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Complex Effects in Discrete Time Prey-Predator Model with Harvesting On Prey

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---ABSTRACT---
This paper investigates complex effects in discrete time prey-predator model with harvesting on prey. The fixed points and their local stability were analyzed. Harvesting is strong impact on the dynamics evaluation of population .To a certain extent it can control the long term stationary density of population efficiently. Bifurcation diagram has been obtained for selected range of different parameters. As some parameters varied the model exhibited chaos as long time behavior.

KEYWORDS: *Bifurcation, Harvesting, Local Stability, Prey-Predator model*

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I. INTRODUCTION

Among all mathematical models, prey-predator model have received much attention during the last few decades due to its wide range of application. The dynamic relationship between prey-predator and their have long had been one of the dominant themes due to its universal prevalence and importance. The prey-predator interaction has been described firstly by two pioneers Lotka [1] and Volterra [2] in two independent works. After them, more realistic prey-predator models were introduced by Holling [3] suggesting three kinds of functional responses for different species to model the phenomena of predation. As is well known in the theory of population dynamical models there are two kinds of mathematical models: The continuous –time models described by differential equations, and the discrete time models described by difference equations, In recent years more and more attention is being paid to discrete time population models. The reasons are as follows first the discrete time models are more appropriate than the continuous time models when populations have non-overlapping generations, or the number of population is small. Second we can get more accurate numerical simulations results from discrete time models. Moreover the numerical simulations of continuous time models are obtained by discretetizing models. At last discrete time models are rich dynamical behaviours. Never less, some of works by {4],[5],[6],[7][8]. Showed that for discrete time prey-predator models the dynamics can produce a much richer set of patterns than those observed in continuous time models.[9] analyzed prey-predator model incorporating a prey refuge and independent harvesting on either species. Several authors improved by the basic Lotka-Volterra model by introducing Allee effect and functional response [10], [11], [12] Harvesting has a strong impact on the dynamic evaluation of a population subject to it depending on the native of the harvesting strategy the long run stationary density of population may be significantly smaller than long run stationary density of population in the absence of harvesting. Therefore while a population can in the absence of harvesting to free extinction risk, harvesting can read to incorporation of a positive.

II. MATHEMATICAL MODEL

This paper consider the following system difference equation which describes interaction between two species prey-predator

$$\begin{aligned} x(n+1) &= rx(n)(1-x(n)-ax(n)y(n)-hx(n) \\ y(n+1) &= (1-c)y(n)+bx(n)y(n) \end{aligned} \qquad (1)$$

Where r represents the natural growth of the prey and a represents' effect of predation on the prey c represents natural death rate of predator and b represents the efficiency and propagation rate of the predator in the presents of the prey and h represents harvesting effect

III. FIXED POINT AND LOCAL STABILITY

We now study the existence of fixed point of the system (1) particularly we are interested non-negative or interior fixed point. To begin with we list all possible fixed points.
By simple computation of the above algebraic system it was found that there are three non-negative fixed points
i) $E_0(0,0)$ is the origin.

ii) $E_1(\frac{r-(1+h)}{r},0)$ is axial fixed point in the absent of predator exist for $r > 1+h$

iii) $E_2(x^*, y^*)$ is the interior fixed point where $x^* = \frac{c}{b}$ $y^* = \frac{r(b-c)-b(1+h)}{ab}$

IV. DYNAMIC BEHAVIOR OF THE MODEL

Now we investigate the local behavior of the model (1) around each of the above fixed points. The local stability analysis of the model (1) can be studied by computing the variation matrix to each fixed point. The jacobian matrix of the equation (1) at the state variable is given by

$$J(x,y) = \begin{pmatrix} r(1-2x)-ay-h & -ax \\ by & bx+(1-c) \end{pmatrix} \tag{2}$$

The characteristic equation of the jacobian matrix can be written as

$$\lambda^2 - Tr\lambda + Det = 0 \tag{3}$$

Where Tr is the trace and Det is the determinant of the Jacobian matrix $J(x,y)$ which is defined as
$Tr = [r(1-2x)-(ay+h)+bx+(1-c)]$ and $Det = [(r(1-2x)-h)(bx+(1-c)-ay+ayc]$

Hence the model (1) is dissipative dynamical system is if $|[(r(1-2x)-h)(bx+(1-c)]-ay+ayc| < 1$
Conservative dynamical one if and only if

$|[(r(1-2x)-h)(bx+(1-c)]-ay+ayc| = 1$ and is un dissipated dynamical system otherwise.

In order to study the stability of fixed points of the model, we first give following lemma which can be easily proved by relation between roots and co-efficient of quadratic equation (3)

Non-linear systems are much harder to analyze since in most cases they do not possesses quantitative solution even when explicit solution are available they are often too complicated to provide much insight. One of the most useful techniques for analyzing non-linear system quantitavely is the linearised stability technique the stability of the system is investigated by obtaining eignvalues of the jacobian matrix is associated with fixed points in order to study the stability of the fixed point model we first give the following theorem

Theorem: Let $p(\lambda) = \lambda^3 + B\lambda^2 + C\lambda + D$ be the roots of $p(\lambda) = 0$.Then the following statements are true

a) If every root of the equation has absolute value less than one, then the fixed point of the System is locally asymptotically stable and fixed point is called a sink
b) If at least one of the roots of equation has absolute value greater than one then the fixed point of the system is unstable and fixed point is called saddle
c) If every root of the equation has absolute value greater than then the system is a source.
d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one .If there exists a root of equation with absolute value equal to one then the fixed point is called Non-hyperbolic.

Proposition: 1 The fixed point E_0 of the system is locally asymptotically stable if $r-h<1$ and $0<c<2$;unstable if $r-h<1$ and $c>2$ otherwise.

Proof: By linearizing system (1) at E_0 , one obtain the Jacobian

$$J(E_0)=\begin{pmatrix} r-h & 0 \\ 0 & 1-c \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_0 are $\lambda_1 = r-h$ and $\lambda_2 = 1-c$

Thus it clear that E_0 is locally asymptotically stable when $r-h<1$ and $0<c<2$,unstable when $r-h<1$ and $c>2$.

Proposition: 2 The fixed point E_1 of the system is locally asymptotically stable if $1<r-h<3$ and $br<\dfrac{cr}{r-(1+h)}$ otherwise unstable fixed point.

Proof: one can see that the Jacobian matrix for E_1

$$J(E_1)=\begin{pmatrix} 2-(r-h) & \dfrac{a(1+h-r)}{r} \\ 0 & b(\dfrac{r-(1+h)}{r})+(1-c) \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_1 are $\lambda_1 = 2-(r-h)$ and $\lambda_2 = b(\dfrac{r-(1+h)}{r})+(1-c)$

Hence E_1 is locally asymptotically stable when $1<r-h<3$ and $br<\dfrac{cr}{r-(1+h)}$ and unstable when $1<r-h<3$ and $br>\dfrac{cr}{r-(1+h)}$.

Proposition: 3

The fixed point $E_2(x^*,y^*)$ of the system is locally asymptotically stable if $\dfrac{b(c+h-4)}{(c+h)(b-(2+c+h))}<r<\dfrac{b}{b-c(1+h)}$

Proof:

We now investigate the local stability and bifurcation of interior point E_2. The Jacobian matrix (1) at E_2 has the form

$$J(E_2)=\begin{pmatrix} 1-\dfrac{cr}{b} & \dfrac{-ac}{b} \\ \dfrac{r(b-c)-b(1+h)}{a} & 1 \end{pmatrix} trJ(E_2)=2-\frac{rc}{b}$$

(4)

$$DetJ(E_2)=1-c(1+h)+cr(1-\frac{(1+c)}{b}) \qquad (5)$$

By solving the characteristic equation (3) at the interior fixed point E_2 the roots (Eigen Values at E_2) will be

$$\lambda_{1,2}=\frac{Tr(J(E_2))\pm\sqrt{(Tr(J(E_2)2-4DetJ(E_2))}}{2}.$$

$$\lambda_{1,2}=(1-\frac{rc}{b})\pm\sqrt{(1-\frac{rc}{2b})^2-c((r-\frac{r}{b}(1+c)))-(1+h)}$$

Where $Tr(J(E_2))$ and $DetJ(E_2)$ given by equations (4) and (5). The Eigen Values in numerical can be used to classify different type of bifurcation.

V. NUMERICAL SIMULATION

In this section we give the numerical simulations to verify our theoretical results proved in the previous section by using MATLAB programming. We also confirm the results by visual representation of the system for some values of parameters. We provide some numerical evidence for quantitative dynamic behavior of the map. Bifurcation diagram are consider in following cases

[1] Fixing parameters a=2, b=3, c=0.8, h=0 and varying r
[2] Fixing parameters a=1.5, b=3. c=0.9, h=0.02 and varying r
[3] Fixing parameters a=1.3, b=3.34, c=0.9, r=2 to 3.6 and varying h
[4] Fixing parameters a=1.3, b=3.34, c=0.9, r=2 to 3.8 and varying h

The case (1) bifurcation diagram of the map(1) in x-y plane is showing the dynamical behavior of the prey –predator model system as a varying and fixing parameters a=2,c= a=2, b=3, c=0.8, h=0. From Figure (1) one can see that the orbit with initial condition (0.4, 0.3) approaches a stable fixed point E_2 for $r < 2.69$ and a discrete Hopf bifurcation occurs at r=2.69.As r increases the interior fixed point becomes unstable through a discrete Hopf bifurcation and the behavior of prey-predator model becomes chaotic. It means that for large values of natural growth rate of the prey species r, a system converge always to complex dynamics .In the case of $r > 2$ one observes a discrete Hopf bifurcation occurs and complex dynamics behavior begin to appear for $2 < r < 3.8$.

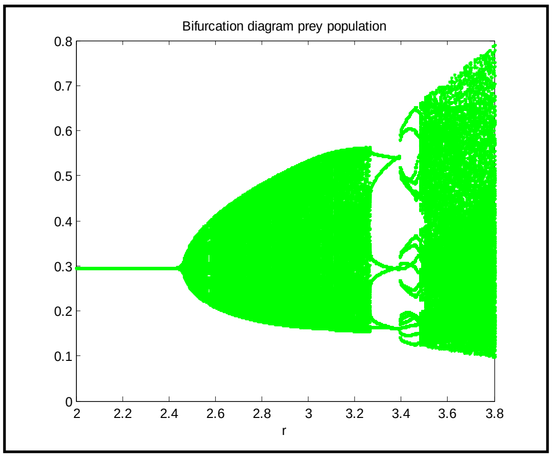

FIG (i)

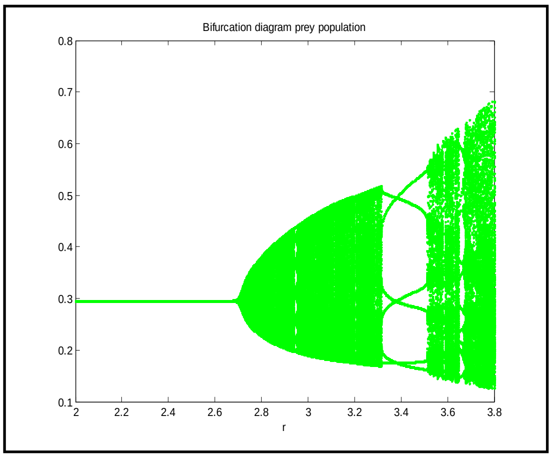

FIG (ii)

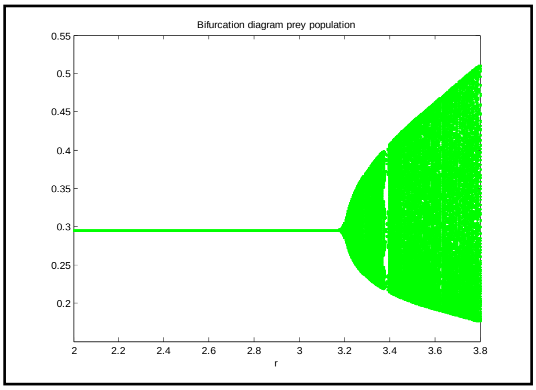

FIG (iii)

A bifurcation diagram with respect to r in Fig2 while others parameters are fixed as follows a=1,5,b=3 ,c=0.9,h=0.02 .also bifurcation diagram with respect to a plotted in figure 3 others parameters are fixed as follows a=1.3, b=3.34,c=0.9, r=2 to 3.6 from Figures (1-3) the change of parameter a affect of stability of the system under goes Hopf bifurcation.

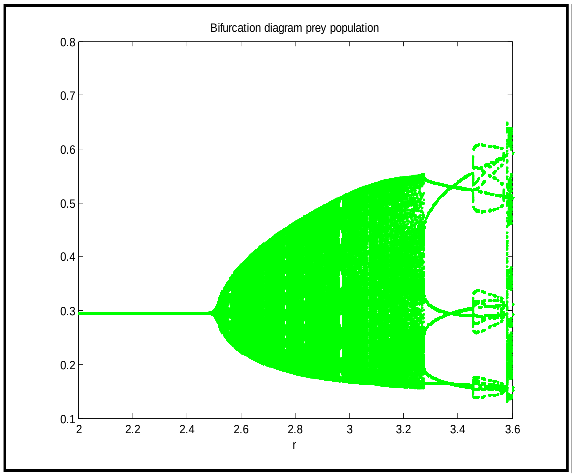

FIG (iv)

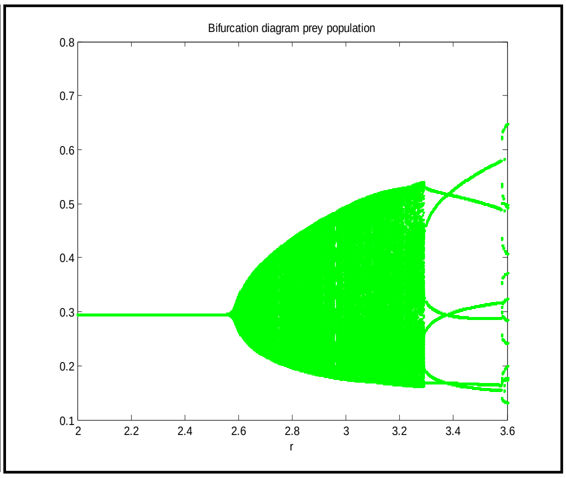

FIG (v)

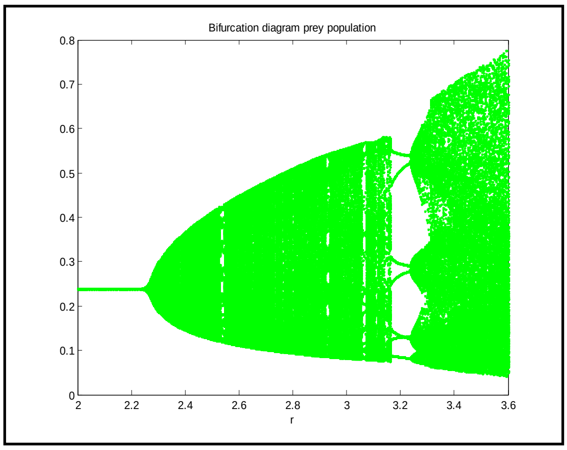

FIG (vi)

VI. CONCLUSION:

In this paper we analyzed the complex effects in discrete time prey-predator system. The stability of fixed points, bifurcation and chaotic behavior are investigated in this system. The influence of the main parameters ($r,a,b,c,h)$ on the local stability is studied. We observed that under certain parametric condition the interior fixed point enters discrete Hopf bifurcation phenomenon. It also been observed that when harvesting activity is not considered then the equilibrium of prey-predator model (1) is stable and when harvesting activity of prey is taken into consideration then the population size of the predator decreased and the natural stable equilibrium of the model becomes unstable.

REFERENCES

[1]. Abd-Elalim A. Elsadany, H. A. EL-Metwally, E. M. Elabbasy, H. N. Agiza, *Chaos and bifurcation of a nonlinear discrete prey-predator system, Computational Ecology and Software, 2012, 2(3):169-180.*

[2]. Leah Edelstein-Keshet, *Mathematical Models in Biology*, SIAM, Random House, New York, 2005.

[3]. Marius Danca, Steliana Codreanu and Botond Bako, *Detailed Analysis of a Nonlinear Prey-predator Model, Journal of Biological Physics 23:* 11-20, 1997.

[4]. J.D.Murray, *Mathematical Biology I: An Introduction*, 3-e, *Springer International Edition*, 2004.

[5]. Robert M.May, *Simple Mathematical Models with very complicated dynamics,Nature*, 261, 459 – 67(1976).

[6]. Saber Elaydi, *An Introduction to Difference Equations*, Third Edition, *Springer International Edition, First Indian Reprint*, 2008.

[7]. L.M.Saha, Niteesh Sahni, Til Prasad Sarma, *Measuring Chaos in Some Discrete Nonlinear Systems, IJEIT, Vol. 2,* Issue 5, Nov-2012.

[8]. Sophia R.J.Jang, Jui-Ling Yu, *Models of plant quality and larch bud moth interaction*, Nonlinear Analysis, doi:10.1016/j.na.2009.02.091.

[9]. Xiaoli Liu, Dongmei Xiao, *Complex dynamic behaviors of a discrete-time predator prey system,* Chaos, Solutions and Fractals 32 (2007) 8094.

[10]. Madhusudanan.V, Anitha.K, Vijaya.S, Gunasekaran.M, *Dynamics In Discrete Time Prey-Predator System With Quadratic Harvesting On Prey*. International Journal of Emerging Science and Engineering (IJESE) ISSN: 2319–6378, Volume-X, Issue-X