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A note on estimation of parameters of multinormal distribution with constraints

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---**ABSTRACT---**

Mardia et al (1989) considered the problem of estimating the parameters of nonsingular multivariate normal distribution with certain constraints. Nagmur (2003) considered the problem of estimating the mean sub-vector of non-sigular multivariate normal distribution with certain constraints. In this paper we try to estimate mean sub-vector under some different constraints and submatrix of ∑ with certain constraints for a nonsingular multivariate normal distribution.

Keywords: *Likelihood Function, Maximum Likelihood Estimator, Non-singular Multivariate Normal Distribution, Constraints.*

I. INTRODUCTION

Mardia et al (1989) considered the estimation of parameters of non-singular multivariate normal distribution with and without constraints. Nagnur (2003) tried to obtain the Mle of sub mean vector of the distribution which can be useful in some practical problems. Mardia et al (1989) considered two type of constraints on the mean vector μ . Estimator, Non-singular Multivariate Normal

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or of parameters of non-singular multivariate normal

or obtain the Mie of sub mean v *n* $x_1x_2x_3x_4x_5x_6x_7x_8x_9x_9x_9x_1x_1x_2x_3x_4x_5x_6x_7x_8x_9x_9x_9x_9x_1x_1x_2x_3x_4x_5x_6x_7x_8x_9x_1x_1x_2x_3x_4x_5x_1x_2x_3x_4x_4x_5x_1x_2x_3x_4x_4x_5x_1x_2x_3x_4x_4x_5x_1x_2x_3x_4x_4x_5x_1x_2x_2x_2x_3x_4x_4x$

i. $\mu = k\mu_0$ i.e. μ is known to be proportional to a known vector μ_0 . For example, the elements of *x* could represent a sample of repeated measurements, in which case $\mu = k1$

ii. Another type of constraint is $R\mu = r$, where R and *r* are pre-specified

The first type of constraint was considered by Nagnur (2003) for sub mean vector of the distribution. In this paper, we consider type of constraint for sub mean vector and give the explicit expression for the estimators. Mardia et al (1989) also considered constraint on variance-covariance matrix Σ , viz $\Sigma = k\Sigma 0$ where $\Sigma 0$ is known. We consider constraint on sub matrix of Σ and obtain its estimator with constraints.

2. ML Estimator of µ :

Let $\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n$ be n iid observation from $N_p(\underline{\mu}, \Sigma)$ population, where Σ is positive definite matrix. Suppose that x , \sim and Σ are partitioned as follows :

1 ² *^x x x* , ¹ ² , 11 12 ¹¹ : ^r ^x r, ²² : ^s ^x ^s with r + s = p. Let *^x* and 1 *n i i x x x ⁱ ^S ^x* **'**be the mean vector and SSP matrix based on the sample observations 1 2 , ,...........

SSP matrix is partitioned as

A note on estimation of parameters of multinormal distribution with constra
\n
$$
\overline{\underline{x}} = \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \end{pmatrix}
$$
\n
$$
S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}
$$
\n
$$
S_{11} : r \times r, S_{22} : s \times s \text{ with } r + s = p.
$$

where S_{11} : r x r , S_{22} : s x s , with $r + s = p$.

A note on estimation of parameters of multinormal distribution with constraints
 $\begin{pmatrix} \frac{1}{x_1} \\ \frac{1}{x_2} \end{pmatrix}$ $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$
 $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$
 $S = \begin{pmatrix} \frac{1}{x_1} & \frac{1}{$ A note on estimation of parameters of multinormal distribution with constraints
 $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$
 $S \cdot S_{22}$: $S \times S$, with $r + s = p$.

is to estimate $\mu : r \times 1$ under the constraint $R\mu_1 = r$, where Following

Figure 11 S_{12}
 S_{21}
 S_{22}
 S_{21}
 \rightarrow $\left[\begin{array}{ccc} 11 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 &$ of parameters of multinormal distribution with constraints
 $= \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$

the constraint $R\mu_1 = r$, where R and r are pre-specified.

Instraint may be achieved by augmenting the log likelihood wi Ford parameters of multinormal distribution with constraints
 $\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$
 \therefore constraint $R\mu_1 = r$, where R and r are pre-specified.

traint may be achieved by augmenting the log likelihood Our problem is to estimate μ : $r \times 1$ under the constraint $R\underline{\mu}_1 = \underline{r}$, where R and \underline{r} are pre-specified. Maximizing the log likelihood subject to this constraint may be achieved by augmenting the log likelihood with a Lagrangian expression, i.e. we maximize A note on estimation of parameters of multinormal distribution with co

1

1

1

2
 $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$
 S_{22} : s x s, with r + s = p.

0 estimate μ : r × 1 under the constraint R μ ₁ = r , wher

$$
L^+ = L - n \underline{\lambda}^{\prime} (R \underline{\mu} - \underline{r}) \tag{2.1}
$$

where λ is a vector of Lagrangian multipliers and L is given by

$$
L = -\frac{np}{2}\log 2\pi - \frac{n}{2}\log |\Sigma| - \frac{1}{2}tr\Sigma^{-1} S - \frac{n}{2}\left(\frac{\overline{x}}{2} - \underline{\mu}\right) S^{-1} \left(\frac{\overline{x}}{2} - \underline{\mu}\right)
$$
 (2.2)

Case-1 (Known)

A note on estimation of parameters of multinormal distribution with constraints
 $\overline{x} = \left(\frac{\overline{x}_1}{\underline{x}_2}\right)$ $S = \left(\frac{S_{11} - S_{12}}{S_{22}}\right)$
 $\left(\frac{1}{\underline{x}_2}\right)$ $S = \left(\frac{S_{11} - S_{12}}{S_{22}}\right)$
 $\left(\frac{1}{\underline{x}_2}\right)$, $\left(\frac{1}{\underline{x}_2$ With Σ assumed to be known, to find m.l.e.'s of $\sum_{i=1}^{\infty}$ we are required to find λ for which the solution to $\partial L^+_{\mathcal{O}_{-1}} = \underline{0}$ satisfies the constraint R $\underline{\sim}_1 = \underline{r}$.

'

Observe that L^+ can be expressed as

$$
\overline{\underline{x}} = \left(\frac{\overline{x}_1}{\underline{x}_2}\right)
$$
\n
$$
\overline{\underline{x}} = \left(\frac{\overline{x}_1}{\underline{x}_2}\right)
$$
\nwhere $S_{11} : r \times r$, $S_{22} : s \times s$, with $r + s = p$.
\nOur problem is to estimate $\underline{\mu} : r \times 1$ under the constraint $R_{1k_1} = r$, where R and r are pre-specified.
\nMaximuming the log likelihood subject to this constraint may be achieved by augmenting the log likelihood with a Lagrangian expression, i.e. we maximize
\n
$$
\Gamma' = \Gamma - n \lambda' (R\mu - r)
$$
\nwhere λ is a vector of Lagrangian multipliers and L is given by
\n
$$
L = -\frac{np}{2} \log 2\pi - \frac{n}{2} \log |\Sigma| - \frac{1}{2} tr \Sigma^{-1} S - \frac{n}{2} \left(\frac{x}{2} - \underline{\mu}\right) \Sigma^{-1} \left(\overline{x} - \underline{\mu}\right)
$$
\nCase-1 (d Know)
\nWith Σ assumed to be known, to find m le.³ of \underline{z}_1 we are required to find λ for which the solution to
\n
$$
\frac{\partial L'}{\partial z_{-1}} = 0
$$
 satisfies the constraint $R_{-1} = r$.
\nObserve that L' can be expressed as
\n
$$
L' = -\frac{np}{2} \log 2f - \frac{n}{2} \log |\Sigma| - \frac{1}{2} tr \Sigma^{-1} S - \frac{n}{2} \left\{ \left(\overline{x} - \underline{z}_1\right) \Sigma^{11} \left(\overline{x} - \underline{z}_1\right) \right\}
$$
\n
$$
+ \frac{n}{2} \left\{ \left[2\left(\overline{x}_1 - \underline{\mu}_1\right) \Sigma^{12} \left(\overline{x}_1 - \underline{\mu}_2\right) + \left(\overline{x}_2 - \underline{\mu}_2\right) \Sigma^{21} \left(\overline{x}_2 - \underline{\mu}_2\right) - n\lambda^* \left(R\underline{\mu}_1 - \underline{\mu}\right) \right\} \right\}
$$
\nNow
\nNow
\n
$$
\frac{\partial L'}{\partial \underline{\mu}_1} =
$$

Now

$$
\frac{\partial L^+}{\partial \underline{\mu_1}} = n \Sigma^{11} \left(\overline{\underline{x}}_1 - \underline{\mu}_1 \right) + n \Sigma^{12} \left(\overline{\underline{x}}_2 - \underline{\mu}_2 \right) - nR' \underline{\sqsubseteq} = 0 \tag{2.4}
$$

and

$$
\frac{\partial L^+}{\partial \underline{\mu}_2} = n \Sigma^{22} \left(\overline{\underline{x}}_2 - \underline{\mu}_2 \right) + n \Sigma^{21} \left(\overline{\underline{x}}_1 - \underline{\mu}_1 \right) = 0 \tag{2.5}
$$

From (2.5) we get

$$
\left(\overline{\underline{x}}_2 - \underline{z}_2\right) = -\left(\Sigma_{22}\right)^{-1} \Sigma^{21} \left(\overline{\underline{x}}_1 - \underline{z}_1\right)
$$
\n(2.6)

Substituting for $\left(\frac{x}{2}, -\frac{z}{2}\right)$ in (2.4), we get

A note on estimation of parameters of multinormal distribution with constraints
\n
$$
n\Sigma^{11}\left(\frac{1}{\underline{x}}_1 - \frac{1}{\underline{x}_1}\right) - n\Sigma^{12}\left(\Sigma^{22}\right)^{-1}\Sigma^{21}\left(\frac{1}{\underline{x}_1} - \frac{1}{\underline{x}_1}\right) - n\overline{R}^1 = 0
$$
\n
$$
\Sigma^{11} - \Sigma^{12}\left(\Sigma^{22}\right)^{-1}\Sigma^{21} = \Sigma_{11}^{-1},
$$
\n
$$
n\Sigma_{11}^{-1}\left(\frac{1}{\underline{x}_1} - \frac{1}{\underline{x}_1}\right) = n\overline{R}^1\}.
$$
\n(2.7)

Since

$$
\Sigma^{11} - \Sigma^{12} (\Sigma^{22})^{-1} \Sigma^{21} = \Sigma_{11}^{-1}.
$$

A note on estimation of parameters of multinormal distribution with constraints
\n
$$
n\Sigma^{11}\left(\frac{1}{X_1} - \frac{1}{-1}\right) - n\Sigma^{12}\left(\Sigma^{22}\right)^{-1}\Sigma^{21}\left(\frac{1}{X_1} - \frac{1}{-1}\right) - n\overline{K}^1\right) = 0
$$
\nSince
\n
$$
\Sigma^{11} - \Sigma^{12}\left(\Sigma^{22}\right)^{-1}\Sigma^{21} = \Sigma_{11}^{-1},
$$
\nwe get
$$
n\Sigma_{11}^{-1}\left(\frac{1}{X_1} - \frac{1}{-1}\right) = n\overline{K}^1\frac{1}{2}.
$$
\n(2.7)
\nThus $\left(\frac{X_1}{X_1} - \frac{1}{-1}\right) = \sum_{i=1}^n \overline{K}^i\right),$
\nPre-multiplying by R gives $\left(\overline{K_{11}^T} - \overline{L}\right) = \left(\overline{K\Sigma}_{11}R^i\right)\underline{1}$ if the constraint $\overline{R}_{-1} = \underline{I}$ is to be satisfied. Thus,
\nwe take,
\n
$$
\underline{1} = \left(\overline{K\Sigma}_{11}R^i\right)^{-1}\left(\overline{K_{11}^T} - \underline{L}\right) \text{ so}
$$
\n
$$
\hat{1} = \overline{X}_1 - \sum_{i=1}^n \overline{K}^i\underline{1}
$$
\nFrom (6), the ML estimator of $\hat{1} \leq 1$ is
\n
$$
\hat{1} = 2\overline{X}_2 + \left(\Sigma^{22}\right)^{-1}\Sigma^{21}\left(\overline{X}_1 - \frac{1}{-1}\right)
$$
\n(2.9)
\nCase 2 (**unknown**):
\nWhen the covariance matrix is not known, we have to estimate $\hat{1} \cdot \hat{1} \cdot \hat{1} = 1$ and $\hat{1} = 1$ is the M. estimator of $\hat{1} = 1$ (2.10)
\nand $\hat{1} = 1$ is the M. estimator of $\hat{1} = 1$ and $\hat{2} = \frac{S^2}{B}$, where $S^* = \sum_{i=1}^n \left(\frac{1}{\Delta i} - \$

we take,

we get
$$
n\Sigma_{11}^{-1}(\overline{X}_1 - \overline{X}_1) = nR^2 \cdot 1
$$
.
\nThus $(\underline{X}_1 - \underline{X}_1) = \sum_n R^2 \cdot 1$, (2.7)
\nThus $(\underline{X}_1 - \underline{X}_1) = \sum_n R^2 \cdot 1$, (2.7)
\n $\text{Pre-multiplying by R gives } (R\overline{X}_1 - \underline{r}) = (R\Sigma_n R^2) \underline{1}$ if the constraint $R_{\overline{X}_1} = \underline{r}$ is to be satisfied. Thus,
\n $\hat{\underline{r}}_1 = \overline{X}_1 - \Sigma_{11} R^2 \underline{1}$ (2.8)
\n $\hat{\underline{r}}_2 = \overline{X}_2 - \Sigma_{11} R^2 \underline{1}$ (2.8)
\nFrom (6), the ML estimator of \underline{r}_2 is
\n $\hat{\underline{r}}_2 = \overline{X}_2 + (\Sigma^2)^{-1} \Sigma^2 (\overline{X}_1 - \hat{\underline{r}}_1)$ (2.9)
\nCase 2 (**unknown**):
\nWhen the covariance matrix is not known, we have to estimate $\underline{r}_1 \cdot \underline{r}_2$ and using the likelihood
\nfunction (2.3) The ML estimator of is
\nfunction (2.3) The M. estimator of $\hat{\underline{r}}_1$ under the restriction R $\underline{\mu}_1 = r$.
\nTo estimate $(\underline{r}_1 \cdot \underline{r}_2)$ the likelihood equations are given by (2.4) and (2.5). Now we have
\n $\hat{\Sigma} = \frac{S^2}{n} = \frac{S}{n} + (\overline{X} - \frac{\hat{n}}{n})(\overline{X} - \frac{\hat{n}}{n})$ (2.10)
\nFrom (2.11), we have
\n
$$
I = \sum_{i=1}^{n-1} \frac{S}{n} + \sum_{i=1}^{n-1} (\overline{X} - \frac{\hat{n}}{n}) (\overline{X} - \frac{\hat{n}}{n})
$$
 (2.12)

From (6), the ML estimator of $\frac{1}{2}$ is

$$
\frac{1}{2} = \overline{\underline{x}}_2 + \left(\Sigma^{22}\right)^{-1} \Sigma^{21} \left(\overline{\underline{x}}_1 - \frac{1}{2}\right)
$$
 (2.9)

Case 2 (unknown) :

function (2.3) The ML estimator of is

$$
L = (4\Omega_{11}R)^{2} \quad (\Lambda_{21}^{2} - \frac{1}{2})^{3,50}
$$
\n
$$
\frac{1}{\Omega_{1}} = \frac{1}{2}I - \Sigma_{11}R^{2} \quad (2.8)
$$
\nFrom (6), the ML estimator of $\frac{1}{2}$ is\n
$$
\frac{1}{\Omega_{2}} = \frac{1}{2} \times \left(\sum_{i=1}^{2} \frac{1}{i}\right)^{2} \sum_{i=1}^{2} \left(\frac{1}{2}I - \frac{1}{2}I\right)
$$
\n
$$
\text{Case 2 (unknown)}:
$$
\nWhen the covariance matrix is not known, we have to estimate\n $\frac{1}{2}I \cdot \frac{1}{2}$ and using the li function (2.3) The ML estimator of is\n
$$
\hat{\Sigma} = \frac{S^{*}}{n}, \text{ where } S^{*} = \sum_{i=1}^{n} \left(\frac{1}{2}I - \frac{\hat{\mu}}{\mu}\right)\left(\frac{1}{2}I - \frac{\hat{\mu}}{\mu}\right)^{1}
$$
\nand\n
$$
\frac{\hat{\mu}}{\mu} \text{ is the ML estimator of } \frac{1}{2} \quad \text{under the restriction R } \mu_{1} = r.
$$
\nTo estimate\n
$$
(\frac{1}{2}I \cdot \frac{1}{2}) \text{ the likelihood equations are given by (2.4) and (2.5). Now we have\n
$$
\hat{\Sigma} = \frac{S^{*}}{n} = \frac{S}{n} + \left(\frac{1}{2} - \frac{\hat{\mu}}{\mu}\right)\left(\frac{1}{2} - \frac{\hat{\mu}}{\mu}\right)^{1}
$$
\nFrom (2.11), we have\n
$$
I = \hat{\Sigma}^{-1} S_{n} + \hat{\Sigma}^{-1} \left(\frac{1}{2} - \hat{\mu}\right)\left(\frac{1}{2} - \hat{\mu}\right)^{1}
$$
\n(2.12)
$$

and $\hat{\mu}$ is the ML estimator of \sim under the restriction R $\mu_1 = \mathbf{r}$.

$$
\hat{\Sigma} = \frac{S^*}{n} = \frac{S}{n} + \left(\frac{1}{\underline{x}} - \frac{\hat{\lambda}}{\underline{\mu}}\right) \left(\frac{1}{\underline{x}} - \frac{\hat{\lambda}}{\underline{\mu}}\right)
$$
\n(2.11)

From (2.11), we have

$$
\frac{1}{2} = \frac{1}{2} + (2^{22})^{-1} 2^{21} (\frac{1}{21} - \frac{2}{21})
$$
\n(2.9)
\nsee 2 (unknown):
\nthen the covariance matrix is not known, we have to estimate $\frac{1}{2!}, \frac{1}{2}$ and using the likelihood
\nction (2.3) The ML estimator of is
\n
$$
= \frac{S^*}{n}, \text{ where } S^* = \sum_{i=1}^n \left(\frac{x_i - \frac{2}{14}}{2!} \right) \left(\frac{x_i - \frac{2}{14}}{2!} \right)
$$
\n(2.10)
\n
$$
\frac{2}{14} \text{ is the ML estimator of } \frac{1}{14} \text{ under the restriction R } \underline{\mu}_1 = \mathbf{r}.
$$
\nestimate $(\frac{1}{2}, \frac{1}{2})$ the likelihood equations are given by (2.4) and (2.5). Now we have
\n
$$
= \frac{S^*}{n} = \frac{S}{n} + (\frac{1}{2} - \frac{2}{14}) (\frac{1}{2} - \frac{2}{14})
$$
\n(2.11)
\n
$$
\text{Sum (2.11), we have}
$$
\n
$$
I = \hat{\Sigma}^{-1} S_n + \hat{\Sigma}^{-1} (\frac{1}{2} - \hat{\mu}) (\frac{1}{2} - \hat{\mu})
$$
\n(2.12)
\nwhere $\hat{\Sigma} = (\hat{\Sigma}_1, \hat{\Sigma}_2)$ is the solution of (2.4) and (2.5) after replacing d^{ij} by $\hat{\Sigma}^{ij}$.
\n2.12)
\n
$$
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Since \tilde{C}_1 and \tilde{C}_2 have to satisfy these equations, from (2.12) we get the equations

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\n
$$
\frac{1}{2} \text{ and } \frac{1}{2}
$$
\nhave to satisfy these equations, from (2.12) we get the equations
\n
$$
\underline{\mu}^{\prime} = \underline{\mu}_{1}^{\prime} \hat{\Sigma}^{11} S_{11} / n + \mu_{1}^{\prime} \hat{\Sigma}^{12} S_{21} / n
$$
\n(2.13)\n
$$
O = \hat{\Sigma}^{21} S_{11} / n + \hat{\Sigma}^{22} S_{21} / n
$$
\n(2.14)\n
$$
O(\mu) = \sum_{i=1}^{n} \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2}
$$

$$
O = \hat{\Sigma}^{21} S_{11} / n + \hat{\Sigma}^{22} S_{21} / n
$$
\n(2.14)

From above equations it follows that

A note on estimation of parameters of multinormal distribution with constraints
\nSince
$$
\tilde{z}_1
$$
 and \tilde{z}_2 have to satisfy these equations, from (2.12) we get the equations
\n
$$
\underline{\mu}^* = \underline{\mu}_1 \cdot \hat{\Sigma}^{11} S_{11} / n + \mu_1 \cdot \hat{\Sigma}^{12} S_{21} / n
$$
\n(2.13)
\n
$$
Q = \hat{\Sigma}^{21} S_{11} / n + \hat{\Sigma}^{22} S_{21} / n
$$
\n(2.14)
\nFrom above equations it follows that
\n
$$
\tilde{z}_1 \left(\frac{S_{1}}{n} \right)^{-1} = \tilde{z}_1 \left(\hat{\Sigma}^{11} - \hat{\Sigma}^{12} \left(\hat{\Sigma}^{22} \right)^{-1} \hat{\Sigma}^{21} \right)
$$
\n
$$
= \tilde{z}_2 \left(\hat{\Sigma}^{11} \right)
$$
\nHence, from (2.7), we have
\n
$$
\left(\tilde{x}_1 - \tilde{z}_1 \right) = S_{11} R^* \sum_{i} f n
$$
\n(2.16)
\nWe multiply (2.16) by R we get
\n
$$
\left(\tilde{x}_2 - \tilde{z}_2 \right) = \left(R S_{11} R^* \right) \sum_{i} f n
$$
\n(2.17)
\nprovided the constraints $R\mu_1 = r$ is to be satisfied.
\nThus we have
\n
$$
\tilde{L} = n \left(R S_{11} R^* \right)^{-1} \left(R \tilde{X}_1 - \tilde{z}_2 \right)
$$
\n
$$
\hat{L} = n \left(R S_{11} R^* \right)^{-1} \left(R \tilde{X}_1 - \tilde{z}_2 \right)
$$
\n
$$
\hat{L} = \tilde{L} = \tilde{L} - S_{11} R^* \left(R S_{11} R^* \right)^{-1} \left(R \tilde{X}_1 - \tilde{z}_2 \right)
$$
\n
$$
\hat{L} = \tilde{L} = \tilde{L} \left(S^* \right)^{-1} S^{11} \left(\tilde{L} \tilde{L} - \tilde{L} \right)
$$
\nThe ML estimator of \tilde{L}

Hence, from (2.7), we have

$$
\left(\frac{x}{x_1} - \frac{x}{-1}\right) = S_{11}R' \frac{1}{n}
$$
\n2.16. The multiplication (2.16) by P, we get

Pre multiplying (2.16) by R we get

$$
\left(R_{\underline{X}_1} - \underline{r}\right) = \left(RS_{11}R'\right)\ \frac{1}{n} \tag{2.17}
$$

provided the constraints $R\mu_1 = r$ is to be satisfied. Thus we have

$$
\underline{\mathbf{I}} = n \left(R \underline{\mathbf{S}}_{11} R^{\dagger} \right)^{-1} \left(R \underline{\overline{\mathbf{X}}}_{1} - \underline{\mathbf{r}} \right) \tag{2.18}
$$

Hence, from (2.7), we have
\n
$$
= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} R^{2} \frac{1}{2} / n
$$
\n(2.16)
\n
$$
R\overline{X}_{1} - \overline{X}_{1} = \int_{0}^{1} R^{2} \frac{1}{2} / n
$$
\n(2.17)
\nprovided the constraints $R\mu_{1} = \underline{r}$ is to be satisfied.
\nThus we have
\n
$$
\frac{1}{L} = n (RS_{11}R)^{-1} (R\overline{X}_{1} - \underline{r})
$$
\n
$$
\frac{\hat{\mu}}{\mu_{1}} = \frac{1}{K_{1}} - S_{11}R^{2}(RS_{11}R)^{-1} (R\overline{X}_{1} - \underline{r})
$$
\n(2.18)
\n
$$
\frac{\hat{\mu}}{\mu_{2}} = \frac{1}{K_{2}} - S_{11}R^{2}(RS_{11}R)^{-1} (R\overline{X}_{1} - \underline{r})
$$
\n(2.19)
\nThe ML estimator of μ_{2} is
\n
$$
\frac{\hat{\mu}}{\mu_{2}} = \frac{1}{K_{2}} + (S^{22})^{-1} S^{21} (\frac{1}{K_{1}} - \frac{\hat{\mu}}{\mu_{1}})
$$
\n3. ML Estimators of
\nAccording to Mardia et al (1989), the likelihood function of p-variate normal distribution with constraints
\n
$$
\Sigma = k \Sigma_{0}
$$
, where Σ_{0} is known, is
\n
$$
2n^{-1}I(x, -k) = -p \log k - \log |2f\Sigma_{0}| - k^{-1}r
$$
\n(3.1)
\nwhere $\alpha = tr \Sigma_{0}^{-1} S + (\frac{1}{K} - \mu_{0})^{-1} \sum_{0}^{-1} (\frac{1}{K} - \mu_{0})^{-1} \text{ is independent of k.}$
\nOur problem is to obtain estimate of k for the constraint $\frac{d}{dt} = k\overline{d}_{110}$, where $\frac{d}{dt}_{110}$ is known and

$$
\hat{\hat{\underline{\mu}}}_2 = \overline{\underline{x}}_2 + \left(S^{22}\right)^{-1} S^{21} \left(\overline{\underline{x}}_1 - \hat{\underline{\mu}}_1\right)
$$

3. ML Estimators of :

According to Mardia et al (1989), the likelihood function of p-variate normal distribution with constraints

$$
2n^{-1}l(x, \sim, k) = -p \log k - \log |2f\Sigma_0| - k^{-1}r
$$
\n(3.1)

where $\alpha = tr \Sigma_0^{-1} S + (\bar{x} - \underline{\mu}_0)^2 \Sigma_0^{-1} (\bar{x} - \underline{\mu}_0)$ is independent of k.

Our problem is to obtain estimate of k for the constraint $d_{11} = k d_{110}$, where d_{110} is known and

$$
A note on estimation of parameters of multinormal distribution with constraints
$$
\n
$$
d_{11}: r x r is a submatrix of d. If we let d_0 = \begin{pmatrix} k d_{110} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}, then
$$
\n
$$
|\Sigma_0| = k' |\Sigma_{110}| | \Sigma_{22}| |\Sigma_{21} \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{22}^{-1} \Big| \left(\sum_{j=0}^{i} C_j \left(\frac{-1}{k} \right)^{i-j} \right)
$$
\n(3.2)
\nwhere $C_j = tr_j \Big(d_{21} d_{110}^{-1} d_{12} d_{22}^{-1} \Big)^{-1}$
\nFurther, by considering d_0^{-1} as
\n
$$
\Sigma_0^{-1} = \begin{pmatrix} \Sigma_{1102}^{-1} & \frac{-1}{k} \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{210}^{-1} \\ -\Sigma_{22}^{-1} \Sigma_{21}^{-1} \Sigma_{12}^{-1} & \Sigma_{210}^{-1} \end{pmatrix}
$$
\nwhere
\n $d_{110,2} = k d_{110} - d_{12} d_{22}^{-1} d_{21}, \quad \Sigma_{21.0} = \Sigma_{22} - \frac{1}{k} \Sigma_{21} \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{21}^{-1}, we get$ \n
$$
tr \Sigma_0^{-1} S = tr \Sigma_{22}^{-1} S_{22} + \frac{1}{k} tr \Big(\Sigma_{110}^{-1} S_{11} - \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{22}^{-1} S_{21} - \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} S_{12} - \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} S_{22} \Big) + O(k^{-2})
$$
\nSince,
\n
$$
\Sigma_{110,2}^{-1} = k \Sigma_{210}^{-1} + O(k^{-2})
$$
\nand
\n
$$
\Sigma_{22.10}^{-1} = \Sigma_{22}^{-1} - \frac{1}{k} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} + O(k^{-2}),
$$

Further, by considering d_0^{-1} as

$$
\Sigma_0^{-1} = \begin{pmatrix} \Sigma_{110.2}^{-1} & \frac{-1}{k} \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{22.10}^{-1} \\ -\Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110.2}^{-1} & \Sigma_{22.10}^{-1} \end{pmatrix}
$$

where

$$
\Sigma_{0} = k^{r} |\Sigma_{110}| |\Sigma_{22}| |\Sigma_{21} \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{22}^{-1}| \left(\sum_{j=0}^{s} C_{j} \left(\frac{-1}{k} \right)^{s-j} \right)
$$
\n(3.2)
\nwhere $C_{j} = tr_{j} (d_{21} d_{110}^{-1} d_{12} d_{22}^{-1})^{-1}$
\n
$$
\Sigma_{0}^{-1} = \begin{pmatrix} \Sigma_{1102}^{-1} & \frac{-1}{k} \Sigma_{110}^{-1} \Sigma_{12}^{-1} \Sigma_{2210}^{-1} \\ -\Sigma_{22}^{-1} \Sigma_{21}^{-1} \Sigma_{1102}^{-1} & \Sigma_{2210}^{-1} \end{pmatrix}
$$
\nwhere
\n
$$
d_{110.2} = k d_{110} - d_{12} d_{22}^{-1} d_{21}, \quad \Sigma_{22.10} = \Sigma_{22} - \frac{1}{k} \Sigma_{21} \Sigma_{110}^{-1} \Sigma_{12}; \quad \text{we get}
$$
\n
$$
tr \Sigma_{0}^{-1} S = tr \Sigma_{22}^{-1} S_{22} + \frac{1}{k} tr (\Sigma_{110}^{-1} S_{11} - \Sigma_{110}^{-1} \Sigma_{12} \Sigma_{22}^{-1} S_{21} - \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} S_{12} - \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} S_{22}) + O(k^{-2})
$$
\n(3.3)
\nSince, $\Sigma_{1102}^{-1} = \frac{1}{k} \Sigma_{110}^{-1} + O(k^{-2})$
\nand
\n
$$
\Sigma_{22.10}^{-1} = \Sigma_{22}^{-1} - \frac{1}{k} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} + O(k^{-2})
$$
\n(3.4)
\n
$$
\Sigma_{22.10}^{-1} = \Sigma_{21}^{-1} - \frac{1}{k} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} + O(k^{-2})
$$
\n(3.5)
\nWe have
\n
$$
\overline
$$

Since,
$$
\Sigma_{110.2}^{-1} = \frac{1}{k} \Sigma_{110}^{-1} + O(k^{-2})
$$
 (3.4)

and

$$
\Sigma_{22.10}^{-1} = \Sigma_{22}^{-1} - \frac{1}{k} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{110}^{-1} + O\left(k^{-2}\right) \quad , \tag{3.5}
$$

We have

$$
\left|\Sigma_{0}\right| = k' \left|\Sigma_{10}\right| \left|\Sigma_{22}\right| \left|\Sigma_{21}\Sigma_{10}^{-1} \Sigma_{22}\Sigma_{12}^{-1} \Sigma_{12}\Sigma_{22}^{-1}\right| \right|
$$
\n
$$
\text{where } C_{j} = tr_{j}\left(d_{21}d_{110}^{-1}d_{12}d_{22}^{-1}\right)^{-1}
$$
\n
$$
\text{Furthermore, by considering } d_{0}^{-1} \text{ as}
$$
\n
$$
\sum_{0}^{-1} = \begin{pmatrix} \sum_{102}^{-1} & \sum_{102}^{-1} \sum_{102}^{-1} \\ -\sum_{22}^{-1} \sum_{11}^{-1} & \sum_{102}^{-1} \sum_{2210}^{-1} \end{pmatrix}
$$
\nwhere\n
$$
d_{110,2} = k d_{110} - d_{12}d_{22}^{-1}d_{21}, \quad \Sigma_{2210} = \Sigma_{22} - \frac{1}{k} \Sigma_{110} \Sigma_{110}^{-1} \Sigma_{12}.
$$
\n
$$
\text{where}
$$
\n
$$
d_{110,2} = k d_{110} - d_{12}d_{22}^{-1}d_{21}, \quad \Sigma_{2210} = \Sigma_{22} - \frac{1}{k} \Sigma_{210} \Sigma_{110}^{-1} \Sigma_{12} - \Sigma_{220}^{-1} \Sigma_{210}^{-1} \Sigma_{100}^{-1} \Sigma_{22}^{-1} \Sigma_{210}^{-1} \Sigma_{
$$

Using results (3.2), (3.3) and (3.6) in (3.1) we get

$$
l^{+} = 2n^{-1}l(x, \underline{\sim}, k) = Const. - (p+r)\log k - \frac{1}{k}r^{+} + O(k^{-2})
$$
\n(3.7)

where

$$
\sum_{0}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{1}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
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\n
$$
\sum_{1}^{1} (\overline{x} - \mu) =
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\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
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\n
$$
\sum_{1}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{3}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{1}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{3}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{4}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{5}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{5}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{1}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{2}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{3}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{4}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{5}^{1} (\overline{x} - \mu) =
$$
\n
$$
\sum_{5
$$

and

$$
b_{s-1} = \frac{c_{s-1}}{c_s} = \frac{tr_{s-1} \left(\sum_{21} \sum_{110}^{-1} \sum_{12} \sum_{22}^{-1} \right)^{-1}}{\left| \left(\sum_{21} \sum_{110}^{-1} \sum_{12} \sum_{22}^{-1} \right)^{-1} \right|}
$$
(3.8)

Thus, if μ is known, then the mle of *k* is

$$
\hat{k} = \frac{r^*}{p+r}
$$
\n(3.9)

If μ is unknown and unconstrained, then the mle's of μ is \bar{x} and that of k is $\frac{m}{2}$ for r , together these gives

$$
\hat{k} = \frac{b_{s-1} + tr\Sigma_{22}^{-1}S_{22}}{p+r}
$$
\n(3.10)

A *note on estimation of parameters of multinorm*

known and unconstrained, then the mle's of μ is $\frac{1}{\chi}$ and that of k is $\hat{k} = \frac{b_{s-1} + tr\sum_{22}^{-1}S_{22}}{p+r}$

for the constraint $d_{22} = k d_{220}$, where d_{220} A note on estimation of parameters of multinormal distribution with constraints

and unconstrained, then the mle's of μ is $\frac{1}{\mu}$ and that of k is $\frac{n}{p} + r$, together these gives
 $\frac{1}{p} + tr \sum_{22}^{-1} S_{22}$
 $\frac{$ Note that for the constraint $d_{22} = k d_{220}$, where d_{220} is known, the mle's of \hat{k} for known μ is $S/(p+s)$ and that of for unknown $\underline{\mu}$ is $S^*/(p+s)$ where *raultinormal distribution with constraints*

at of k is $\Gamma^* / p + r$, together these gives

(3.10)

the mle's of \hat{k} for known μ is $S / (p + s)$
 $S^* = d_{r-1} + tr \Sigma_{11}^{-1} S_{11}$ with ltinormal distribution with constraints

of k is $\Gamma^* / p + r$, together these gives

(3.10)

mle's of \hat{k} for known μ is $S / (p + s)$
 $= d_{r-1} + tr \Sigma_{11}^{-1} S_{11}$ with

H µ is unknown and unconstrained, then the mle's of µ is
$$
\frac{1}{X}
$$
 and that of *k* is $r''/p + r$, together these gives
\n
$$
\hat{k} = \frac{b_{r-1} + r r \sum_{r2} S_{r2}}{p + r}
$$
\n(3.10)
\nNote that for the constraint $Q_{22} = kQ_{220}$, where Q_{220} is known, the mle's of \hat{k} for known µ is $S/(p + s)$
\nand that of for unknown u is $S''/(p + s)$ where
\n $\beta = d_{r-1} + r r \sum_{r1} S_{r1} + (\frac{r}{X_1} - \mu_1) \sum_{r1}^{r-1} (\frac{r}{X_1} - \mu_1)$, $S'' = d_{r-1} + r r \sum_{r1}^{r-1} S_{r1}$ with
\n $d_{r-1} = |d_{12}Q_{22}^{-1}d_{21}d_{11}^{-1}|r_{r-1} (d_{12}d_{22}^{-1}d_{21}d_{11}^{-1})|$
\n4. Illustrative Example:
\n1. Estimation of sub mean vector with constraint $R\mu_1 = r$.
\n4. Illustrative Example:
\nwhere recorded. The sample means vector and covariance matrix are
\nwere recorded. The sample means vector and covariance matrix are
\n $\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = \begin{pmatrix} 2.36 \\ \bar{x}_2 \\ 2.76 \end{pmatrix}$
\n
$$
S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 0.0735 & 0.1937 & 0.2156 & 0.3072 \\ 0.12156 & 0.1969 & 2.1420 & 0.1525 \\ 0.3072 & 0.1057 & 0.1525 & 0.136 \end{pmatrix}
$$

\nNote that here \bar{x} and S are unconstrained miles of µ and Z. However, from other information we know
\nthat

4. Illustrative Example :

1. Estimation of sub mean vector with constraint $R_{\mu} = r$.

For 47 female cats the body weights (kgs), heart weights (gms), lungs weights (gms) and Kidney weights (gms) were recorded. The sample mean vector and covariance matrix are

$$
\overline{x} = \begin{pmatrix} x_1 \\ \overline{x}_2 \\ \overline{x}_3 \\ \overline{x}_4 \end{pmatrix} = \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_2 \end{pmatrix} = \begin{pmatrix} 2.36 \\ 9.20 \\ 3.56 \\ 2.76 \end{pmatrix}
$$

known and unconstrained, then the mles of µ is
$$
\frac{x}{\Delta}
$$
 and that of *k* is $\Gamma''/p + r$, together these gives
\n
$$
\hat{k} = \frac{b_{x+1} + rY\sum_{22}^{x+2}S_{22}}{p+r}
$$
\n(3.10)
\nfor the constraint d₂₂ = kd₂₂₀, where d₂₂₀ is known, the mles of \hat{k} for known µ is S/(p+s)
\nof for unknown µ is S^{*}/(p+s) where
\n $\beta = d_{-1} + tr\sum_{11}^{x+1}S_{11} + (\overline{x}_1 - \mu_1) \sum_{11}^{x+1} (\overline{x}_1 - \mu_1)$, S^{*} = d_{r-1} + $tr\sum_{11}^{x+1}S_{11}$ with
\nd_{r-1} = |d₁₂d₂₂⁻¹d₂₁d₁₁⁻¹| tr_{r-1} (d₁₂d₂₂⁻¹d₂₁d₁₁⁻¹)
\n
$$
f(t) = \sum_{n=1}^{n} \frac{2.36}{n}
$$
\n(2.36)
\n
$$
f(t) = \sum_{n=1}^{n} \frac{2.36}{n}
$$
\n(2.376)
\n
$$
f(t) = \sum_{n=1}^{n} \frac{2.36}{3.56}
$$
\n(2.376)
\n
$$
S = \begin{pmatrix} S_{11} & S_{12} \\ \overline{S}_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 0.0735 & 0.1937 & 0.2156 & 0.3072 \\ 9.2156 & 0.1969 & 2.1420 & 0.1575 \\ 0.1969 & 2.1420 & 0.1525 \\ 0.3072 & 0.1057 & 0.1525 & 2.0136 \end{pmatrix}
$$
\nNote that here \overline{x} and S are unconstrained rules of

Note that here $\frac{1}{x}$ and S are unconstrained mle's of μ and Σ . However, from other information we know

$$
\sim = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 10 & 0 \\ 3 & 5 \\ 2 & 5 \end{pmatrix}
$$

and

that

$$
\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{pmatrix} = \begin{pmatrix} \frac{\bar{x}_1}{x_1} \\ \frac{\bar{x}_2}{x_2} \end{pmatrix} = \begin{pmatrix} 2.36 \\ 9.20 \\ 3.56 \\ 2.76 \end{pmatrix}
$$
\n
$$
S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 0.0735 & 0.1937 & 0.2156 & 0.3072 \\ 0.1937 & 1.8040 & 0.1969 & 0.1057 \\ 0.2156 & 0.1969 & 2.1420 & 0.1525 \\ 0.3072 & 0.1057 & 0.1525 & 2.0136 \end{pmatrix}
$$
\nNote that here \bar{x} and S are unconstrained mles of \underline{u} and Σ . However, from other information we know that
\n
$$
\bar{x} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 10 & .0 \\ 3.5 \\ 2.5 \end{pmatrix}
$$
\nand
\n
$$
\Sigma = \begin{pmatrix} \Sigma_{\text{II}} & \Sigma_{\text{II}} \\ \Sigma_{\text{II}} & \Sigma_{\text{II}} \end{pmatrix} = \begin{pmatrix} 0.07810 & 0.15620 & 0.21745 & 0.31033 \\ 0.15620 & 1.56200 & 0.17767 & 0.07322 \\ 0.21745 & 0.17767 & 2.15436 & 0.17338 \\ 0.31033 & 0.07322 & 0.17338 & 2.04885 \end{pmatrix}
$$
\nWith above given information we estimate sub-mean vector $\Sigma_1 = 2 \times 1$ under the constraint $R_{\Sigma_1} = \underline{r}$ where $R : 2 \times 2$ and $\underline{r} : 2 \times 1$ are pre-specified as follows:
\n
$$
\text
$$

 $R: 2 \times 2$ and $r: 2 \times 1$ are pre-specified as follows :

$$
R = \begin{pmatrix} 0.45000 & 0.12500 \\ 0.12500 & 0.91875 \end{pmatrix} and \qquad \underline{r} = \begin{pmatrix} 2.37500 \\ 9.50000 \end{pmatrix}
$$

For unknown Σ , we have

A note on estimation of parameters of multinormal distribution with constraints
\n
$$
R = \begin{pmatrix} 0.45000 & 0.12500 \\ 0.12500 & 0.91875 \end{pmatrix} and \qquad r = \begin{pmatrix} 2.37500 \\ 9.50000 \end{pmatrix}
$$
\nFor unknown Σ , we have
\n
$$
\frac{1}{\hat{r}} = \pi \begin{pmatrix} R_{\text{S}_1} & R \end{pmatrix}^{-1} \begin{pmatrix} R_{\overline{X}_1} - \overline{r} \end{pmatrix} = \begin{pmatrix} -2.25821 \\ -0.056431 \end{pmatrix}
$$
\n
$$
\hat{r}_1 = \overline{\Delta}_1 - S_1 R^1 \underline{r} = \begin{pmatrix} 2.49993 \\ 10.00006 \end{pmatrix}
$$
\nand
\n
$$
\hat{r}_2 = \overline{\Delta}_2 + \left(s^{22} \right)^{-1} s^2 \begin{pmatrix} \overline{\Delta}_1 & -\hat{\overline{\Delta}}_1 \\ \overline{\Delta}_1 & -1 \end{pmatrix}
$$
\n
$$
= \overline{\Delta}_2 - S_{21} S_{11}^{-1} \begin{pmatrix} \overline{\Delta}_1 & \overline{\Delta}_1 \\ \overline{\Delta}_1 & -1 \end{pmatrix}
$$
\n
$$
= \begin{pmatrix} 3.84642 \\ 3.10968 \end{pmatrix}
$$
\nFor known Σ , we have
\n
$$
\frac{1}{\Sigma} = \begin{pmatrix} -0.0804 \\ -0.1706 \end{pmatrix}, \frac{\overline{\Delta}_1}{\Sigma_1} = \begin{pmatrix} 2.4999 \\ 10.0006 \end{pmatrix}, \frac{\overline{\Delta}_2}{\Sigma_2} = \begin{pmatrix} 3.9219 \\ 3.1205 \end{pmatrix}
$$
\n2. Estimation of sub-matrix Σ_1 of Σ with constraint $\Sigma_{11} = k \Sigma_{10}$.
\nFor $\Sigma_{110} = \begin{pmatrix} 0.1 & 0.2 \\ 0.2 & 2.0 \end{pmatrix}$, we have for known μ , $\hat{k} = 0.49426$ and for unknown μ ,

$$
\begin{aligned}\n\frac{1}{2} &= \frac{1}{2} x_2 + \left(s^{22} \right)^{-1} s^{21} \left(\frac{1}{2} - \frac{1}{2} \right) \\
&= \frac{1}{2} x_2 - S_{21} S_{11}^{-1} \left(\frac{1}{2} - \frac{1}{2} \right) \\
&= \left(\frac{3.84642}{3.10968} \right)\n\end{aligned}
$$

$$
\underline{\lambda} = \begin{pmatrix} -2.0804 \\ -0.1706 \end{pmatrix}, \hat{\Xi}_1 = \begin{pmatrix} 2.4999 \\ 10.0006 \end{pmatrix}, \hat{\Xi}_2 = \begin{pmatrix} 3.9219 \\ 3.1205 \end{pmatrix}
$$

2. Estimation of sub-matrix Σ_{11} of Σ with constraint $\Sigma_{11} = k \Sigma_{110}$.

$$
z_1 = \underline{x}_1 - S_{11}R' \underline{1} = \begin{pmatrix} 2.75556 \\ 10.00096 \end{pmatrix}
$$

\nand
$$
\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}^{-1} S^2 \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ -2 \end{pmatrix}
$$

\n
$$
= \frac{1}{2} \begin{pmatrix} 2 \\ 2 \end{pmatrix} S_{11}^{-1} \begin{pmatrix} 2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}
$$

\nFor known Σ , we have
\n
$$
\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2.0804 \\ -0.1706 \end{pmatrix}, \frac{2}{-1} = \begin{pmatrix} 2.4999 \\ 10.0006 \end{pmatrix}, \frac{2}{-2} = \begin{pmatrix} 3.9219 \\ 3.1205 \end{pmatrix}
$$

\n2. Estimation of sub-matrix Σ_{11} of Σ with constraint $\Sigma_{11} = k \Sigma_{110}$.
\nFor $\Sigma_{110} = \begin{pmatrix} 0.1 & 0.2 \\ 0.2 & 2.0 \end{pmatrix}$, we have for known μ , $\hat{k} = 0.49426$ and for unknown μ , we have $\hat{k} = 0.4886$

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