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Anti-Synchronization of the Bullard and Rikitake Dynamo Systems via Nonlinear Active Control

Edwin A. Umoh, Jacob O. Ebozoje
[1,] *Department of Electrical Engineering Technology*
Federal Polytechnic Kaura Namoda, Nigeria
[2,] *Department of Science Laboratory Technology*
Federal Polytechnic Kaura Namoda, Nigeria

ABSTRACT
In this paper, we present the anti-synchronization of two chaos-exhibiting systems-Bullard and Rikitake dynamos using nonlinear active control techniques in a master-slave topology. Nonlinear active control laws were derived and added to the algebraic structure of the Bullard slave system and the Lyapunov stability criteria was applied to verify the negative definiteness of the error dynamics as a condition for antisynchronization of the two systems. Simulation results confirmed the effectiveness of the approach in coupling the dynamics of the systems.

Keywords - *Antisynchronization, Bullard, Chaos, Lyapunov stability, Rikitake*

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I. INTRODUCTION

During the past half of a century since the discovery of chaos in weather by the physicist, Edward Lorenz [1] and the experimental control of chaos by other physicists such as Ott et al [2] and Pyragas [3], intense research has continued to spurn out literature on the ubiquity of chaos in various natural and man-made systems such as economics [4], psychology [5], ecology [6], food [7], . Applications of chaos real life designs have continued to grow even as new chaotic systems have continued to be evolved through research over the years. Chaos is a phenomenon of nonlinear systems which are extremely sensitive to perturbation in their parametric structures leading to unpredictability in their future evolution. Chaos synchronization or anti-synchronization occurs when two dissipative chaotic systems are coupled such that, in spite of the exponential divergence of their nearby trajectories, synchrony or anti-synchrony is achieved in their chaotic behaviours as $t \to \infty$. Different methods have been developed to synchronize or anti-synchronize chaotic systems. These include Fourier series expansion [8], adaptive control [9], active control [10], fuzzy control [11], impulsive control [12], and sliding mode control [13] among others. A simplified architecture for anti-synchronization of two chaotic systems is depicted in Fig. 1. The nonlinear control laws synchronize or anti-synchronize the time series trajectories of the two coupled systems according to some laws governing the choice synchronization scheme where $x(0) = x_1(0), x_2(0), x_3(0) \neq y(0) = y_1(0), y_2(0), y_3(0)$. Synchronization has found application in secure communication and other systems while studies on anti-synchronization of chaos have been found to be useful in mitigating electrical power outage and system management [14]-[15].

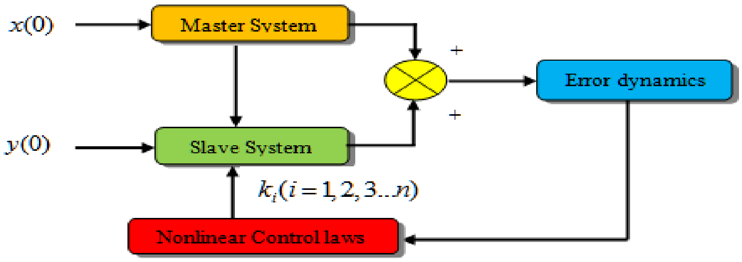


Figure 1: Simplified architecture for antisynchronization of two chaotic systems

II. DESCRIPTION OF THE RIKITAKE AND BULLARD DYNAMO SYSTEMS

The Rikitake system is is a simplified dynamic model which attempts to explain the irregular polarity switching of the earth's geomagnetic field. The physics of the Rikitake system has been studies by various authors [16]-[17], while other authors have studies synchronization of the system using various methods [18] - [19]. Mathematically, the system is represented by three coupled-differential equations given as follows:

$$\begin{aligned} x_1' &= x_2 x_3 - \psi x_1 \\ x_2' &= x_1 x_3 - \pi x_1 - \psi x_2 \\ x_3' &= 1 - x_1 x_2 \end{aligned}$$

(1)

Where x_1, x_2, x_3 are state variables, $\psi, \pi > 0$ are positive constants. For values of $\psi = 2,\ \pi = 3.4641$, the system evolves the 2-D phase portraits and state trajectories shown in Fig.2 and Fig. 3..

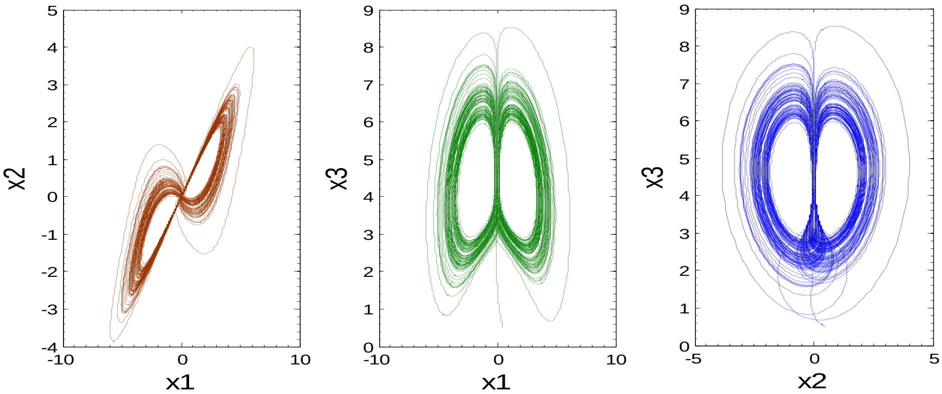


Figure 2. 2D Phase portraits of the Rikitake dynamo system

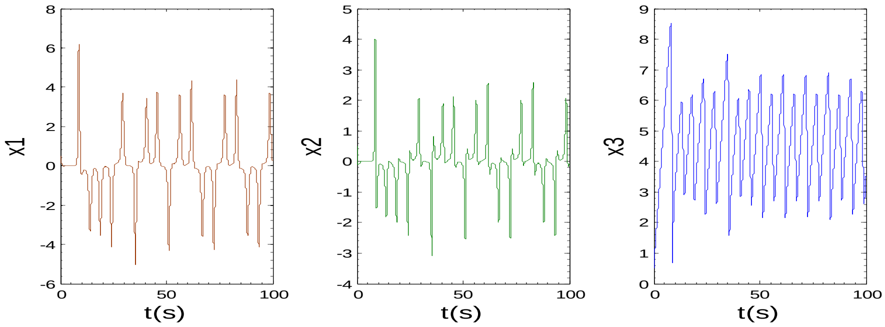


Figure 3. State trajectories of the Rikitake dynamo system

The Bullard dynamo system is also related to the dynamic problems associated with the earth's core and was first studied in detail by E.C. Bullard [20]. The physics of the system may be found in [21]-[22]. The algebraic structure of the system is represented by three coupled differential equations given as:

$$\begin{aligned} y_1' &= y_1(y_2 - 1) - \Theta y_3 \\ y_2' &= \xi - \gamma y_2 - \varphi y_1^2 \\ y_3' &= \Theta y_1 - \gamma y_3 \end{aligned}$$

(2)

Where y_1, y_2, y_3 are state variables, $\Theta, \xi, \gamma, \varphi > 0$ are positive constants. For values of $\Theta = 2,\ \xi = 20,\ \gamma = 2,\ \varphi = 4$, the system evolves the 2-D phase portraits and state trajectories shown in Fig.4 and Fig.5.

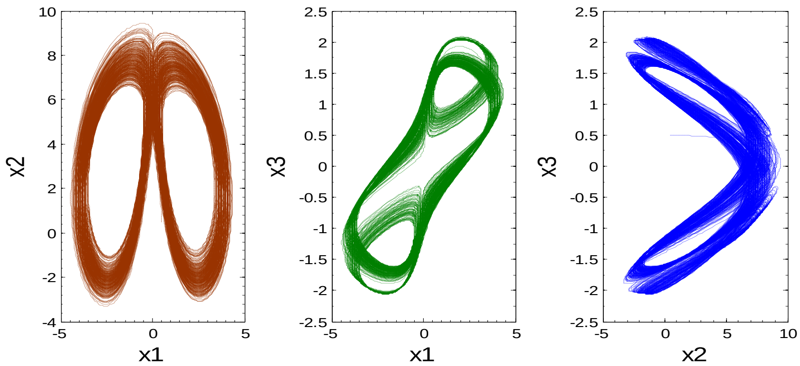


Figure 4: 2D Phase portraits of the Bullard dynamo system

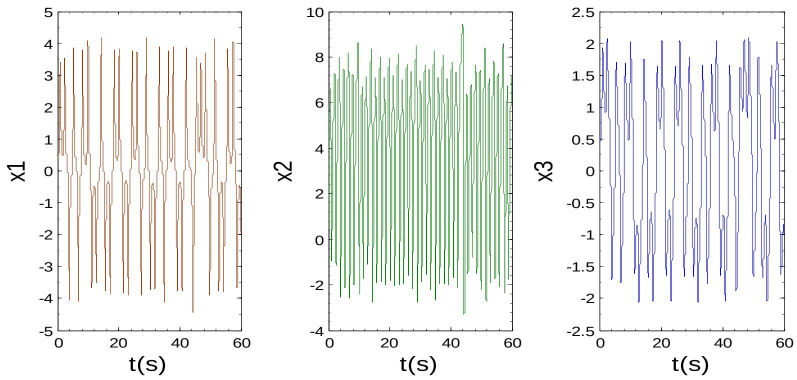


Figure 5: State trajectories of the Bullard dynamo system

III. ANTISYNCHRONIZATION OF THE RIKITAKE AND BULLARD SYSTEMS

In this section, the anti-synchronization problem is presented. While the design of nonlinear controllers for antisynchronization of chaotic system is not a new challenge, it is however interesting as it has continued to elicit interest from researchers due to its potential applications in engineering and non-engineering systems. It can therefore be safely said that every (anti)synchronizable chaotic system is a potential candidate for multi-dimensional applications in such fields as biomedical engineering, astronomy, economics and finance, automotive engineering, radar engineering amongst numerous other fields. Let eq. (1) be the master system and eq. (2) be the slave system. The controlled slave system can be represented as

$$\begin{aligned} y_1' &= y_1(y_2 - 1) - \Theta y_3 + u_1 \\ y_2' &= \xi - \gamma y_2 - \varphi y_1^2 + u_2 \\ y_3' &= \Theta y_1 - \gamma y_3 + u_3 \end{aligned}$$

(3)

Where u_1, u_2, u_3 are nonlinear controllers to be designed. Let the antisynchronization error be represented as

$$e = (e_1, e_2, \ldots e_n)^T = y + x$$

(4)

Where $x = (x_1, x_2, \ldots x_n)^T$, $y = (y_1, y_2, \ldots y_n)^T$ and

$e_1 = y_1 + x_1;\ e_2 = y_2 + x_2;\ e_3 = y_3 + x_3;\ e_n = y_n + x_n$. The objective of the study is to design the nonlinear active controllers $u_i,\ i = 1, 2, 3$ such that the trajectories y_1, y_2, y_3 of the slave system can come into anti-synchrony with trajectories x_1, x_2, x_3 of the master system, subject to different initial conditions $x(0) \neq y(0)$ such that

$$\lim_{t \to 0} \|e\| = \lim_{t \to 0} \|y(t, y(0)) + x(t, x(0))\| = 0$$

(5)

By adding (3) to (1) using the relation in (4), the error dynamics becomes

$$\begin{aligned} e_1' &= y_1 y_2 - y_1 - \Theta y_3 + x_2 x_3 - \psi x_1 + u_1 \\ e_2' &= \xi - \gamma y_2 - \varphi y_1^2 + x_1 x_3 - \pi x_1 - \psi x_2 + u_2 \\ e_3' &= \Theta y_1 - \gamma y_3 + 1 - x_1 x_2 + u_3 \end{aligned} \tag{6}$$

By simplifying (6) and using (4), the error dynamics becomes

$$\begin{aligned} e_1' &= y_1 y_2 - e_1 - \Theta e_3 + x_2 x_3 + (1 + \Theta - \psi) x_1 + u_1 \\ e_2' &= \xi - \gamma e_2 + \gamma x_2 - \varphi y_1^2 + x_1 x_3 - \pi e_1 + \pi y_1 - \psi e_2 + \psi y_2 + u_2 \\ e_3' &= \Theta e_1 - \Theta x_1 - \gamma e_3 + \gamma x_3 + 1 - x_1 x_2 + u_3 \end{aligned} \tag{7}$$

From (7), the nonlinear active control laws are given as

$$\begin{aligned} u_1 &= -y_1 y_2 + e_1 + \Theta e_3 - x_2 x_3 - (1 + \Theta - \psi) x_1 - \chi^1 \\ u_2 &= -\xi + \gamma e_2 - \gamma x_2 + \varphi y_1^2 - x_1 x_3 + \pi e_1 - \pi y_1 + \psi e_2 - \psi y_2 - \chi^2 \\ u_3 &= -\Theta e_1 + \Theta x_1 + \gamma e_3 - \gamma x_3 - 1 + x_1 x_2 - \chi^3 \end{aligned}$$

(8)

Where $\chi^i, i = 1, 2, 3$ is given as

$$\begin{bmatrix} \chi^1 \\ \chi^2 \\ \chi^3 \end{bmatrix} = \Delta \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

(9)

And Δ is a diagonal matrix whose diagonals elements $diag[\zeta_{11}, \zeta_{22}, \zeta_{33}]$ constitutes the feedback coefficients of the controllers, such that

$$\begin{bmatrix} \chi^1 \\ \chi^2 \\ \chi^3 \end{bmatrix} = \begin{bmatrix} \zeta_{11} & 0 & 0 \\ 0 & \zeta_{22} & 0 \\ 0 & 0 & \zeta_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

(10)

By inserting the elements of (10) in (8) and using (9), the error dynamics reduced to

$$e_1' = -\zeta_{11}e_1$$
$$e_2' = -\zeta_{22}e_2$$
$$e_3' = -\zeta_{33}e_3$$

(11)

The Lyapunov stability criteria is used to verify the asymptotic convergence of the error dynamics as $t \to \infty$.

We choose the Lyapunov function candidate $V(.)$ such that $V(.) > 0$; $\dot{V}(.) \le 0$. We choose the following candidate

$$V(e_1, e_2, e_3) = \frac{\gamma}{2\Theta}(e_1^2 + e_2^2 + e_3^2)$$

(12)

By using (11) in the partial derivative of (12), we have

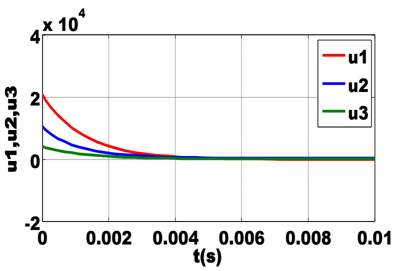
$$\dot{V}(e_1, e_2, e_3) = -\zeta_{11}e_1^2 - \zeta_{22}e_2^2 - \zeta_{33}e_3^2$$

(13)

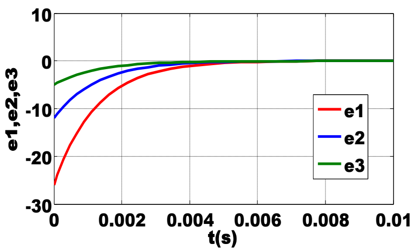
Which is Hurwitz for all $\zeta_{ij} > 0$. Consequently, the error dynamics will converge asymptotically to the origin as $t \to \infty$.

IV. SIMULATION RESULTS

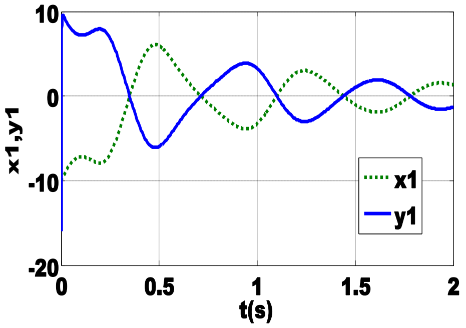
The Rikitake dynamo (1), Bullard dynamo (2) and the nonlinear control law (8) were simulated in MATLAB environment for the following parameters $\Theta = 2,\ \xi = 20,\ \gamma = 2,\ \varphi = 4$ and $\psi = 2,\ \pi = 3.4641$ for initial conditions $[x_1(0), x_2(0), x_3(0)] = [-10, -7, -4]$ and $[y_1(0), y_2(0), y_3(0)] = [-16, -5, -1]$. The initial conditions of the antisynchronization error dynamics becomes $[e_1(0), e_2(0), e_3(0)] = [-26, -12, -5]$. The resultant plots are given in the following figures.



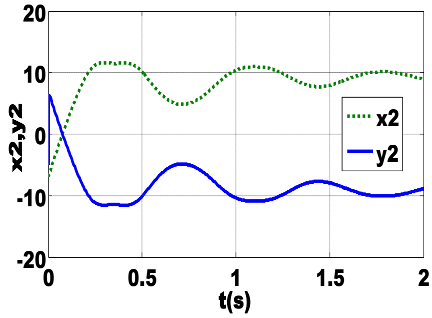
(a)



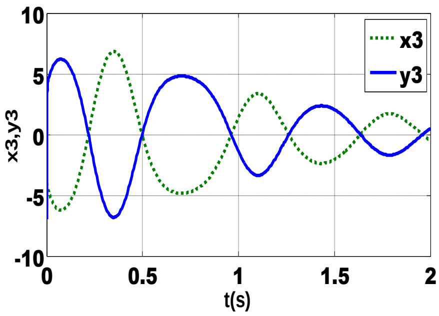
(b)



(c)



(d)



(e)

Figure 6: Simulated results of the antisynchronized systems - (a) plot of the asymptotically converged error dynamics; (b) Plot of the converged control laws; (c) Antisynchronized trajectories x1-y1; (d) Antisynchronized trajectories x2-y2 and (e) Antisynchronized trajectories x3-y3.

V. CONCLUSION

In this paper, the exponential divergent trajectories of the chaotic Bullard and Rikitake dynamos were antisynchronized using nonlinear active control laws. Lyapunov stability criterion was applied to the error dynamics inorder to test for asymptotic convergence. The partial derivative of the Lyapunov function candidate was Hurwitz, and as a result, the error dynamics and active control laws asymptotically converged in transient time. In power system engineering, antisynchronization has found usefulness in mitigation of power outage.

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