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Numerical Analysis of Wave Function controlled by (1+1)-Dimensions External Trapping Potentials in BEC Experiments

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----------**ABSTRACT**----------
This paper will focus on numerical analysis and the behavior of wave function in Bose-Einstein condensation (BEC) controlled by (1+1) external trapping potentials which are usually used in experiments that lead to produced BEC in ultra cold gases. Two mixed types of trapping potentials are used in this analysis. The first one is a harmonic oscillator potential (HOP) assume to be applied parrale to the propagation axis, and the second is optical lattice potential OLP assume to be applied normal to the propagation axis. The cases of slowly and rapidly varying in anisotropy term for HOP are considered. Although these numerical analyses give us the overall view of the region of confinement that the external trapping potentials have employed but also shows that the anisotropy term in the mathematical formula of HOP play a major part in term of values and shape of the wave function of condensation in a confinement region. The nonlinearity term in this analysis is kept constant, while the time interval is of order of 0.0002, with space step is of order of 0.0025. Both the accuracy and the stability of this solution are remarkable.

KEYWORDS: *Laser cooled atom, BEC atom, Trapping, Atom laser, Quantum Oscillator*

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I. INTRODUCTION

A simplest example for the realization of Bose-Einstein Condensation (BEC) is an ideal gas which is consisting of non-interaction Bose particles. This system consider as a fictitious system since every realistic Bose gas shows some level of particle-particle interaction. This simple model, first studied theoretically by A. Einstein [1], correctly describes important basic properties of actual non-ideal (interacting) Bose gas. Bose-Einstein condensation then has become a widely studied research topic among physicists and applied mathematicians since its first experimental realization of (BEC) in ultra cold atomic gases by a sequence of experiments in 1995 by Anderson *et al*. (vapor of rubidium) [2], and Davis *et al*. (vapor of sodium) [3], In these experiments the atoms were confined in magnetic traps and cooled down to low temperatures at an order of Micro-kelvins. For the detail discussions see also [4-6]. After the recognition of Bose-Einstein condensates in dilute quantum gases a great deal of attention to the dynamics of nonlinear excitations in matter waves, such as dark [7,8] and bright solitons [9], vortices [10-15], super-vortices [16], was strained. In this paper, we solve numerically the Gross-Pitaevskii equation and then analyze the effect anisotropy of the gas on wave function in Bose-Einstein condensation which is controlled by (1+1) dimensions harmonic oscillator potential propagates along x-axis plus optical lattice potential propagates along the y-axis.

II. THEORY

Bose-Einstein condensate, composed of Bosonic atoms all in the same quantum state, behaves very much like a classical electromagnetic field obeying Maxwell's equations arises as an assembly of photons all in the same quantum state. The equation of motion of the Bosonic atoms is the Gross-Pitaevskii equation

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+\frac{4\pi\hbar^2 a}{m}|\Psi(r,t)|^2\right]\Psi(r,t) \qquad (1)$$

It is worth to mention that, there are phenomena in which the quantized nature of this field is important—for example, when two Bose-Einstein condensates collide at a sufficiently high velocity, a *halo* of elastically scattered atoms is produced [17,18]. This phenomenon is a direct effect of the fact that the quantized field consists of interacting particles. Unfortunately the Gross-Pitaevskii equation with initial conditions corresponding to two Bose-Einstein condensates does not predict this scattering. To treat this phenomenon theoretically we have two choices. In the first, the Gross-Pitaevskii equations for the two condensate wave packets are modified (either phenomenologically [19], or on the basis of a method of approximating quantum

field theory [20]) to give an elastic scattering loss term. While these methods yield equations of motion which allow for depletion of the condensate wave functions, they do not include a description of the scattered atoms, and hence cannot describe the effects of bosonically stimulated loss. In the second class of treatments [21] the quantum field theory is linearized about the condensate, yielding equations of motion linear in the fluctuation operators. This method shows that the process is essentially one of four-wave mixing between the two condensate fields and pairs of quantized fluctuations—however, as a linearized theory; it can deal only with perturbatively small amounts of scattering and cannot simultaneously account for depletion of the condensate. Both of these formalisms are valid only in the limit of weak scattering, but fail for large scattered fractions. Moreover the phenomenon of super-fluidity requires that there be interactions between the particles.. Super-fluidity can be defined in a number of ways, but perhaps the most powerful is the connection of super-fluidity with the presence of an order parameter or macroscopic wave function which describes all the super-fluid particles. As in any scalar field of complex numbers, a macroscopic wave function ψ(r) contains precisely two degrees of freedom at each point in space, which may be interpreted in terms of the local super-fluid density n(r) and the local super-fluid phase φ(r), that is,

$$\psi(r) = \sqrt{n(r)}\exp\,(i\varphi(r)) \tag{2}$$

Since the global phase of a wave function is not measurable and plays no physical role, it is only the variation in space of the super-fluid phase that is of physical significance. To make this clear, we point out that for the simple situation of a de Broglie matter wave of a free particle of mass m with wave vector $\boldsymbol{k}$, the wave function is simply a plane wave proportional to exp(i$\boldsymbol{k}$.$\boldsymbol{r}$). In this case it is the gradient of the quantum mechanical phase multiplied by $\hbar/m$ which leads to the particle's velocity, that is, $\hbar\boldsymbol{k}/m$. We define the super-fluid velocity in analogous manner from the gradient in the phase of the macroscopic wave function $v = \hbar/m\nabla\varphi$. In order to solve equation (1) numerically along the X-Axis one can rewrite it in more convenient form [22-24].

$$\frac{\partial\psi(x,t)}{\partial t} = \frac{i\hbar}{2m}\nabla^2\psi(x,t) - \frac{1}{\hbar}V(\boldsymbol{r})\psi(x,t) - \frac{1}{\hbar}g|\psi(x,t)|^2\psi(x,t) \tag{3}$$

The Crank-Nicolson Scheme for equation (3) is:

$$\frac{\psi_i^{n+1}-\psi_i^n}{k} = \frac{i\hbar}{2m\hbar^2}[\psi_{i=1}^{n+1} - 2\psi_i^{n+1} + \psi_{i-1}^{n+1} - 2\psi_i^n + \psi_{i-1}^n] - \frac{1}{\hbar}V_i^n\psi_i^n - \frac{1}{\hbar}g|\psi_i^n|^2\psi_i^n,\ i = 1,2,3,\ldots \tag{4}$$

Where *k* is the time interval and *h* is the space step. This scheme is unconditionally stable, time reversible, conserve the total particle number but it is not time transverse-invariant. A compares tests with fully implicit and fully explicit finite difference methods are carried out but not include in this paper. Reader can refer to references [25], and [26] for a mathematical analysis of finite differences methods for Schrodinger equations in semi-classical regimes. In this work, we will analysis the wave function under the action of a typical optical lattice trapping potentials which are widely used in current experiments $V_{olpt}(x) = S_x E_x sin^2(\hat{q}_\tau x)$, where $\hat{q}_x = 2\pi/\lambda_x$ is the angular frequency of the laser beam, with wavelength λ_x, that creates the stationary 2D periodic lattice, $E_\tau=(\hbar^2\hat{q}_x^2)/2m$ is the recoil energy, and S_x is a dimensionless parameter characterizing the intensity of the laser beam. The optical lattice potential has periodicity $T_x=\pi/\hat{q}_x=\lambda_x/2$ along the x-axis. The choices for the scaling parameters t_0 and x_0, the dimensionless potential $V(x)$, the energy unit $E_0 = \hbar/t_0 \rightarrow E_0 = \hbar^2/mr_0^2$, and the interaction parameter $g = 4\pi a_s N/r_0$ for external optical lattice trapping potentials are reads as follow: $t_0 = 1/\omega_r$, $r_0 = \sqrt{\hbar/m\omega_r}$, $k_r = 2\pi^2 r_0^2 S_x/\lambda_x^2$, $q_x = 2\pi r_0/\lambda_x$.

III. RESULT AND DISCUSSION

It is worth to imagine that the atoms are tightly confined in two directions and can be successfully describe the wave function by one-dimension by Appling optical Lattice potentials over lapping the harmonic potential along the x-axia. The time interval used in this solution is 0.00020 and the space step is 0.002500 more over the non linearity term (G) is fixed initially to the value 12.5484. The most factors which affect this numerical solution are the stability since a constant amplification in one time step turns into an exponential amplification over time. In addition to this classical stability requirement, we would also like that the norm of the system is unchanged. In the present case this corresponds to conservation of the particle number and that the energy is unchanged. These considerations from the physical properties of the system some time do not fulfill the norm and energy preservation properties. The careful adjustments between the time interval and space step will reflect that the physical properties of this system is satisfied and the result of this numerical solution can be explained satisfactory.

The contour level of optical lattice potential over lapping harmonic oscillators for anisotropy value equal to 0.5 is shown in figure (1a). One can conclude from this figure that the potential takes the form of elliptical shape with zero potential at a centre of the distribution, the maximum potential localized at the edge of the working area as expected. The corresponding contour level of the wave function for the same value of anisotropy is shown in figure (1b). The maximum value of wave function is at the centre of the propagation field, and the distribution of the wave function is in harmony with the applied external trapping potentials. As the anisotropy of the working gas increases to 1.0 the distribution of the trapping potential change its shape from elliptical like shape to a circular shape as shown in figure (2a) this means that the two trapping potential exerts an equal forces on the condensation and the wave function of this condensation follow same circular distribution. Again the force applied at the centre of condensation due to the effects of external trapping potentials is zero and at the edge of the working field are maximum. The corresponding wave function has its maximum at the centre as seen in figure (2b). The concentric circles around the center of the propagations field in this figure are equi-potentials lines of constant potential energy. The radial lines are lines of steepest descent that depict the gradient of the potential energy surface. The slope of a radial line at any point on the plane is proportional to the force that a particle would experience there. Figures (3a) and (3b) shows the contour levels of the potential and the corresponding wave function of the condensation for the anisotropy value of 1.5. It is clear from this figure that the force exerts by the harmonic trapping potential along the X-axis is much higher than the force exert by the optical lattice potential along the Y-axis, however the result of this differences of force that the concentrated circles for both the potentials and the wave function are pulled down toward the X-Axis. Finally in order to understand the bahaviour of the energy, chemical potentials and wave function with the anisotropy one can look to figures (4), and figure (5). There are almost linear relationships between the energy, chemical potential from one side with the anisotropy on the other side, this linearity become more clear at higher values of anisotropy. This picture is not applied to the wave function where the exponential increases of the wave function with the anisotropy is clearer than the expected from this solution. This result make one much confidence a bout the stability and the accuracy of our numerical solution that employed in this analysis.

IV. CONCLUSION

Although the theories which describe the the wave function under the action of harmonic oscillator and optical lattice potential are working very well to some extent, but this result shows that care must be taken in to account in order to explain the experimental results more accurately. The limitation of different factor in theory need more study in parallel with the available of experimental data. The numerical solution for this study satisfied the stability and accuracy condition, this means we can extend our analysis to much more complicated cases in the next future.

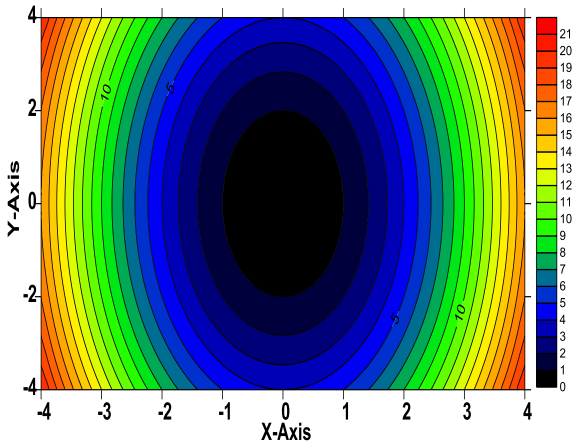

Figure (1a) The Contour Levels of the (1+1) Trapping Potentials G = 12.5484, anisotropy = 0.5

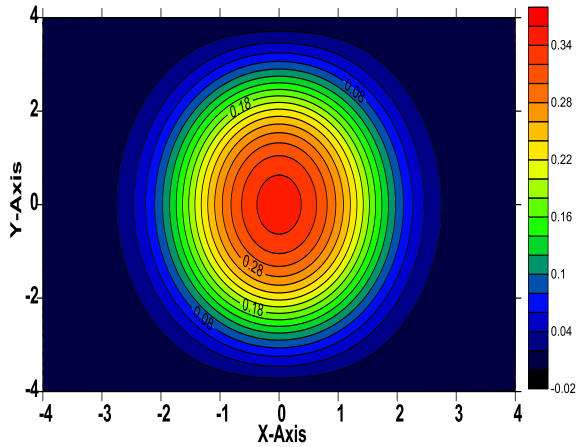

Figure (1b) The Contour Levels of the Wave Function G = 12.5484, anisotropy = 05

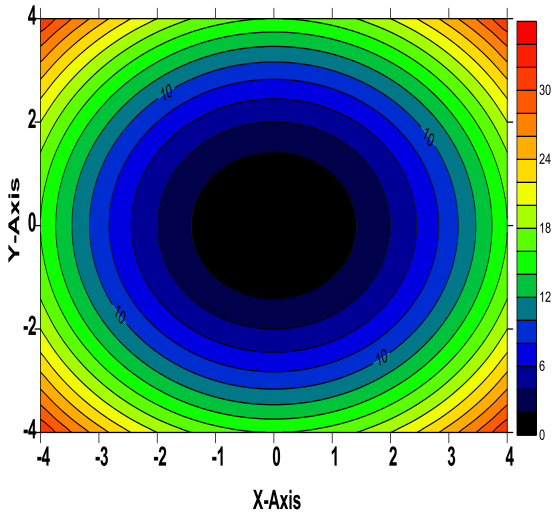

Figure (2a) The Contour Levels of the (1+1) Trapping Potentials G = 12.5484, anisotropy = 01

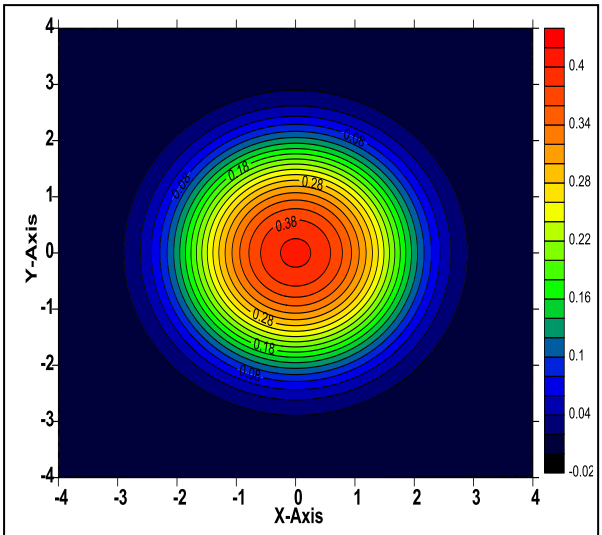

Figure (2b) The Contour Levels of the Wave Function G = 12.5484, anisotropy = 01

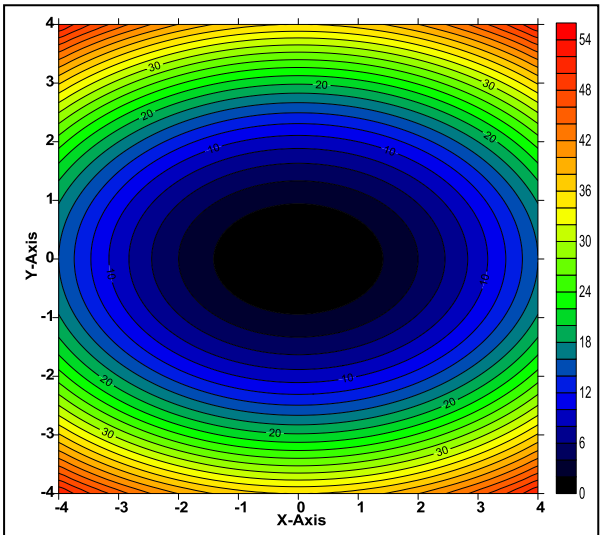

Figure (3a) The Contour Levels of the (1+1) Trapping Potentials G = 12.5484, anisotropy = 1.5

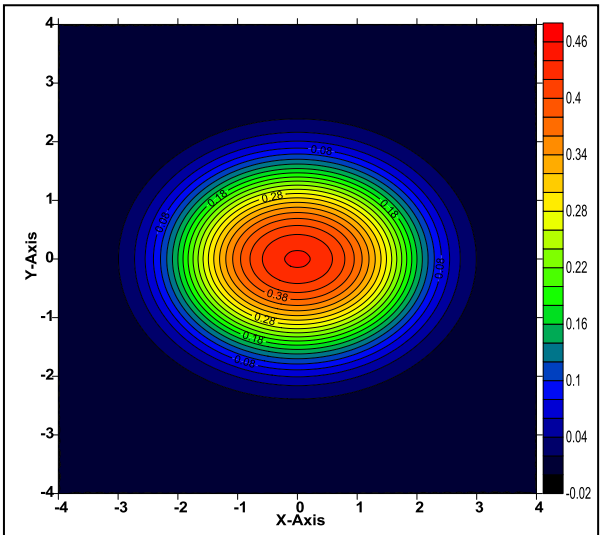

Figure (3b) The Contour Levels of the Wave Function G = 12.5484, anisotropy = 1.5

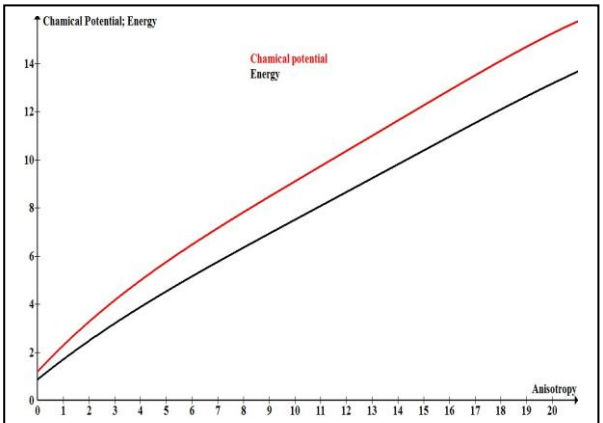

Figure (4) Chemical potential and Energy VS Anisotropy

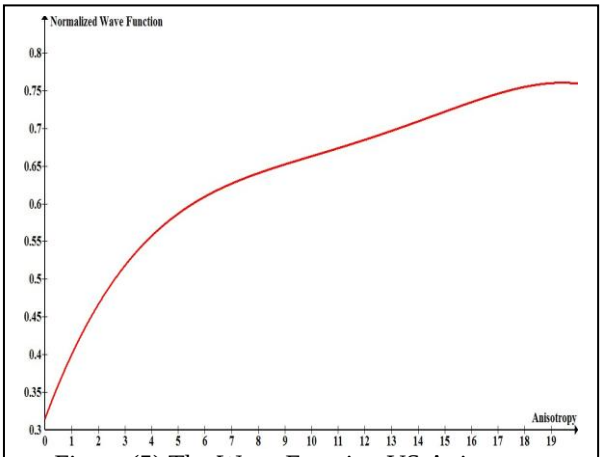

Figure (5) The Wave Function VS Anisotropy

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