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Identification of Isomorphism amongst Kinematic Chains and Inversions Using Link - Link Connectivity and Adjacency Values

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----------**ABSTRACT**----------

The present work deals with problem of detection of isomorphism which is frequently encountered in structural synthesis of kinematic chains. Using link-link connectivity and link- adjacency values, a new method has been proposed to reveal simultaneously whether chain is isomorphic. Two new invariants, called first adjacency link value (FALV) and second adjacency link value (SALV), are developed for identifying distinct mechanisms of a planer kinematic chain. Another two invariants, called maximum first adjacency link value (MFALV) and maximum second adjacency link value (MSALV), which are the by – products of the same method, has been proposed to detect isomorphism among the kinematic chains. These invariants takes into account the connectivity value of links and type of joints of kinematic chain and are used as the composite identification number of a kinematic chain and mechanisms. The proposed method is easy to compute, reliable and capable of detecting isomorphism in all types of kinematic chains. This study will help kinematician / designer to select the best kinematic chain and mechanism to perform the specific task at conceptual stage of design. The proposed method is applied to 9-links, 2DOF kinematic chains.

Key-Words: *Inversion of kinematic chain, Isomorphism, Joint value, Kinematic chain, Link value.*

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I. INTRODUCTION

A major problem encountered during the structural synthesis of kinematic chains is the detection of isomorphism among the kinematic chains. Various methods have been reported by the researchers in the past for detection of isomorphism among the kinematic chains along with the methods to detect distinct inversions (1-10). Each method has its own merits and demerits and involves lots of complex mathematics. There is always a need of simple reliable and efficient method for detecting isomorphism among the kinematic chains. How ever, almost much of the methods reported so far are based on 1st adjacency of links. 1st adjacency of a link deals with links that are directly connected to it. These methods do not take care of higher order adjacency of links so these methods do not specify necessary and sufficient conditions in the present work, this aspect is given more importance and two structural invariants, 1st adjacency link value (FALV) and 2nd adjacency link value (SALV) are developed on the basis of linked connectivity and link values to identify distinct inversions of a given chain. Another two invariants, maximum1st adjacency link value (MFALV) and maximum 2nd adjacency link value (MSALV) which are the by-products of the same methods has been proposed to detect isomorphism among the kinematic chains.

II. TERMINOLOGY

Following definitions are to be understood clearly before applying this method.

2.1 Link Connectivity

It is a numerical value given for each link which is based on its connectivity to other links. For example, consider the adjacency matrix of kinematic chain 1 which is given below.

$$\begin{vmatrix} 3 & 0 & 3 & 3 & 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 & 0 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 & 2 & 0 & 0 \\ 3 & 3 & 0 & 3 & 0 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 \end{vmatrix}$$

All the diagonal elements represent degree of the corresponding links. If a link 'j' is directly connected to link 'i' then it is represented by the degree of link 'j' and if link 'j' is not directly connected to link 'i' then it is represented by a number equal to zero (0).

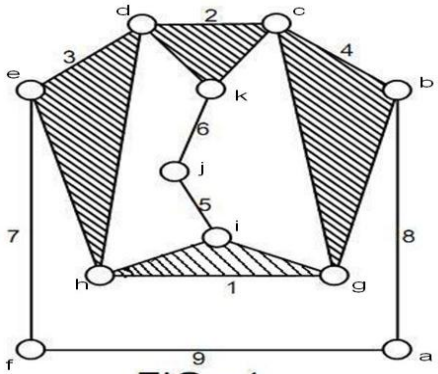

FIG . 1

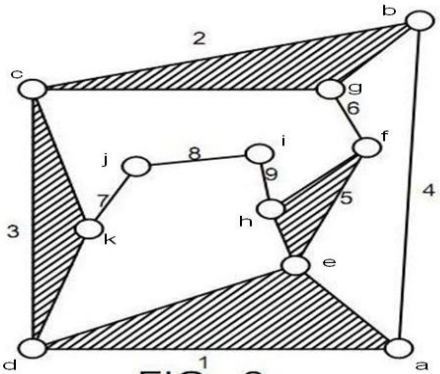

FIG . 2

However link connectivity of link 1 is represented by:

$L_{C1} = 3 + 2 \times 10^{-3} + 1 \times 10^{-2} = 3.012$

Similarly, link connectivity for other links of kinematic chain 1 are:

$L_{C2} = 3.012$, $L_{C3} = 3.012$, $L_{C4} = 3.012$, $L_{C5} = 2.011$, $L_{C6} = 2.011$, $L_{C7} = 2.011$, $L_{C8} = 2.011$, $L_{C9} = 2.02$

Similarly, link connectivity for all the links of kinematic chain 2 are:

$L_{C1} = 3.012$, $L_{C2} = 3.021$, $L_{C3} = 3.011$, $L_{C4} = 2.002$, $L_{C5} = 3.021$, $L_{C6} = 2.002$, $L_{C7} = 2.011$, $L_{C8} = 2.02$, $L_{C9} = 2.011$

2.2 Joint Value

For a particular joint it is defined as the ratio of algebraic sum of all the link connectivity to the number of links connected at that joint. It is denoted by Jv.

J_v = (Σ all link connectivity of all links connected) / number of links connected

For example, joint value of joint 'a' of kinematic chain 1 is given by:

$J_a = (L_{C8} + L_{C9}) / 2$

$= 2.0155$

Similarly, joint value for other joints of kinematic chain 1 is:

$J_b = 2.5115$, $J_c = 3.012$, $J_d = 3.012$, $J_e = 2.5115$, $J_f = 2.0155$, $J_g = 3.012$, $J_h = 3.012$, $J_i = 2.5115$, $J_j = 2.011$, $J_k = 2.5115$

Similarly, joint value for all the joints of kinematic chain 2 is:

$J_a = 2.507$, $J_b = 2.5115$, $J_c = 3.016$, $J_d = 3.0115$, $J_e = 3.0165$, $J_f = 2.5115$, $J_g = 2.5115$, $J_h = 2.516$, $J_i = 2.0155$, $J_j = 2.0155$, $J_k = 2.511$

2.3 Link Value

For a link it is defined as sum of joint values of all the joints of that link. For example, link value of link 1 of kinematic chain as shown in figure 1, is given by:

$L_{V1} = J_g + J_h + J_i$

$= 8.5355$

Similarly, link value of other links of kinematic chain 1 is:
L_{V2} = 8.5355 , L_{V3} = 8.5355 , L_{V4} = 8.5355 , L_{V5} = 4.5225 , L_{V6} = 4.5225 , L_{V7} = 4.527 , L_{V8} = 4.527 , L_{V9} = 4.031
Similarly, link value of all the links of kinematic chain 2 is:
L_{V1} = 8.535, L_{V2} = 8.039 , L_{V3} = 8.5385 , L_{V4} = 5.0185 , L_{V5} = 8.044 , L_{V6} = 5.023 , L_{V7} = 4.5265 , L_{V8} = 4.031 , L_{V9} = 4.5315

III. NEW STRUCTURAL INVARIANTS

Considering all essential features of the kinematic chains, two new structural invariants called first adjacency link value (FALV) and second adjacency link value (SALV) are proposed. They are invariants for a kinematic chain because they are independent of the labeling of links and joints of a chain.

3.1 First Adjacency Link Value (FALV)

For a link it is defined as the sum of link values of all the links that are directly connected to it. For example, FALV of link 1 of kinematic chain, as shown in figure, is given by:
$L_{f1} = L_{V3} + L_{V4} + L_{V5}$
= 21.5935
Similarly, FALV for other links of kinematic chain 1 are given by:
L_{f2} = 21.5935, L_{f3} = 21.598, L_{f4} = 21.598, L_{f5} = 13.058, L_{f6} = 13.058, L_{f7} = 12.5665, L_{f8} = 12.5665, L_{f9} = 9.054
Similarly, FALV for all the links of kinematic chain 2 are given by:
L_{f1} = 21.601, L_{f2} = 18.58, L_{f3} = 21.1005, L_{f4} = 16.574, L_{f5} = 18.0895, L_{f6} = 16.083, L_{f7} = 12.5695, L_{f8} = 9.058, L_{f9} = 12.075

3.2 Second Adjacency Link Value (SALV)

For a link it is defined as the sum of first adjacency link values of all the links that are directly connected to it. For example, SALV of link 1 of kinematic chain, as shown in figure1, is given by:
$L_{S1} = L_{f3} + L_{f4} + L_{f5}$
= 56.254
Similarly, SALV for other links of kinematic chain 1 are given by:
L_{S2} = 56.254, L_{S3} = 55.7535, L_{S4} = 55.7535, L_{S5} = 34.6515, L_{S6} = 34.6515, L_{S7} = 30.652, L_{S8} = 30.652, L_{S9} = 25.133
Similarly, SALV for all the links of kinematic chain 2 are given by:
L_{S1} = 55.764, L_{S2} = 53.7575, L_{S3} = 52.7505, L_{S4} = 40.181, L_{S5} = 49.759, L_{S6} = 36.6695, L_{S7} = 30.1585, L_{S8} = 24.6445, L_{S9} = 27.1475

3.3 Maximum First Adjacency Link Value (MFALV)

For a kinematic chain, it is defined as the maximum of first adjacency link value of all the links of a kinematic chain. For example, MFALV of kinematic chain 1 and 2 are 21.598 and 21.601 respectively.

3.4 Maximum Second Adjacency Link Value (MSALV)

For a kinematic chain, it is defined as the maximum of second adjacency link value of all the links of a kinematic chain. For example, MFALV of kinematic chain 1 and 2 are 56.254 and 55.764 respectively.

IV. TESTS FOR ISOMORPHISM AMONG INVERSIONS

The first and second adjacency values of the various links have the characteristics to declare the number of inversions that can be obtained from a given chain and by fixing which links these inversions are possible. If the first and second adjacency values of two links are identical then the inversions are equivalent and constitute only one distinct mechanism.

V. TESTS FOR ISOMORPHISM AMONG KINEMATIC CHAINS

The invariants of MFALV and MSALV are definitive test for isomorphism among the chains. The test is very simple based on comparison of these two structural invariants. If in comparison these invariants are same then chains are declared to be isomorphic otherwise not. For example:
For chain 1:-
MFALV = 21.598 MSALV = 55.7535
For chain 2:-
MFALV = 21.601 MSALV = 55.764

This method shows that chain 1 and chain 2 are non- isomorphic as the corresponding values of MFALV and MSALV are different. Similar explanation can be given to other chains also.

VI. RESULTS

(1) The proposed structural invariants MFALV and MSALV of kinematic chain are able to detect isomorphism among the kinematic chains. In the present paper, 9 links, 2 DOF kinematic chains have been tested successfully for their non- isomorphism.
(2) Using the presented method the number of mechanisms derived from the family of 9 links, 2 DOF kinematic chains are 254. This is also verified by the results given by A.Dargar. et. al. which is also 254.

VII. CONCLUSION

Authors strongly believe that method is unique as it takes care of nature and all inherent properties of the mechanism. The present method is applicable to planar chains of any size and complexity even the kinematic chain with co-spectral graph and also able to identify all distinct mechanisms derived from a given kinematic chain. It is hoped that the presented method presents a new concept on which a new classification system for distinct mechanisms can be based.

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Table 1: Structural Invariants of 9 links, 2 dof Kinematic Chains

KinematicChain No.	FALV	MFALV	SALV	MSALV
1	21.598/21.598/21.5935/21.5935/13.058/13.058/12.5665/12.5665/9.054	21.598	56.254/56.254/55.7535/55.7535/34.6515/34.6515/30.652/30.652/25.133	56.254
2	21.601/21.1005/18.0895/18.58/16.083/12.5695/12.075/9.058	21.601	55.764/53.7575/52.7505/49.759/40.181/36.6695/30.1585/27.1475/24.6445	55.764
3	21.1065/21.1065/18.586/18.586/16.079/16.079/12.571/12.571/9.054	21.1065	53.2645/53.2645/52.2635/52.2635/37.172/37.172/30.1605/30.1605/25.142	53.2645
4	21.598/21.59819.098/21.5935/21.5935/13.058/13.058/12.5565/12.5565/9.054	21.598	56.2495/56.2495/55.758/55.758/34.6515/34.6515/30.652/30.652/25.133	56.2495
5	21.598/21.598/18.0855/18.0855/16.5795/16.5795/12.075/12.075/9.063	21.598	56.263/56.263/50.2525/50.2525/39.6835/39.6835/27.1485/27.1485/24.15	56.263

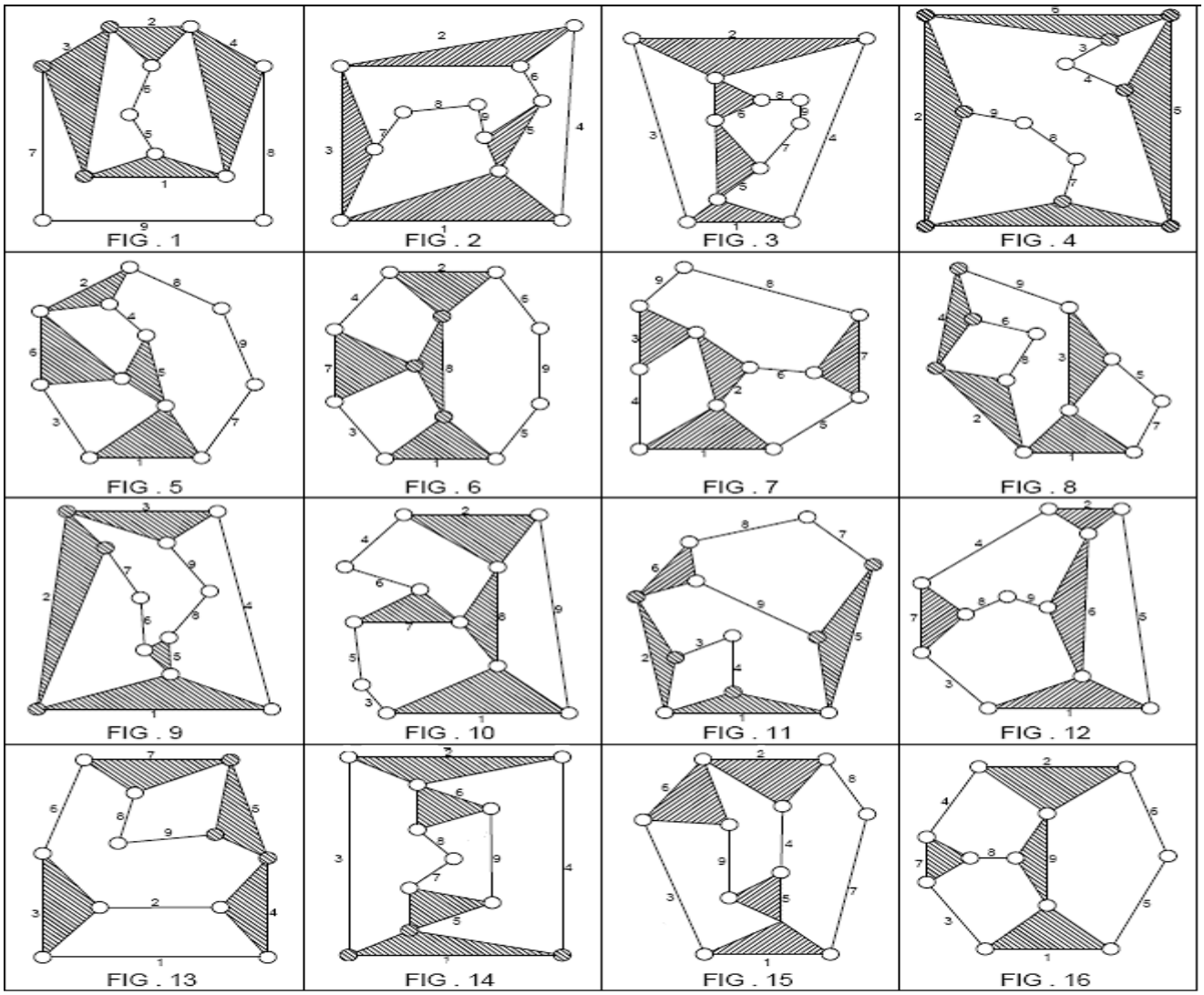
6	24.4695/19.082/18.5905/18.5905/16.0745/16.0745/12.0705/12.0705/9.063	24.4695	56.6185/56.263/52.6145/52.6145/37.6725/37.6725/27.6535/27.6535/24.141	56.6185
7	21.111/18.5905/18.09/16.088/16.0925/15.592/14.582/12.0795/12.58	21.111	52.791/52.773/49.779/43.764/36.6805/35.693/33.1725/30.1695/27.162	52.791
8	21.1065/21.1065/18.09/18.09/16.088/13.067/13.067/12.5665/12.5665	21.1065	52.2635/52.2635/49.761/49.761/33.673/33.673/31.157/31.157/36.18	52.2635
9	21.607/21.102/18.0855/17.5895/16.5795/13.067/12.5755/12.571/12.571	21.607	55.271/52.7595/50.2525/39.6925/46.7535/30.661/33.673/30.1605/30.661	55.271
10	24.123/18.586/18.586/18.09/18.09/16.079/12.571/12.571/12.5665/12.5665	24.123	55.262/52.7685/52.7685/49.265/37.172/31.157/31.15/30.6565/30.6565	55.262
11	21.106/21.106/18.09/18.09/16.088/13.0625/13.0625/12.571/12.571	21.106	52.259/52.259/49.7655/49.7655/36.18/34.169/34.169/30.661/30.661	52.259
12	20.6015/18.595/18.595/16.079/16.079/15.592/15.598/14.5865/13.076/12.075	20.6015	52.2725/52.2725/50.266/43.259/37.19/33.1815/33.1815/32.6765/27.6625	52.2725
13	20.606/18.5995/18.099/15.5875/15.5875/15.592/15.0825/13.0715/12.5665	20.606	51.781/49.77/48.7645/46.767/33.682/33.682/33.1815/33.1725/31.1705	51.781
14	18.0945/18.0945/17.594/17.594/16.097/16.088/16.088/12.5755/12.5755	18.0945	49.77/49.77/46.767/46.767/36.189/36.189/30.1695/30.1695/16.097	49.77
15	18.0945/18.0945/17.594/17.594/16.0925/16.0925/16.088/12.5755/12.5755	18.0945	49.7745/49.7745/46.7625/46.7625/36.189/35.6885/35.6885/30.1695/30.1695	49.7745
16	42.222/36.189/36.189/32.176/31.184/31.184/30.156/25.142/25.142	42.222	52.277/49.274/49.274/47.272/36.189/33.1724/33.1724/30.6655	52.277
17	21.1065/21.1065/18.09/18.09/16.088/13.067/13.067/12.5665/12.5665	21.111	52.2635/52.2635/49.761/49.761/36.18/33.673/33.673/31.157/31.157	52.2635
18	18.099/18.099/15.592/15.592/15.592/15.592/15.096/15.087/15.087	18.099	49.283/49.283/46.28/46.28/30.1785/30.679/30.174/30.679/30.679	49.283
19	18.099/18.099/15.592/15.592/15.592/15.592/15.096/15.087/15.087	18.099	48.256/48.256/46.28/46.28/33.186/33.186/33.186/33.186/30.174	48.256
20	25.14785/24.64195/20.6213//19.1018/17.10475/16.60875/13.0787/13.0742/9.56215	25.14785	46.758/46.258/33.68/32.188/30.163/29.681/27.148/24.646/23.649	46.758
21	24.65095/22.1363/20.1253/20.1253/16.60875/16.09925/16.09025/12.5827/9.56665	24.65095	78.99565/56.34985/53.3329/40.7412/40.7412/38.22655/34.2176/31.70295/29.19145	78.99565
22	21.6694/20.13475/20.1298/20.1298/19.1125/18.1062/16.11725/12.0786/9.5716	21.6694	76.50665/58.3658/51.3246/40.78065/40.78065/39.7756/31.241/28.19585/17.285	76.50665
23	27.1875/22.62285/22.62285/17.1043/17.1043/17.0953/13.0742/13.0742/10.0622	27.1875	79.4543/57.327/57.327/45.2457/39.72715/37.2197/37.2197/35.69705/35.69705	79.4543
24	16.0961/16.0961/16.61145/16.61145/22.63635/	22.63635	31.68225/31.68225/34.2253/34.2253/56.2422/75.98815/39.74115/42.3211/33.2229	75.98815
25	25.14335/20.6213/19.10225/16.6083/16.6083/14.08375/13.57515/12.56515/10.0712	25.14335	80.99225/58.3298/57.32385/43.7487/37.70865/35.3176/35.3176/35.2276/32.6774	80.99225

26	27.153/22.62735/22.6224/17.6003/17.10475/17.0953/13.57425/12.5782/9.56215	27.153	79.9548/57.82255/56.8265/45.24975/40.72725/40.2227/36.71515/32.1895/29.68295	79.9548
27	25.1429/24.64195/20.6213/19.10675/17.0998/16.60875/14.07925/12.07365/9.5671	25.1429	79.4638/77.81895/57.8289/43.7397/42.242738.7167/34.19555/28.6645/28.66035	79.4638
28	22.1444/20.1338/20.1338/20.1338/20.1338/15.6037/15.60378/12.0831/12.0831	22.1444	80.5352/52.3507/52.3507/37.7481/37.7481/37.7481/37.7481//27.6868/27.6868	80.5352
29	24.1495/22.13585/20.11925/17.1088/17.09825/16.10375/15.0742/13.58325/12.56765	24.1495	76.4918/53.8325/48.8203/39.24465/39.2343/37.72875/37.22465/36.72775/32.20165	76.4918
30	27.1575/22.12685/22.12685/17.6003/17.6003/13.0787/13.0787/13.57875/13.57875	27.1575	79.4543/53.81495/53.81495/40.73625/40.73625/39.72715/39.72715//35.20555/35.20555	79.4543
31	25.14345/24.13255/18.0968/17.10925/17.10925/17.0954//14.07035/13.07915/13.07915	25.14345	75.45735/56.2997/51.30175/42.23885/38.2029/37.2117/37.2117/35.20605/35.20605	75.45735
32	21.6484/19.6428/19.6428/19.63875/16.61775/16.1087/15.10365/15.09915/12.58315	21.6484	75.4621/54.34475/47.29605/37.7271/37.7271/36.72705/34.2065/31.6964/31.19735	75.4621
33	24.65085/20.1298/20.1298/17.1043/15.5946/14.5842/13.58775/13.08815/12.07815	24.65085	71.9481/52.33775/46.8057/40.24545/40.24545/38.2386/31.6885/28.68275/26.66235	71.9481
34	21.1344/20.1343/20.1343/19.1162/16.61775/16.61775/15.5962/13.0886/13.0886	21.1344	67.5041/52.8648/39.7934/36.7506/36.7506/31.723/31.723/30.71395/30.71395	67.5041
35	21.1344/20.13925/20.13925/18.1062/18.1062/16.61775/16.61775/13.08365/13.08365	21.1344	73.514/51.32895/51.32895/39.2406/39.2406/34.72395/34.72395/34.21805/34.21805	73.514
36	24.1806/24.1806/22.15565/22.15565/17.12545/17.12545/16.1204/16.1204/10.0802	24.1806	81.60705/81.60705/44.3113/44.3113/39.2811/39.2811/32.23585/32.23585/32.2408	81.60705
37	24.1905/17.1304/17.1304/17.1304/17.1304/17.1304/17.1304/21.14565/21.14565	24.1905	75.5817/75.5817/42.2913/38.27605/38.27605/38.27605/38.27605/38.27605/38.27605/38.27605	75.5817
38	28.18595/28.18592/18.1206/18.1206/18.1206/18.1206/17.1205/17.1205/10.0703	28.18595	81.54765/81.54765/46.30655/46.30655/46.30655/46.30655/46.30655/38.25625/38.25625/34.241	81.54765
39	28.71255/25.192535/25.192535/21.671035/21.671035/22.17203/16.613025/13.09202/11.09002	28.71255	115.89917/63.47709/43.34207/43.325575/43.325575/41.80457/39.80257/39.80257/38.785055	115.89917
40	31.114555/27.184545/22.658535/22.658535/22.162535/22.162535/14.597525/14.597525/10.98102	31.114555	116.826685/60.309605/44.32507/49.84308/49.84308/45.71208/45.71208/42.095575/42.095575	116.826685

Table 2: Inversions of Kinematic Chains Shown in Figures

Kinematic Chain No.	No.of Inversions	Kinematic Chain No.	No.of Inversions
01	05	21	08
02	09	22	08
03	05	23	06
04	05	24	07
05	05	25	08
06	06	26	09
07	09	27	09
08	05	28	04
09	08	29	09
10	06	30	05
11	05	31	07
12	07	32	08
13	08	33	08
14	05	34	06
15	05	35	05
16	06	36	05
17	05	37	03
18	04	38	04
19	04	39	07
20	09	40	06

FIGURES OF 9 LINKS 2 DOF KINEMATIC CHAINS



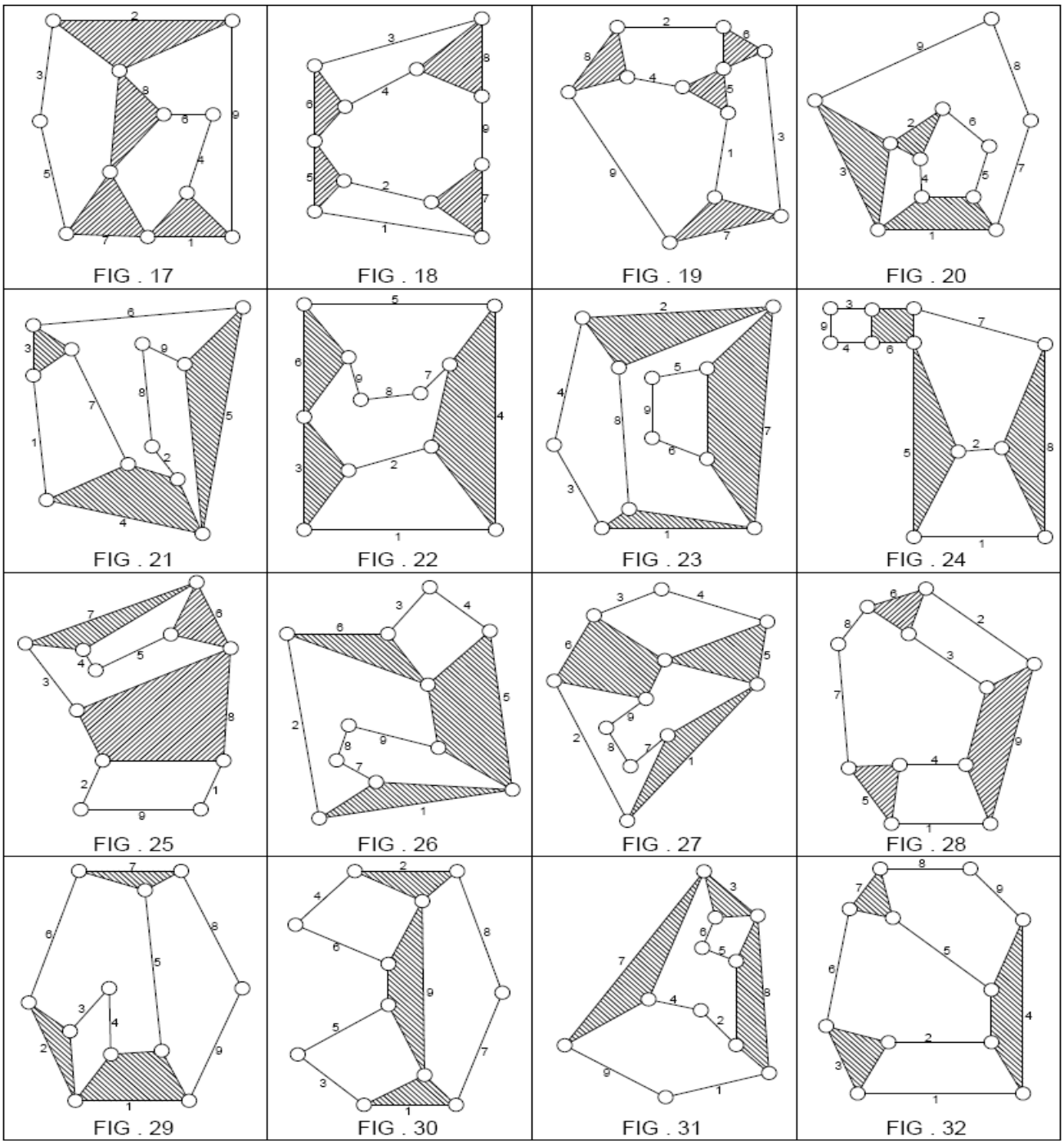


FIG . 17 FIG . 18 FIG . 19 FIG . 20

FIG . 21 FIG . 22 FIG . 23 FIG . 24

FIG . 25 FIG . 26 FIG . 27 FIG . 28

FIG . 29 FIG . 30 FIG . 31 FIG . 32

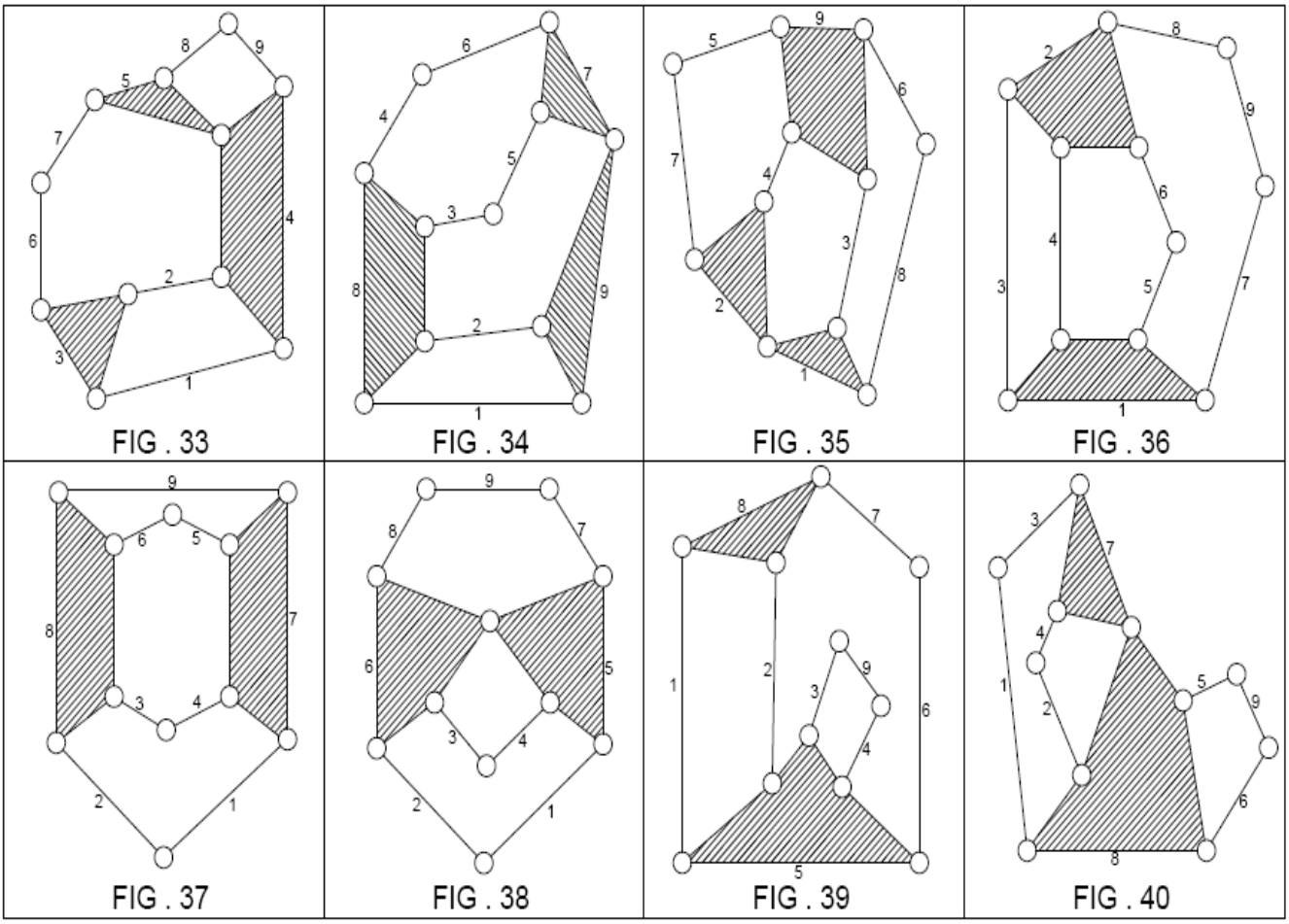


FIG . 33
FIG . 34
FIG . 35
FIG . 36
FIG . 37
FIG . 38
FIG . 39
FIG . 40