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Examining the Effect of Common Fuzzy Control Law in Linear Matrix Inequality Framework with Eigenvalue Optimization Applied To Power Network Load Frequency Control

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---ABSTRACT---
The paper present design of a common control law for R number of Takagi-Sugeno (T-S) fuzzy model rules in the context of Linear Matrix Inequality (LMI) with eigenvalue minimization for load frequency stability control. Simulation results obtained showed that the controller has slightly demerit in performance compared with its counter part that uses separate fuzzy control law for each T-S fuzzy model rule reported by Shehu R. S., Salawu H. and Yusuf M. (2014), T-S Fuzzy Model based State Feedback Control Design for Exponential System Stability, Submitted for Publication in the Tropical journal of Engineering, ATBU, Bauchi, Nigeria

KEYWORDS: *parallel distributed compensation, T-S fuzzy model, linear matrix inequality, eigenvalue minimization, local gain method, common gain method*

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I. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy modeling has evolved to be of prime important in nonlinear systems modeling and control. Discussion on its basic theory and application have been widely reported as in [1],[2] and [3].

A fuzzy based strategy used for controlling dynamic systems whose models are represented in T-S fuzzy frame work is the *Parallel Distributed Compensation* (PDC). It is basically a state feedback control law developed for compensating specific T-S fuzzy rule. For i-th T-S fuzzy rule, an i-th fuzzy control law is design, we may call it local control law for local rule method. Recently robust control synthesis and optimization such as convex constraint control involving Linear Matrix Inequality and exponential stability involving eigenvalue minimization have being incorporated into the PDC design. We have found some works in these areas in [4], [5], [6], [7], [8], [9],[10], [11],[12] ,and [13].

In [12], in an effort to compare robust LMI control and exponential stability we presented a T-S fuzzy system model control using LMI in conjunction with eigenvalue minimization, employing local control law for local T-S fuzzy rule method. Therein we realized a bit more cots involved in determining the required number of state feedback gains for the number of T-S fuzzy rules. In this paper we seek to explore if a common control law with single state feedback gain can be develop for the whole T-S fuzzy rules that would make the algorithm computation simpler and at the same time ensure same or better performance than the local gain method.

II. MATHEMATICAL PRELIMINARY

3.1 The T-S Fuzzy Modeling

We consider a general nonlinear system model of the type

$$\dot{x} = f(x,t) + g(x,t)u + \Delta(x,t) \quad (1)$$

$$y = x(t) \quad (2)$$

Where $x(t) \in \mathbb{R}^n$ is state vector, $u(t) \in \mathbb{R}^m$ is input vector, $\Delta(x,t)$ is external disturbance signal, *y* is output vector, *f* and *g* are smooth nonlinear functions often unknown or difficult to described and therefore need to be represented using some form of knowledge domain- the Takagi-Sugeno (T-S) fuzzy system. Theoretical discussion on the T-S fuzzy system modeling is provided in [1]. The result of applying the principle to (1) and (2) are the state and output models in (3) and (4).

$$\dot{x} = \sum_{i=1}^{R} \xi_i(x)(\boldsymbol{A}_i x + \boldsymbol{B}_i u) \qquad (3)$$
$$y = \sum_{i=1}^{R} \xi_i(x)\boldsymbol{C}_i x \qquad (4)$$

Where

$$\xi_i(x) = \frac{\rho_i}{\sum_{i=1}^{R} \rho_i}$$

Is fuzzy scaling factor, R is number of T-S fuzzy rules, ρ_i is i-th rule strength representing product of fuzzy sets in i-th fuzzy rule expressed as

$$\rho_i = \prod_j^p \Gamma_{i,j}$$

Γ_{ij} is j-th fuzzy set in i-th rule, $\dot{x}(t) = \mathbf{A}_i x(t) + \mathbf{B}_i u(t)$ is local linear system model in response to local input space.

3.2 Fuzzy Control Law

Assuming equal number rules for T-S fuzzy model and control laws we propose a control law with common state feedback gain as,

$$u^j = \xi_j(x) K x(t) \qquad (5)$$

And for R number of rules, the control law is

$$u^j = \sum_{j=1}^{R} \xi_j K x(t) \qquad (6)$$

Putting (6) in (3) yields the closed loop fuzzy system model in (7).

$$\dot{x}(t) = \sum_{i=1}^{R} \sum_{j=1}^{R} \xi_i(x)\xi_j(x)(\mathbf{A}_1 - \mathbf{B}_i \mathrm{K})x(t) \qquad (7)$$

The fuzzy scaling function is

$$\xi_j(x) = \frac{\rho_j}{\sum_j^R \rho_j}$$

Where ρ_j is as used in (3).

3.3 LMI Formulation with Eigenvalue Minimization Involving Common Gain Matrix

We propose the Lyapunov inequality as

$$(\mathbf{A}_i - \mathbf{B}_i \mathbf{K})^T \mathbf{P} + \mathbf{P}(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}) + 2\lambda \mathbf{QP} < 0 \qquad (8)$$

Where λ is a set containing the Eigen values of the local system matrices - $\mathbf{A}_i$ in (1), **Q** is introduced here as a scaling matrix. **P** is positive definite symmetrical Lyapunov matrix to determined. Adopting the same assumptions and dummy matrix variables we used in [14], we write the LMI-eigenvalue minimization formulation as follows:

Minimize λ

$\mathbf{\Phi}, \mathbf{\Omega}$

Subject to

$$\begin{bmatrix} 2\lambda \mathbf{Q\Phi} & \mathbf{A}_i^T \mathbf{\Phi} - \mathbf{\Omega}^T \mathbf{B}_i^T \\ \mathbf{\Phi A}_i - \mathbf{B}_i \mathbf{\Omega} & \mathbf{I} \end{bmatrix} < \mathbf{0} \qquad (9)$$

In Matlab we can check for the feasible **P** matrix, if found the common state feedback gain is calculated from the relation

$$\mathbf{K} = \mathbf{\Omega} \quad \mathbf{\Phi} \qquad (10)$$

Where $= \mathbf{P}^{-1}$, $\mathbf{\Omega}$ is a dummy matrix variable to be determined.

III. APPLICATION

For the purpose of comparison with local gain method reported, here also, we consider the power network model originally used in [15], in (11) and (12) as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\zeta\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\Delta P_L \qquad (11)$$

$$y = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (12)$$

Where $x \in \mathbb{R}^n$ is n sized state vector, ΔP_L is the per unit input load deviation. The parameters ω_n and ζ are the natural frequency and the damping ratio of the system expressed as

$$\omega_n = \sqrt{\frac{\pi f_o P_s}{H}} \qquad (13)$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_o}{H P_s}} \qquad (14)$$

D is damping constant, *H* is per unit inertia constant, f_o is the operating frequency in Hz and *P*s is the synchronizing power coefficient.

4.1 Formulating System T-S Fuzzy Rules

To evolve the T-S fuzzy model of the power network, we make the remarks

Remark I: the network electrical and mechanical parameters are V = 1,0p.u., P_s = 1.9888, H = 8.89, D = 0.138, f_o = 50Hz, δ_o = 17^o [14], min and max load deviation demand is $\Delta P_L = [\Delta P_{Lmin}, \Delta P_{Lmax}]$ = [0.8, 0.87].

Remark II: based on the system transient response test, we set the state universe of discourse as X = [0.0036, 0.36]. Evaluating the system response at different load input, we determine the minimum and maximum possible values of the two states as

$\Delta x_1 = [0.1, 0.36]$, $x_2 = [0.0036, 0.009]$.

We associate two fuzzy sets to each state deviation with following linguistic qualification: Positive Small (PS) and Positive (P). The fuzzy sets membership centers are:

Δx_1

PS: μ_1^1 : [0.0 0.125 0.25]

P: μ_1^2 : [0.125 0.25 0.5]

Δx_2

PS: μ_2^1 : [0 2×10^{-3} 2.5×10^{-3}]

P : μ_2^2 : [2×10^{-3} 5×10^{-3} 0.01]

States fuzzy membership plots over the centers are obtained as shown in Fig.1

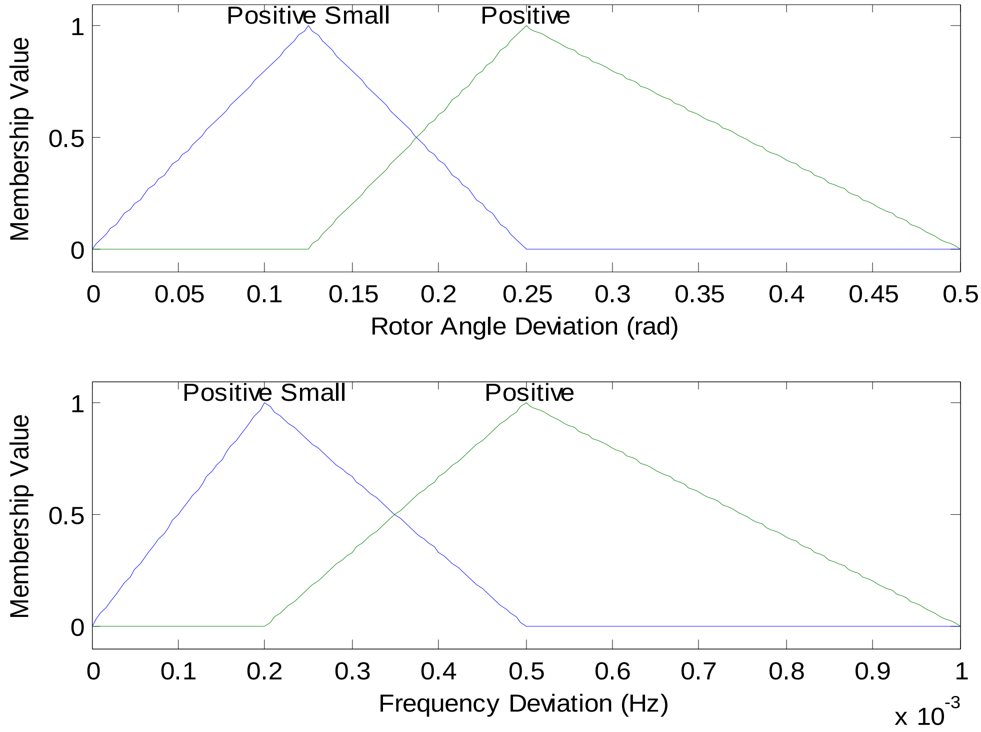

Fig.1: Fuzzy Sets Membership Plots

At the arbitrarily selected operation points

$$\Delta x \quad \exists \left\{ x_{01} = \begin{bmatrix} 0.11 \\ 0.001 \end{bmatrix}, x_{02} = \begin{bmatrix} 0.25 \\ 0.005 \end{bmatrix}, x_{03} = \begin{bmatrix} 0.45 \\ 0.007 \end{bmatrix}, x_{04} = \begin{bmatrix} 0.5 \\ 0.01 \end{bmatrix} \right\}$$

We formulate the four T-S fuzzy rules as follows:

$$\begin{aligned} &\text{IF } x_1 \text{ is } \Gamma_{PS} \text{ AND } x_2 \text{ is } \Gamma_{PS} \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_1 x + \mathbf{B}_1 u \\ &\text{IF } x_1 \text{ is } \Gamma_{PS} \text{ AND } x_2 \text{ is } \Gamma_{P} \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_2 x + \mathbf{B}_2 u \\ &\text{IF } x_1 \text{ is } \Gamma_{P} \text{ AND } x_2 \text{ is } \Gamma_{PS} \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_3 x + \mathbf{B}_3 u \\ &\text{IF } x_1 \text{ is } \Gamma_{P} \text{ AND } x_2 \text{ is } \Gamma_{P} \text{ THEN } \dot{\mathbf{x}} = \mathbf{A}_4 x + \mathbf{B}_4 u \end{aligned} \tag{15}$$

Algorithm

We may list the steps for determining the common gain matrix and implementing the fuzzy closed loop system as follows:

1. T-S fuzzy model and closed loop fuzzy model representing the nonlinear dynamic system in (1) and (2) are formed in (3) and (4).
2. Using (15) we formed a fuzzy control law involving common state feedback gain and closed loop fuzzy system in (5).

In Matlab we can utilize the implemented interior point algorithm to program and check for the feasible **P** matrix and evaluate the LMI over the minimized eigenvalue set.

IV. SIMULATION AND RESULTS

The numerical evaluation of the closed loop fuzzy system in (6) is to be carried out in three cases

Case 1: System response to unit step perturbation for zeroing frequency deviation

Case 2: System response to unit step perturbation over the preset load frequency value of 50Hz under nominal system parameter condition

Case 3: Response at reduced damping constant

Cases 4: System response at reduced damping and negative power coefficient

Table 3 shows the system conditions.

Setting	Nominal	Reduced Damping	Reduced and Negative Power Coefficient (worst case)
Parameter Values	D = 0.318, Ps = 1.988	D = 0.0318	D = 0.0318, Ps = -1.988

Results are obtained. These are shown in figures (2) – (5) respectively.

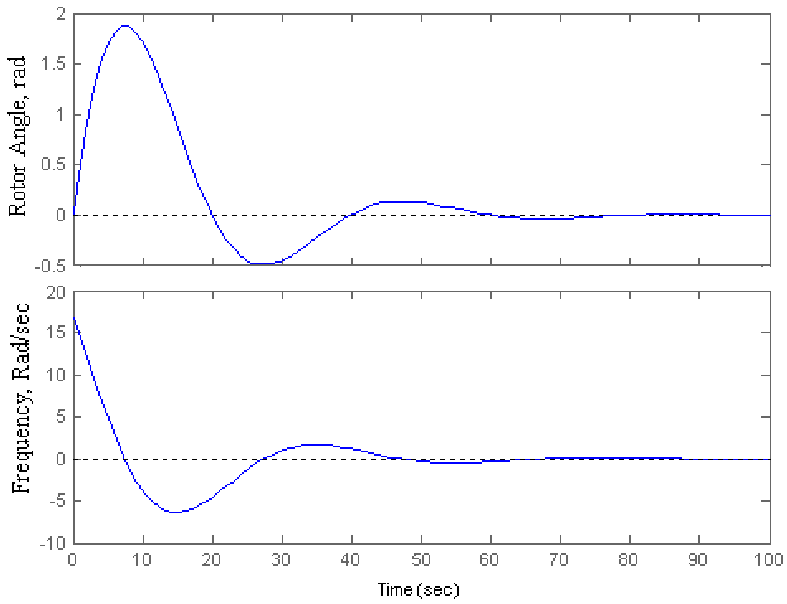

Fig.2: System response for zeroing frequency Deviation at nominal parameter condition

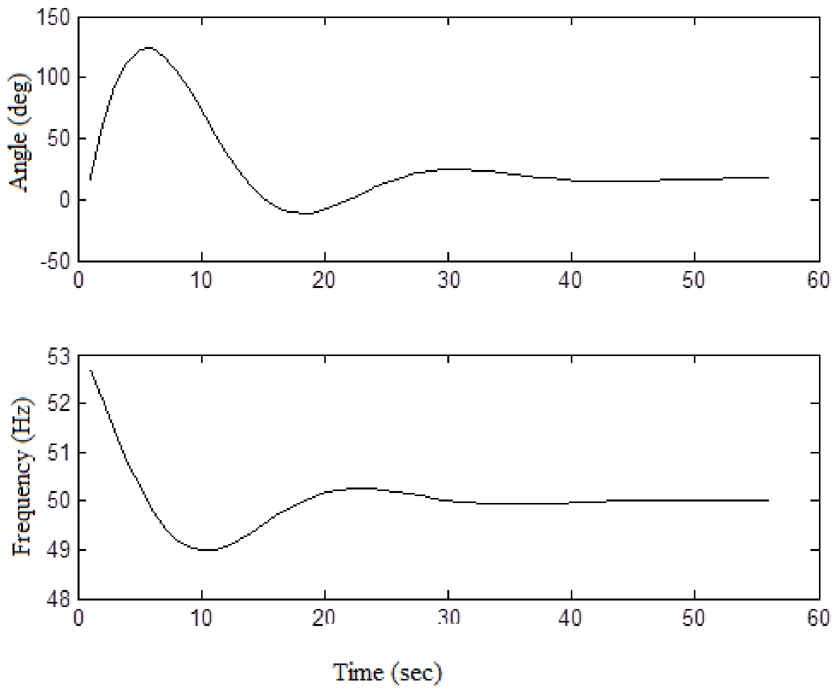

Fig.3: System response for setting reference load frequency at nominal parameter setting

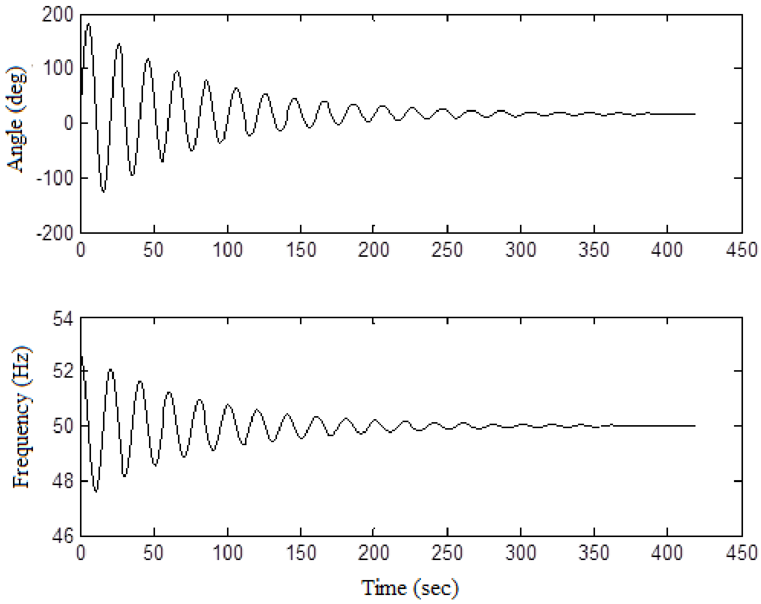

Fig.4: Response at reduced damping constant

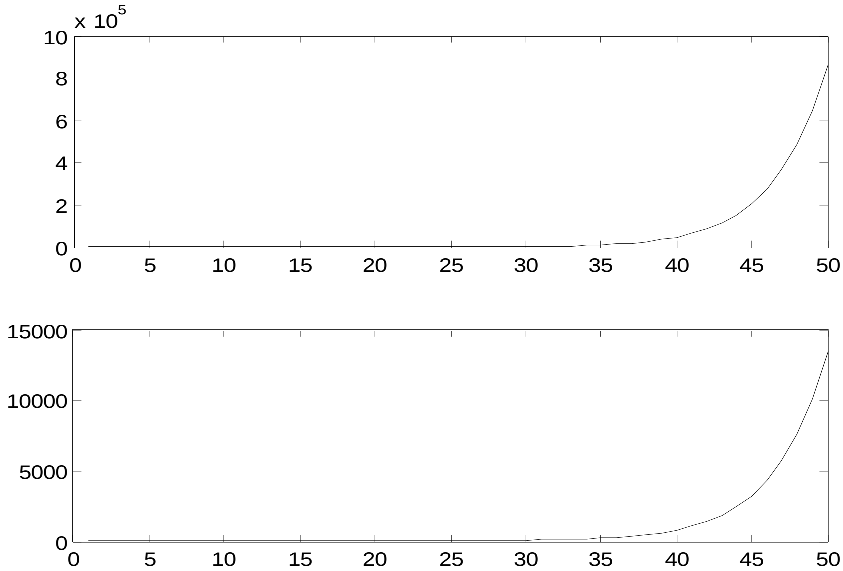

Fig.5: Response at reduced damping and neagtive power coefficient

V. RESULTS DISCUSSION

Response obtained in Fig.2 indicates the viability of the common gain strategy to mitigate load frequency excursion and bring back the deviation to zero of course at nominal parameter values. When evaluated over the desired load frequency the method also mitigate the deviation and maintain the load frequency at 50Hz reference setting at nominal parameter condition as shown in Fig.3. At reduced damping stability is insured by achieving desired 50Hz frequency at steady state as shown in Fig.4, but at high cost of initial oscillations. With local gain method, perfectively damped response was obtained.

At combined reduced damping and negative power coefficient (worst case) both the common controller gain (response shown in Fig.5) and the local controller produced an unstable responses.

VI. CONCLUSION

In the paper we presented design of a single common state feedback fuzzy control law derived from the framework of Linear Matrix Inequality with eigenvalue optimization scheme. The closed loop system is simulated at nominal and varying system parameter conditions. In comparison with the local gain method reported in [14], the common gain method appeared to lack the required ability to dampen frequency excursion when damping constant is reduced. The local gain method therefore is better in performance when faced with varied system conditions than the common gain strategy. For real application, it is recommended that for each T-S fuzzy rule developed, a separate compensation rule should be design for better load frequency control.

REFERENCES

[1] Tanaka K. and Wang H.O., Fuzzy Control Systems Design and Analysis, John & Wiley, New York, US, 2001

[2] Herrra-EspinosaJi J. Salazar-Del-Moral V.M. and Portollo-Mendez A.R., Simulation of Takagi-Sugeno Fuzzy Speed/Frequency Controller for A DC Motor-Alternator with Current Loop, *Journal of Vectorial Relativity*, vol. 4, pp66-76, 2009

[3] Jingzhuo S., Lin L. and Yu Z.(20110,Dynamic Takagi-Sugeno Model for the Control of Ultrasonic Motor, Journal of Control Science and Engineering, vol. 2011, Retrieved from http://dx-doi.org/10.1155/2011/219278 , December, 2013

[4] Grman L. Rosinova D. and Vesely V., Robust Stability Condition for Polytopic Systems, *International Journal of Systems Science*, vol.36, pp 961-973, 2006

[5] Khaber F. Zehar K. and Hamzaoui A.,, State Feedback Controller Design via Takagi-Sugeno Fuzzy Model: LMI Approach, *International Journal of Information and Mathematical Sciences*, pp.148-153, 2006

[6] Abdelkarim A., Ghorbel C. and Benrejeb M., LMI-based Tracking Controller for Takagi-Sugeno Fuzzy Model, *International Journal of Control and Automation*, vol 3, No. 2, pp.21-36, 2010

[7] Tseng C., Chen B. and Uang H., Fuzzy Tracking Control Design for Nonlinear Dynamic System via T-S Fuzzy Model, *IEEE Transactions on Fuzzy Systems*, vol.9, No.3,pp381-392, 2001

[8] Nachidi M., Benzaouiia A.,Tadeo F. and Ait Rami M., LMI-Based Approach for Output-Feedback Stabilization for Discrete-Time Takagi-Sugeno Systems, *IEEE Trans on Fuzzy Systems*, vol.16, no. 5,pp-1188-1196, 2008

[9] Najat O. and Noureddine E., Exponential Stabilization of T-S System for Synchronous Machine without Damper, *International Journal of Mathematical Model and Methods in Applied Sciences*, vol.5, no.8, pp 1406-1413, 2011

[10] Khairy M., Elshafei A.L. and Emara Hassan M., LMI based Design of Constraint Fuzzy Predictive Control, *International Journal of Fuzzy Sets and Systems*, vol. 161, and pp 893-918. 2009

[11] Salem A., Kardous Z. and Braiek N.B., On the State Observer based Stabilization of T-S Systems with Maximum Convergence Rate, *International Journal of Engineering*, vol.3, no.3, pp 293-305, 2008

[12] Chung-Hsum S., Wen-June W. and Wei-Wei L., Linear Control and Parallel Distributed Fuzzy Control Design for T-S Fuzzy Time-Delay System, *International Journal of Fuzzy Systems*, vol.9,no.4, pp.229-235, 2007

[13] Mohamed N., Abdellah B., Ahmed E. and Mohamed B., Further results on the Stabilization of Fuzzy Constrained Control System, Intnl. Journal of Science and Techniques of Automatic Control & Computer Engineering, December, 2007, pp.276-286

[14] Shehu R. S., Salawu H. and Yusuf M. (2014). T-S Fuzzy Model based State Feedback Control Design for Exponential System Stability. Submitted for Publication in the *Tropical journal of Engineering*, ATBU, Bauch, Nigeria.

[15] Sadat, H., Power System Analysis, Tata McGraw-Hill Limited, New Delhi, pp. 461-463, 1999