_{(B)}\rvert$	$\lvert\widehat{e}_{(BM)}\rvert = \lvert Y_i - \widehat{Y}_{i+1(BM)}\rvert$
10.553	11.424	14.02155148	-	2.59755148	-
11.105	13.651	16.1252846	13.52773312	2.47428460	0.12326688
11.834	17.075	18.90358432	16.42929972	1.82858432	0.64570028
12.610	19.828	21.86100625	20.03242192	2.03300625	0.20442192
13.266	22.377	24.36109488	22.32808864	1.98409488	0.04891136
13.605	24.603	25.65306142	23.66896654	1.05006142	0.93403346
13.469	25.132	25.13475036	24.08468894	0.00275036	1.04731106
12.921	25.264	23.04626168	23.04351132	2.21773832	2.22088680
12.191	24.497	20.26415085	22.48188917	4.23284915	2.01511083
11.424	21.665	17.34102893	21.57387807	4.32397107	0.09112193
10.755	16.808	14.79139585	19.11536692	2.01660415	2.30736692
10.398	12.610	13.43082932	15.44743347	0.82082932	2.83743347

Mean of the absolute deviations in column 5 = 2.131860443

& Mean of the absolute deviations in column 6 = 1.134142265

IV. CONCLUSION:

In the fitting of linear regression of monthly mean extremum (maximum and minimum) temperature on the monthly average length of day at the five cities in Assam, the computed values of the mean absolute deviations of the estimated values from the respective observed values, obtained in both types of fitting (namely fitting by Basian principle and fitting by Basian-Markovian principle) have been shown in the following table (**Table: 4.1 and Table: 4.2**).

Table: 4.1- Maximum Temperature

City	Mean of absolute deviation		Comparison
	Basian Principle	Basian – Markovian Principle	
Guwahati	1.255949775	1.155245155	Basian > Basian - Markovian
Dibrugarh	1.463719091	1.095751493	Basian > Basian - Markovian
Dhubri	1.262194493	1.168747773	Basian > Basian - Markovian
Silchar	1.159774993	1.007069435	Basian > Basian - Markovian
Tezpur	1.307337169	1.109283682	Basian > Basian - Markovian

Table: 4.2- Minimum Temperature

City	Mean of absolute deviation		Comparison
	Basian Principle	Basian – Markovian Principle	
Guwahati	2.214837646	1.589418986	Basian > Basian - Markovian
Dibrugarh	2.305926353	1.356749793	Basian > Basian - Markovian
Dhubri	2.145491684	1.343239858	Basian > Basian - Markovian
Silchar	2.176905501	1.324688591	Basian > Basian - Markovian
Tezpur	2.131860443	1.134142265	Basian > Basian - Markovian

It is found that the mean absolute deviation of estimated values from the respective observed values in the case of fitting by Basian-Markovian principle is less than the corresponding mean absolute deviation in the case of fitting by Basian principle in the case of both monthly mean maximum and monthly mean minimum temperature at all the five cities of Assam. Thus one can conclude that the application of Basian-Markovian principle in fitting of linear regression of monthly mean extremum temperature on the monthly average length of day is more accurate than the fitting of the same by applying the Basian principle.

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