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Radiation and Mass Transfer Effects on An Unsteady Mhd Convection Flow of A Micropolar Fluid Past In Infinite Heated Vertical Moving Porous Plate In A Porous Medium

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ABSTRACT

An analysis is presented for the problem of free convection with mass transfer flow for a micropolar fluid via a porous medium bounded by an infinite vertical porous plate in an optically thin environment with time dependent suction in the presence of thermal radiation field in the case of unsteady flow. The plate moves with constant velocity in the longitudinal direction, and the free steam velocity follows an exponentially small perturbation law. A uniform magnetic field acts perpendicularly to the porous surface in which absorbs the micropolar fluid with a suction velocity varying with time. Numerical results of velocity distribution of micropolar fluids are compared with the corresponding flow problems for a Newtonian fluid. It is observed that, when the radiation parameter increases the velocity and temperature decrease in the boundary layer, whereas when Grashof number increases the velocity increases. Also, the results of the skin- friction coefficient, the couple stress coefficient, the rate of the heat and mass transfers at the wall are prepared with various values of fluid properties and flow conditions.

KEYWORDS: *Mass transfer, MHD, Micropolar, Radiation, Sherwood Number, Skin-Friction.*

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NOMENCLATURE:

A Suction velocity parameter
B_0 Magnetic flux density.
C Concentration..
C_f Skin friction coefficient.
C_m Couple stress coefficient
C_p Specific heat at constant pressure .
D Chemical molecular diffusivity
g Acceleration due to gravity.
Gc Modified Grashof number.
Gr Grashof number.
j Microinertia per unit mass.
K Permeability of the porous medium.
k Thermal conductivity
M Magnetic field parameter.
N Model parameter.
n Parameter related to microgyration vector and shear stress.
Nu Nusult number.
R Radiation parameter
Re_x Local Reynolds number
p Pressure.

$\Pr$ Prandtl number.
q_r Radiative heat flux
R Radiation parameter
Sc Schmidt number.
Sh Sherwood number.
t Time.
T Temperature.
u, v Components of velocities along and perpendicular to the plate.
U_0 Scale of free stream velocity.
V_0 Scale of suction velocity.
x, y Distances of along and perpendicular to the plate.

Greek symbols
α Fluid thermal diffusivity
α_1 Absorption coefficient
B Plank's function
β Ratio of vertex viscosity and dynamic viscosity
β_c Coefficient of volumetric expansion with concentration
β_f Coefficient of volumetric expansion of the working fluid
γ Spin gradient viscosity
δ Scalar constant
ε Scalar constant $(<< 1)$
θ Dimensionless temperature
Λ Coefficient of vertex(microrotation) viscosity
φ Dimensionless heat absorption coefficient
μ Fluid dynamic viscosity
ρ Fluid density
σ_c Electrical conductivity.
ν Fluid kinematic viscosity
δ_1 Radiation absorption coefficient
ν_r Fluid dynamic rotational viscosity
τ Friction coefficient
ω Angular velocity vector
λ Frequency

Subscripts
w Wall condition
∞ Free steam condition

Superscripts
$(\ )'$ Differentiation with respect to y.
$*$ Dimensional properties

I. INTRODUCTION

Eringen [1] has proposed the theory of micropolar fluids which takes into account the inertial characteristics of the microstructure particles which are allowed to undergo rotation. This theory may be applied to explain the phenomenon of the flow of colloidal fluids, liquid crystals, fluids with additives, animal blood, etc. The theory of thermomicropolar fluids has been developed by Eringen [2] by extending the theory of micropolar fluids. The boundary layer flow of a micropolar fluid past a semi-infinite plate has been studied by Peddieson and Mcnitt [3]. The flow characteristics of the boundary layer flow of a micropolar fluid over a semi-infinite plate was studied by Ahmadi [4] by taking into account the gyration vector normal to the xy - plane and the micro-inertia effects.Flow and heat transfer for a micropolar fluid through porous media have several practical engineering applications such as transpiration cooling, packed-bed chemical reactors, geothermal systems, crude oil extraction, ground water hydrology and building thermal insulation [5]. Sharma and Gupta [6] considered thermal convection in micropolar fluids in porous medium. Kim [7] presented an analysis of an unsteady convection flow of a micropolar fluid past a vertical porous plate embedded in a porous medium.

We know that the radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic. Recently, the effects of radiation on the flow and heat transfer of a micropolar fluid past a continuously moving plate have been studied by many authors [8-12]. The study of flow and mass transfer for an electrically conducting micropolar fluid past a porous plate under the influence of magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magnetohydrodynamic generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and boundary layer in the field of aerodynamics. Mohammadien and Gorla [13] analyzed the effects of magnetic field on the laminar boundary layer mixed convection flow of a micropolar fluid over a horizontal plate. Chamkha et al. [14] considered the effect of radiation on free convection flow past a semi-infinite vertical plate with mass transfer. El-Hakiem [15] analyzed the effects of magnetic field on natural convection with temperature dependent viscosity in micropolar fluids. Effects of joule heating on the magnetohydrodynamic free convection flow of a micropolar fluid were studied by El-Hakiem et al. [16]. El-Amin [17] solved the problem of MHD free convection and mass transfer flow in micropolar fluid with constant suction. Kim [18] analyzed an unsteady MHD mixed convection with mass transfer flow of a micropolar fluid past a vertical moving porous plate via a porous medium. Raptis et al. [19] investigated a steady MHD asymmetric flow of an electrically conducting fluid past a semi-infinite stationary plate in the presence of radiation. Mahmoud [20] studied the radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity. But the effect of thermal radiation and mass transfer on the non- Newtonian fluids with magnetic field has not received any attention.

The objective of the present chapter is to analyze the radiation effects on hydrodynamic heat and mass transfer flow of an incompressible micropolar fluid past a moving semi-infinite heated vertical porous plate in a porous medium with time-dependent suction. It is assumed that the free stream to consist of a mean velocity over which is superimposed an exponentially varying with time. The equations of continuity, linear momentum, angular momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. The behavior of the velocity, microrotation, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the physical parameters.

II. MATHEMATICAL ANALYSIS

We consider a two-dimensional unsteady flow of a laminar, incompressible, electrically conducting and micropolar fluid in an optically thin environment past a semi- infinite heated vertical moving porous plate embedded in a uniform porous medium in the presence of thermal radiation is considered. The x^* - axis is taken along the vertical porous plate in an upward direction and y^* - axis is taken normal to the plate. The applied magnetic field is considered in the direction perpendicular to the plate. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible [21]. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equations. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. Following Yamamoto and Iwamura [22], the porous medium is regarded as an assemblage of small identical spherical particles fixed in space. Due to the semi-infinite plane surface assumption, the flow variables are functions of y^* and the time t^* only. Now under the usual Boussinesq's approximation, the equation of mass, linear momentum, micro-rotation, energy and diffusion can be written as

Continuity:

$$\frac{\partial v^*}{\partial y^*} = 0, \qquad (1)$$

Linear momentum:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \left(\nu + \nu_r\right)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f\left(T - T_\infty\right) + g\beta_C\left(C^* - C_\infty^*\right) - \nu\frac{u^*}{K^*} - \frac{\mu^2\sigma_C}{\rho}B_0^2 u^* + 2\nu_r\frac{\partial \omega^*}{\partial y^*}, \quad (2)$$

Angular momentum:

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma\frac{\partial^2 \omega^*}{\partial y^{*2}}, \quad (3)$$

Energy:

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{k}\frac{\partial q_r^*}{\partial y^*}\right), \quad (4)$$

$$\frac{\partial q^*}{\partial y^*} - 3\alpha_1^2 q_r^* - 16\sigma_s\alpha_1 T_\infty^3\frac{\partial T}{\partial y^*} = 0 \quad (5)$$

Diffusion:

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D^*\frac{\partial^2 u^*}{\partial y^{*2}}, \quad (6)$$

where x^*, y^* and t^* are the dimensional distances along and perpendicular to the plate and dimensional time respectively. u^*, v^* are the components of dimensional velocities along x^* and y^* directions, respectively, ρ is the density, ν is the kinematic viscosity, υ_r is the kinematic rotational viscosity, g is the acceleration of gravity, β_f and β_C is the coefficients of volumetric thermal and concentration expansion of the fluid , c_p is the specific heat at constant pressure, σ_c is the fluid electrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, j^* is the micro inertia density, ω^* is the component of the angular velocity vector normal to the $x^* y^*$-plane, γ is the spin gradient viscosity, α is the effective thermal diffusivity of the fluid, k is the effective thermal conductivity, q_r is the radiative heat flux, T is the dimensional temperature, C^* is the dimensional concentration of the fluid and D^* is the chemical molecular diffusivity.
The third term on the right hand side of the momentum equation (2) denotes thermal and concentration buoyancy effects, the fifth is the bulk matrix linear resistance, that is, Darcy term and the sixth is the MHD term. Also, the second term on the right hand side of the energy equation (4) represents the radiative heat flux. Equation (5) is the differential approximation for radiation under fairly broad realistic assumptions. In one space coordinate y^*, the radiative heat flux q^* satisfies this nonlinear differential Eq. [5]. It is assumed that the porous plate moves with constant velocity in the longitudinal direction, and the free stream velocity follows an exponentially small perturbation law.
Under these assumptions, the appropriate boundary conditions for the velocity, microrotation, temperature and concentration fields are

$$u^* = u_p^*, T = T_W + \varepsilon(T_w - T_\infty)e^{\delta^* t^*}, \omega^* = -n\frac{\partial u^*}{\partial y^*}, C^* = C_w^* + \varepsilon(C_w^* - C_\infty^*)e^{\delta^* t^*} \ at\ y^* = 0 \quad (7)$$

$$u^* \to U_\infty^* = U_0(1 + \varepsilon e^{\delta^* t^*}), T \to T_\infty, \omega^* \to 0, C^* \to C_\infty^* \ as\ y^* \to \infty$$

where u_p^*, C_w and T_w are the wall dimensional velocity, concentration and temperature, respectively, C_∞ and T_∞ are the free stream dimensional concentration and temperature, respectively, δ^* is a constant, ε is small less than unity and U_0 is a scale of free steam velocity. The boundary condition for microrotation variable ω^* describes its relationship with the surface stress. In this equation, the parameter n is a number between 0 and 1 that relates the microgyration vector to the shear stress. The value $n=0$ corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value $n=0.5$ is indicative of weak concentrations, and when $n=1$ flows believed to represent turbulent boundary layers (Rees and Bassom, [24]).

From the continuity Eq. (1), the suction velocity normal to the plate can be written as following form:

$$v^* = -V_0(1+\varepsilon A e^{\delta^* t^*}), \tag{8}$$

where A is a real constant, ε and εA are small less than unity, and V_0 is the scale of suction velocity which is a non-zero positive constant . The negative sign indicates that the suction is towards the plate. Outside the boundary layer, Equation (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\upsilon}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 U_\infty^*. \tag{9}$$

Since the medium is optically thin with relatively low density and (α_1 absorption coefficient) <<1, the relative heat flux given by Equation (4), in the spirit of Cogley et al. [23], becomes

$$\frac{\partial q_r^*}{\partial y^*} = 4\alpha_1^2\left(T^* - T_\infty\right) \tag{10}$$

where

$$\alpha_1^2 = \int_0^\infty \delta_1 \lambda \frac{\partial B}{\partial T^*} \tag{11}$$

where B is plank's function.
On the introducing the non-dimensional quantities

$$u = \frac{u^*}{U_0},\ v = \frac{v^*}{V_0},\ y = \frac{V_0 y^*}{\nu},\ U_\infty = \frac{U_\infty^*}{U_0},\ U_p = \frac{u_p^*}{U_0},\ \omega = \frac{\nu}{U_0 V_0}\omega^*,\ t = \frac{V_0^2}{4\nu}t^*,\ \theta = \frac{T-T_\infty}{T_w - T_\infty},\ C = \frac{C^* - C_w^*}{C_w^* - C_w^*},$$

$$\delta = \frac{4\nu}{V_0^2}\delta^*,\ j = \frac{V_0^2}{\nu^2}j^*,\ K = \frac{\nu^2}{kV_0^2},\ M = \frac{\sigma_c B_0 \nu}{\rho V_0^2},\ R^2 = \frac{4\alpha^2}{\rho C_p k V_0^2}\left(T_w - T_\infty\right),\ \Pr = \frac{\nu\rho C_p}{k} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}, \tag{12}$$

$$Gr = \frac{\nu\beta_f g(T_w - T_\infty)}{U_0 V_0^2},\ Gc = \frac{\nu\beta_C g(C_w^* - C_\infty^*)}{U_0 V_0^2}\ \ Sc = \frac{\nu}{D^*}$$

Furthermore, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma = (\mu + \frac{\Lambda}{2})j^* = \mu j^*\left(1 + \frac{1}{2}\beta\right);\ \beta = \frac{\Lambda}{\mu} \tag{13}$$

where β denotes the dimensionless viscosity ratio, in which Λ is the coefficient of gyro-viscosity (or vertex viscosity).
In view of Equations (5) and (8)-(13), the governing Equations (2)-(4) and (6) reduce to the following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1+\varepsilon A e^{\delta t}\right)\frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU_\infty}{dt} + (1+\beta)\frac{\partial^2 u}{\partial y^2} + G_r\theta + G_c C + N\left(U_\infty - u\right) + 2\beta\frac{\partial \omega}{\partial y}. \tag{14}$$

$$\frac{1}{4}\frac{\partial \omega}{\partial t} - \left(1+\varepsilon A e^{\delta t}\right)\frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2}, \tag{15}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \left(1+\varepsilon A e^{\delta t}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial y^2} - R^2\theta\right), \tag{16}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1+\varepsilon A e^{\delta t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \tag{17}$$

where $N = \left(M + \frac{1}{K} \right),\ \eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}$

and $Gr, Gc, \Pr$, K, M, R and Sc denote the Grashof number, solutal Grashof number, prandtl number, permeability of the porous medium, magnetic field parameter, Radiation parameter and the Schmidt number, respectively.

The boundary conditions (8) are than given by the following dimensionless equations:

$$u = U_p,\ \theta = 1 + \varepsilon e^{\delta t},\ \omega = -n\frac{\partial u}{\partial y},\ C = 1 + \varepsilon e^{\delta t}\ at\ y = 0,$$

$$u \to 1 + \varepsilon e^{\delta t},\ \theta \to 0,\ w \to 0,\ C \to 0\ as\ y \to \infty \tag{18}$$

III. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we may represent the velocity, microrotation, temperature and concentration in the neighbourhood of the porous plate as

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{\delta t} u_1(y) + O(\varepsilon^2) + \cdots \\ \omega &= \omega_0(y) + \varepsilon e^{\delta t} \omega_1(y) + O(\varepsilon^2) + \cdots \\ \theta &= \theta_0(y) + \varepsilon e^{\delta t} \theta_1(y) + O(\varepsilon^2) + \cdots \\ C &= C_0(y) + \varepsilon e^{\delta t} C_1(y) + O(\varepsilon^2) + \cdots \end{aligned} \tag{19}$$

By Substituting Equation (19) into Equations (13)-(16), and equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of $0(\varepsilon^2)$, we obtain the following pairs of equations for $(u_0, \omega_0, \theta_0, C_0)$ and $(u_1, \omega_1, \theta_1, C_1)$.

$$(1+\beta)u_0'' + u_0' - Nu_0 = -N - G_r\theta_0 - G_c C_0 - 2\beta\omega_0' \tag{20}$$

$$(1+\beta)u_1'' + u_1' - (N + \frac{\delta}{4})u_1 = -(N + \frac{\delta}{4}) - Au_0' - G_r\theta_1 - G_c C_1 - 2\beta\omega_1' \tag{22}$$

$$\omega_1'' + \eta\omega_1' - \frac{\delta}{4}\eta\omega_1 = -A\eta\omega_0' \tag{23}$$

$$\theta_0'' + \Pr\theta_0' - R^2\theta_0 = 0 \tag{24}$$

$$\theta_1'' + \Pr\theta_1' - \left(R^2 + \frac{\delta}{4}\Pr \right)\theta_1 = -A\Pr\theta_0' \tag{25}$$

$$C_0'' + ScC_0' = 0 \tag{26}$$

$$C_1'' + ScC_1' - \frac{\delta}{4}Sc\theta_1 = -AScC_0' \tag{27}$$

where the prime denote differentiation with respect to y. The corresponding boundary conditions can be written as

$$u_0 = U_p,\ u_1 = 0,\ \omega_0 = -nu_0',\ \omega_1 = -nu_1',\ \theta_0 = 1,\ \theta_1 = 1,\ C_0 = 1,\ C_1 = 1,\ at\ y = 0$$

$$u_0 = 1,\ u_1 = 1,\ \omega_0 \to 0,\ \omega_1 \to 0,\ C_0 \to 0,\ C_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0,\ \text{as } y \to \infty \tag{30}$$

Without going into details, the solutions of Eqs. (20)-(27) subject to Eq. (30) can be shown to be

$$\begin{aligned} u(y,t) = 1 &+ a_1 e^{-m_1 y} + a_2 e^{-m_3 y} + a_3 e^{-Scy} + a_4 e^{-\eta y} \\ &+ \varepsilon e^{\delta t}\left\{ 1 + b_1 e^{-m_1 y} + b_2 e^{-m_2 y} + b_3 e^{-m_3 y} + b_4 e^{-m_4 y} + b_5 e^{-m_5 y} + b_6 e^{-m_6 y} + b_7 e^{-Scy} + b_8 e^{-\eta y} \right\} \end{aligned} \tag{31}$$

$$\omega(y,t) = c_1 e^{-\eta y} + \varepsilon e^{\delta t}\left\{ c_2 e^{-m_5 y} - \frac{4A\eta}{\delta} c_1 e^{-\eta y} \right\} \tag{32}$$

$$\theta(y,t) = e^{-m_3 y} + \varepsilon e^{\delta t}\left\{e^{-m_4 y} + \frac{4Am_3}{\delta}\left(e^{-m_4 y} - e^{-m_3 y}\right)\right\} \tag{33}$$

$$C(y,t) = e^{-Scy} + \varepsilon e^{\delta t}\left\{e^{-m_6 y} + \frac{4ASc}{\delta}\left(e^{-m_6 y} - e^{-Scy}\right)\right\} \tag{34}$$

The skin-friction, the couple stress coefficient, the Nusselt number and the Sherwood number are important physical parameters for this type of boundary layer flow. These parameters can be defined and determined as follows.

$$\tau_w^* = (\mu + \Lambda)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} + \Lambda \omega^*\Big|_{y^*=0},$$

$$C_f = \frac{2\tau_w^*}{\rho U_0 V_0}$$

$$= 2\left[1 + (1-n)\beta\right]\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= -2\left[1+(1-n)\beta\right](a_1 m_1 + a_2 m_3 + a_3 Sc + a_4 \eta\left[+\varepsilon e^{\delta t}\left\{b_1 m_1 + b_2 m_2 + b_3 m_3 + b_4 m_4 + b_5 m_6 + b_6 m_6 + b_7 Sc + b_8 \eta\right\}\right] \tag{35}$$

$$M_w = \gamma \left.\frac{\partial \omega^*}{\partial y^*}\right|_{y^*=0},$$

$$C_m = \frac{M_w}{\mu j U_0}$$

$$= \left(1 + \frac{1}{2}\beta\right)\left.\frac{\partial \omega}{\partial y}\right|_{y=0} \tag{36}$$

$$= -\left(1 + \frac{1}{2}\beta\right)\left(k_1 \eta + \varepsilon e^{\delta t}\left\{c_2 m_5 + \frac{4A\eta^2 k_1}{\delta}\right\}\right)$$

$$Nu_x = x\frac{\left(\partial T/\partial y^*\right)\Big|_{y^*=0}}{T_w - T_\infty},$$

$$Nu_x \operatorname{Re}_x^{-1} = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0}$$

$$= m_3 + \varepsilon e^{\delta t}\left[m_4\left(1 + \frac{4Am_3}{\delta}\right) - \frac{4Am_3^2}{\delta}\right] \tag{37}$$

$$Sh_x = \frac{j_w x}{D^*(C_w^* - C_\infty^*)}, \text{ where } j_w = -D^*\left.\frac{\partial C^*}{\partial y^*}\right|_{y^*=0},$$

$$Sh_x \operatorname{Re}_x^{-1} = -\left.\frac{\partial C}{\partial y}\right|_{y=0}$$

$$= Sc + \varepsilon e^{\delta t}\left\{\left(1 + \frac{4ASc}{\delta}\right)m_6 - \frac{4ASc^2}{\delta}\right\} \tag{38}$$

where $\operatorname{Re}_x = V_0 x/\nu$ is the Reynolds number.

IV. RESULTS AND DISCUSSION

The formulation of MHD free convection flow and mass transfer of an incompressible electrically conducting micropolar fluid along a semi-infinite heated vertical porous moving plate in a porous medium in the presence of thermal radiation field has been performed in the preceding sections. This enables us to carry out the numerical computations for the distribution of translational velocity, microrotation, temperature and concentration across the boundary layer for various values of the parameters. In the present study we have chosen $t=1$, $\delta=0.01$, $\varepsilon=0.001$ and $A=0.5$, while β, n, Gr, Gc, U_p, M, K, $\Pr$, R and Sc ,are varied over a range, which are listed in the figure legends.

The effect of viscosity ratio β on the translational velocity and microrotation profiles across the boundary layer are presented in Fig. 1. It is noteworthy that the velocity distribution greater for a Newtonian fluid ($\beta=0$) with given parameters, as compared with micropolar fluids until its peak value reaches. The translational velocity shows a decelerating nature near the porous plate as β-parameter increases, and then decays to the relevant free stream velocity. In addition, the magnitude of microrotation at the wall is decreased as β-parameter increases. However, the distributions of microrotation across the boundary layer do not show consistent variations with increment of β-parameter. The translational velocity and the microrotation profiles against spanwise coordinate y for different values of Grashof number Gr and modified Grashof number Gc are described in Figs. 2 and 3 respectively. It is observed that the velocity increases as Gr or Gc increase, but decreases due to microrotation. Here the positive values of Gr corresponds to a cooling of the surface by natural convection. In addition, the curves observed that the peak value of velocity increases rapidly near the wall of the porous plate as Gr or Gc increases, and then decays to the free stream velocity.

Fig. 4 illustrates the variation of velocity and microrotation distribution across the boundary layer for various values of the plate velocity U_P in the direction of the fluid flow. It is obvious that the values of translational velocity and microrotation on the porous plate are increased as the plate velocity increases, and then decayed to the free stream velocity. For different values of the magnetic field parameter M, the translational velocity and microrotation profiles are plotted in Fig. 5. It is obvious that the effect of increasing values of M -parameter results in a decreasing velocity distribution across the boundary layer. Further-more, the results shows the magnitude of microrotation on the porous plate is decreased as M -parameter increases. For different values of the Schmidt number Sc, translational velocity and the microrotation profiles are plotted in Fig. 6. It is obvious that the effect of increasing values of Sc results in a decreasing velocity distribution across the boundary layer. Furthermore, the results show that the magnitude of microrotation on the porous plate is decreased as Sc increases. For different values of the radiation parameter R, the velocity and temperature profiles are plotted in Fig. 7. It is obvious that an increase in the radiation parameter R results in decreasing velocity and temperature within the boundary layer, as well as a decreased thickness of the velocity and temperature boundary layers. This is because the large R-values correspond to an increased dominance of conduction over radiation thereby decreasing buoyancy force (thus, vertical velocity) and thickness of the thermal and momentum boundary layers.

Typical variations in the temperature profiles along the spanwise coordinate are shown in Fig. 8 for different values of Prandtl number $\Pr$. The results show that an increase of Prandtl number results in a decreasing temperature distribution across the boundary layer. Fig. 9 shows the concentration profiles across the boundary layer for various values of Schmidt number Sc. The Figure shows that an increase in Sc results in a decreasing the concentration distribution, because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity. Numerical values for functions proportional to shear stress C_f, wall couple stress C_m, heat and mass transfer rates are given in Tables 1-3. It is note that both the skin–friction coefficient and the wall couple stress coefficient decrease, as Sc increases. In addition, it is interesting to note for micropolar fluids that the skin-friction coefficient decreases as the n-parameter increases, while, an increase in n results in a decrease of the wall couple stress coefficient. From the analytical results, it can be seen that the rates of heat transfer and mass transfer depend only on Prandtl number and Schmidt number, respectively. Therefore, concentration gradient at the porous plate increases as Schmidt number increases. From Tables 2 and 3 it can be seen that C_f and C_m decrease as Pr or R increases, but increases due to the absolute values of the heat transfer rate.

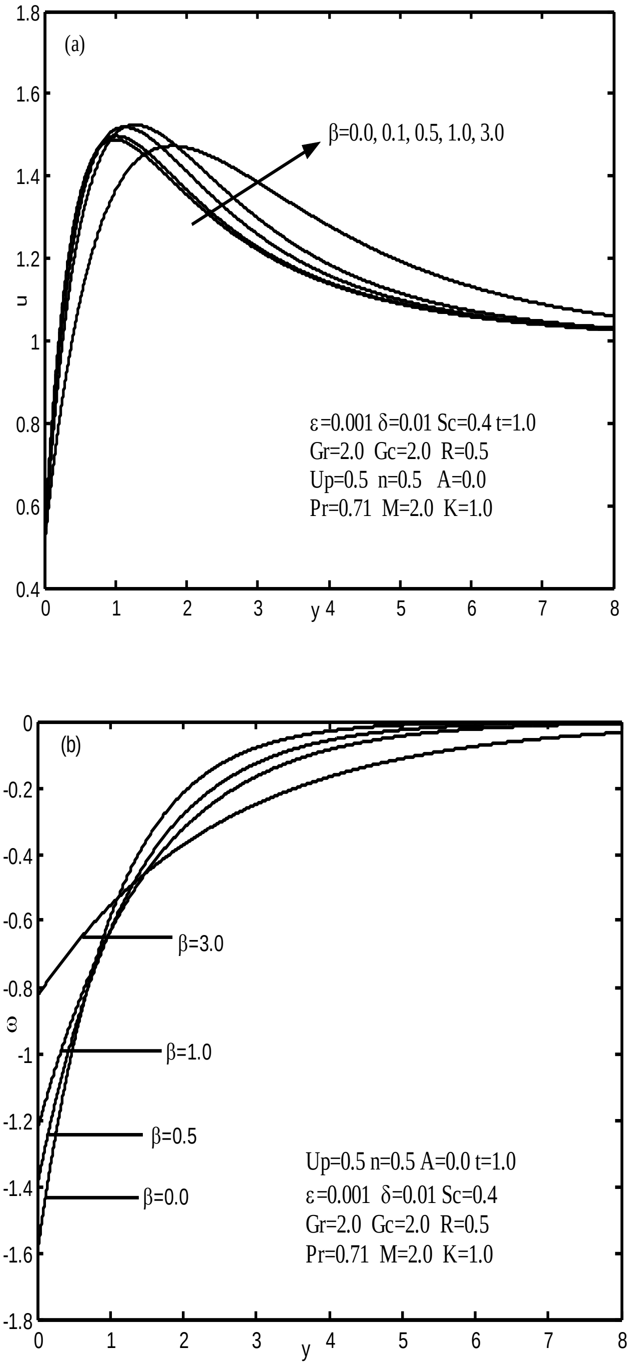


Fig. 1 Velocity and microrotation profiles for various values of β .

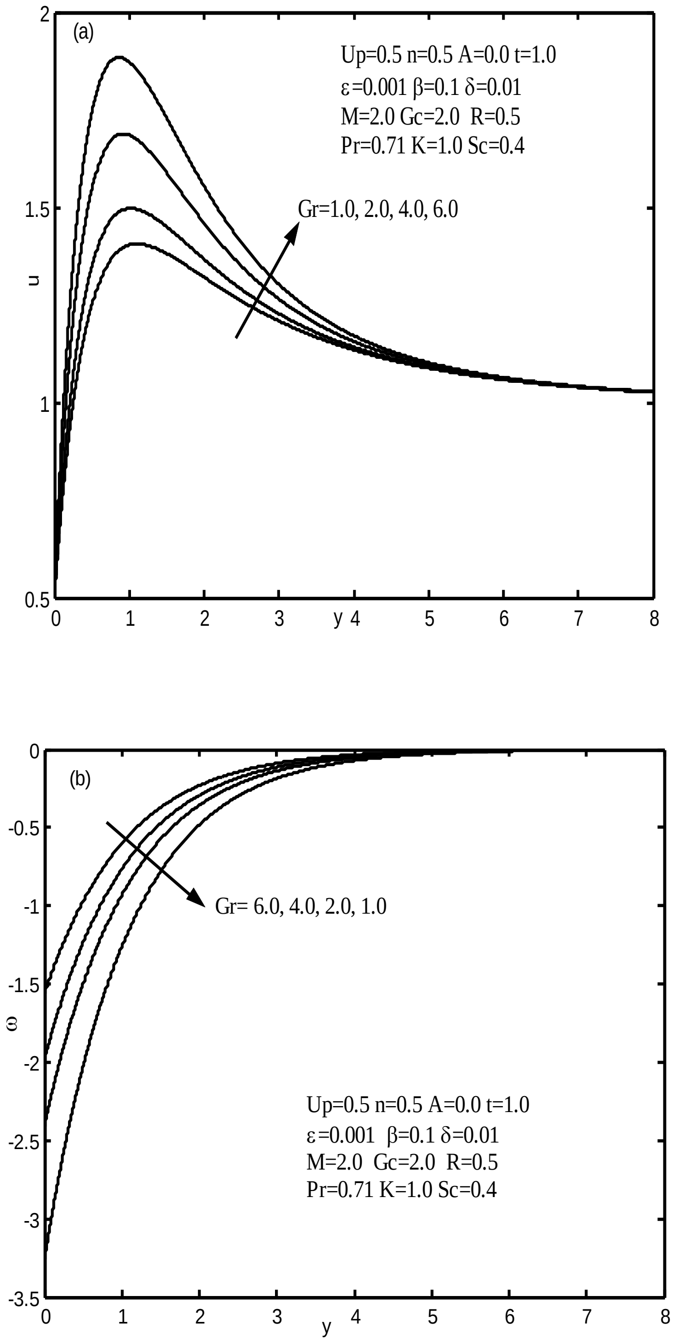


Fig. 2 Velocity and microrotation profiles for various values of Gr.

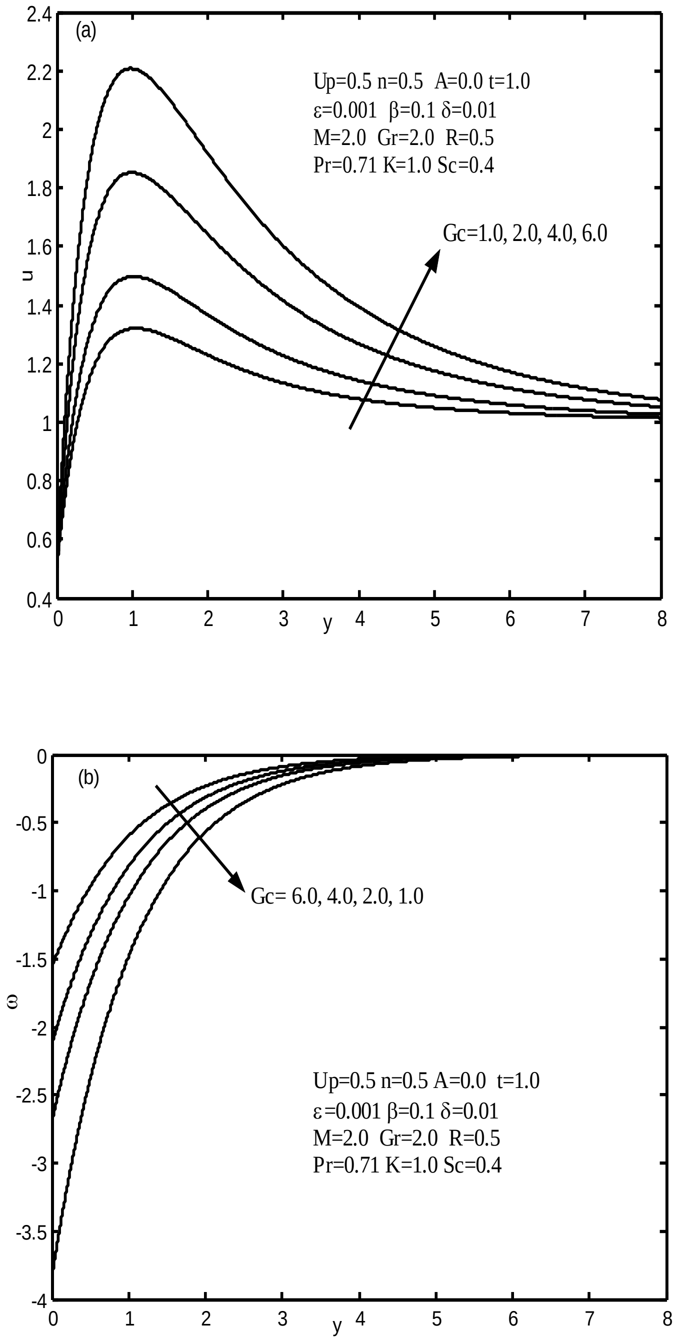
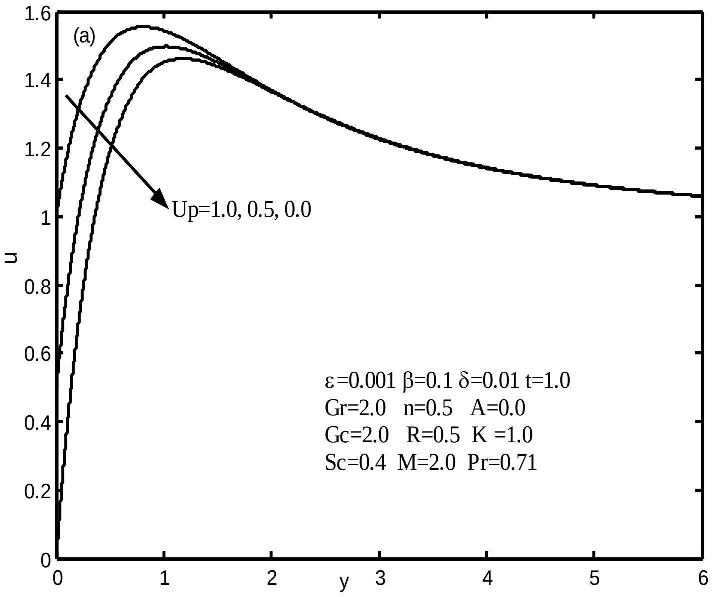


Fig. 3 Velocity and microrotation profiles for various values of Gc.



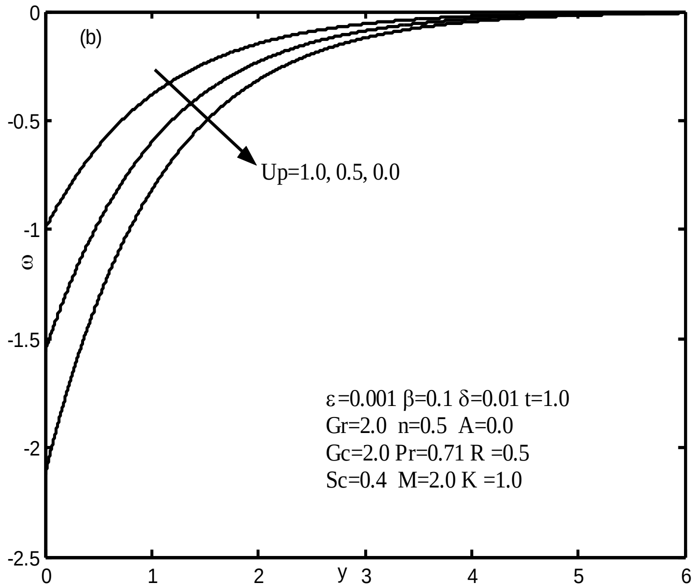


Fig. 4 Velocity and microrotation profiles for various values of Up.

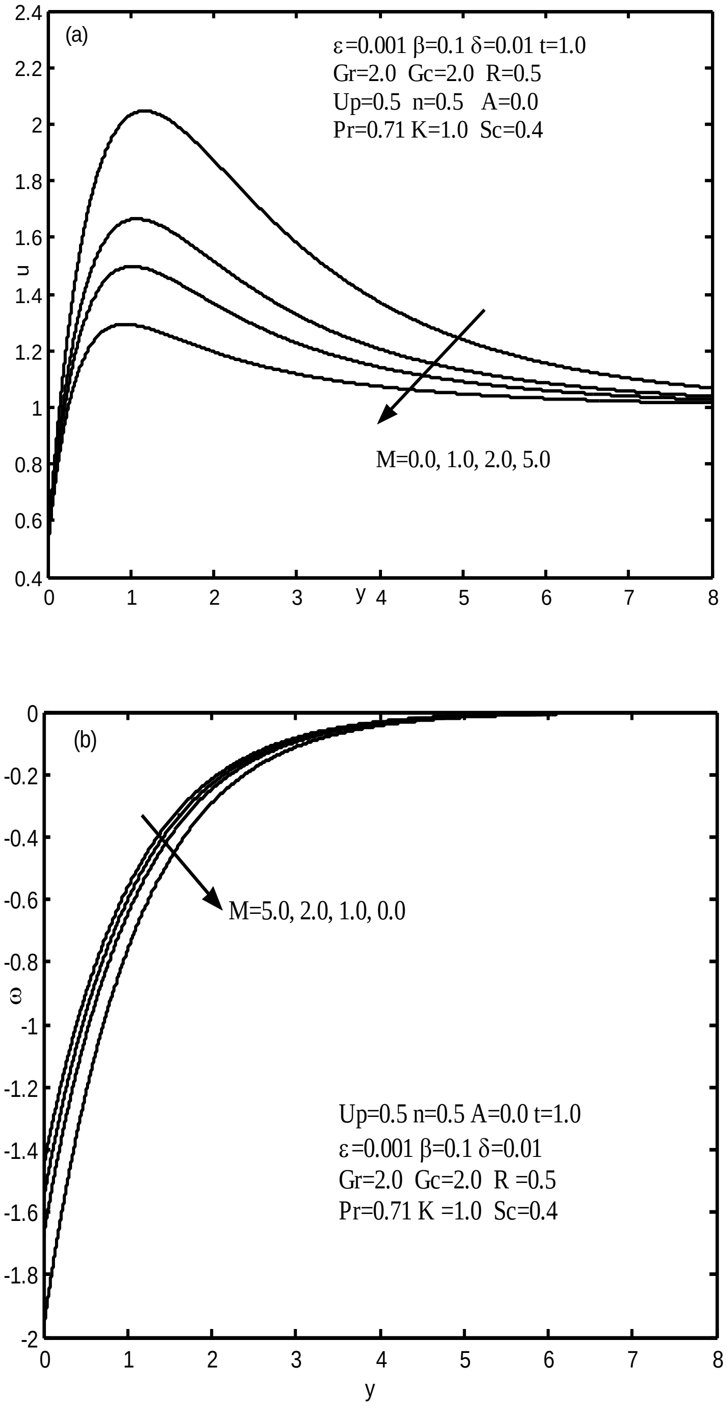
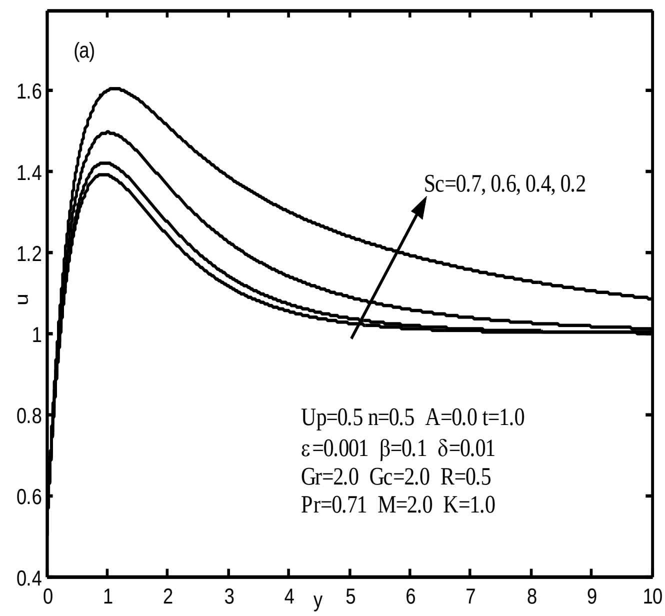


Fig. 5 Velocity and microrotation profiles for various values of M.



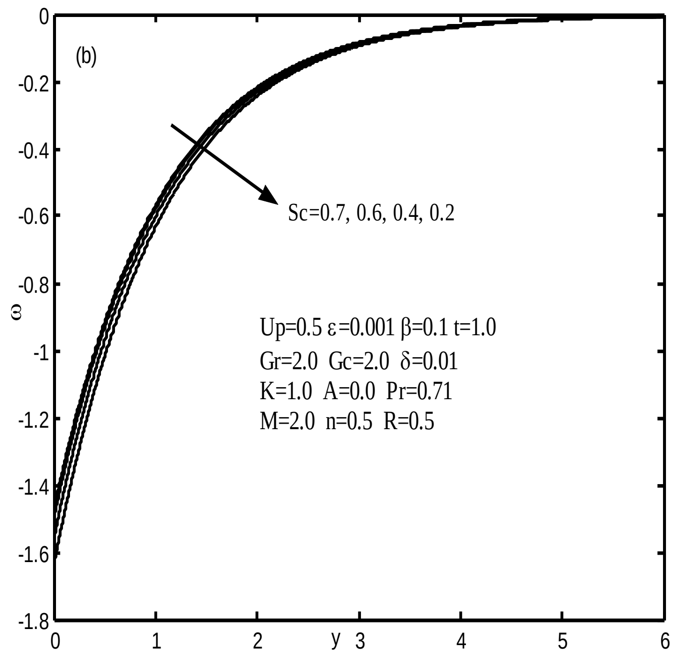
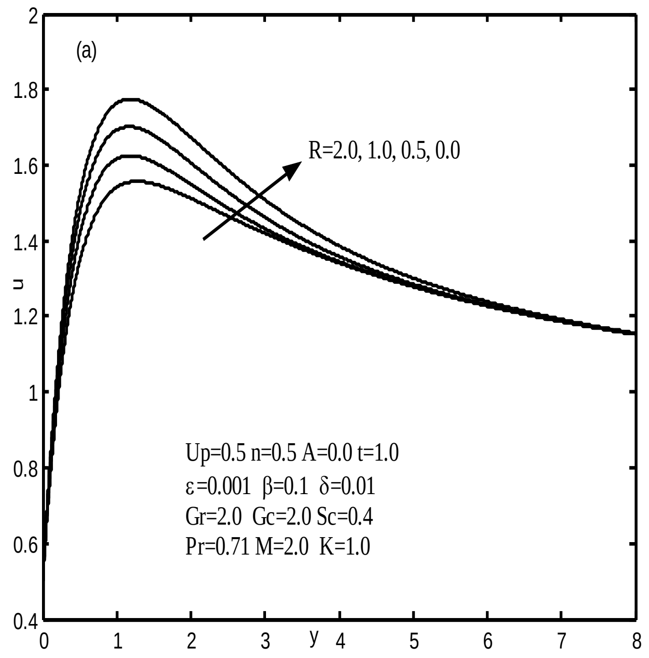


Fig. 6 Velocity and microrotation profiles for various values of Sc.



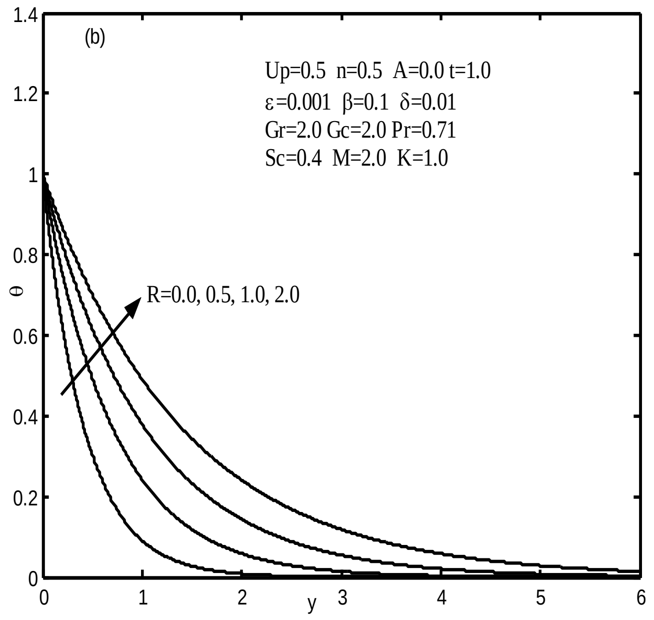


Fig. 7 Velocity and temperature profiles for various values of R.

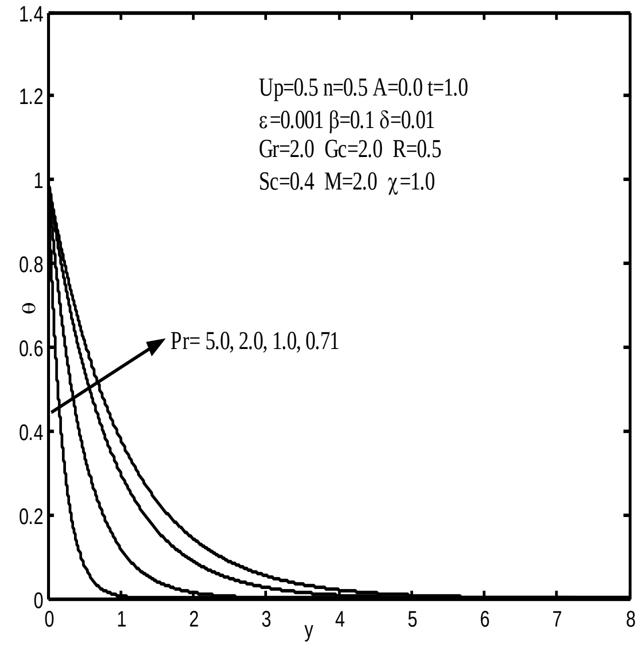


Fig. 8 Temperature profiles for different values of Pr.

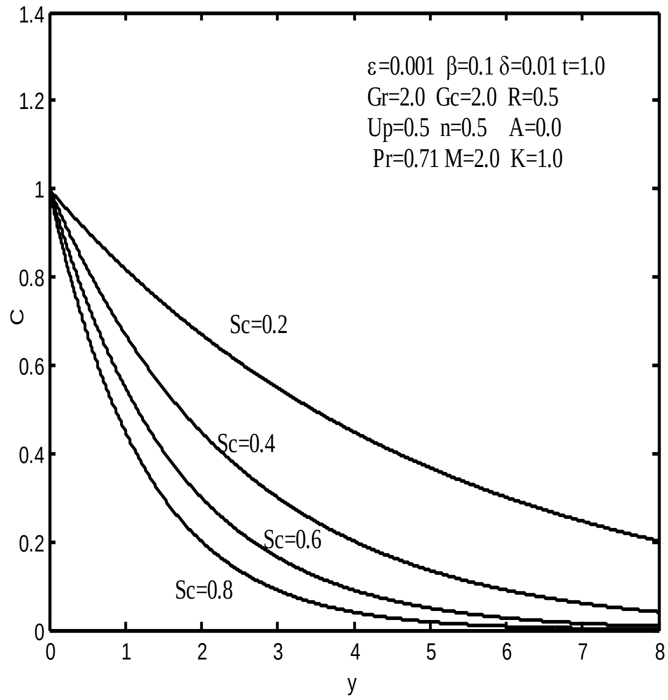


Fig. 9 Concentration profiles for various values of Sc.

TABLE I: Value of C_f, C_m, $\theta^1(0)$ and $C^1(0)$ for various values of β With $t=1$, $\varepsilon=0.001$, $\delta=0.01$, $A=0.0$, $n=0.5$, $M=2.0$, $Sc=0.4$, $\Pr=0.71$ $R=0.5$, $K=1.0$, $Gr=2.0$, $Gc=2.0$ and $U_p=0.5$.

β	n	Sc	C_f	C_m	$-\theta^1(0)$	$-C^1(0)$
0.0	0.0	0.2	6.7214	0	0.9695	0.2002
		0.4	6.4089	0	0.9695	0.4004
		0.6	6.1622	0	0.9695	0.6006
		0.8	5.9623	0	0.9695	0.8008
	0.5	0.2	6.7214	1.6837	0.9695	0.2002
		0.4	6.4089	1.6055	0.9695	0.4004
		0.6	6.1622	1.5437	0.9695	0.6006
		0.8	5.9623	1.4936	0.9695	0.8008
	1.0	0.2	6.7214	3.3674	0.9695	0.2002
		0.4	6.4089	3.2109	0.9695	0.4004
		0.6	6.1622	3.0874	0.9695	0.6006
		0.8	5.9623	2.9873	0.9695	0.8008
1.0	0.0	0.2	8.3628	0	0.9695	0.2002
		0.4	7.8868	0	0.9695	0.4004
		0.6	7.5297	0	0.9695	0.6006
		0.8	7.2520	0	0.9695	0.8008
	0.5	0.2	7.8393	1.3089	0.9695	0.2002
		0.4	7.3930	1.2344	0.9695	0.4004
		0.6	7.0583	1.1786	0.9695	0.6006
		0.8	6.7980	1.1381	0.9695	0.8008
	1.0	0.2	6.9667	3.4904	0.9695	0.2002
		0.4	6.5701	3.2918	0.9695	0.4004
		0.6	6.1622	3.1429	0.9695	0.6006
		0.8	5.9623	3.0270	0.9695	0.8008
3.0	0.0	0.2	10.5367	0	0.9695	0.2002
		0.4	9.8046	0	0.9695	0.4004
		0.6	9.2893	0	0.9695	0.6006
		0.8	8.9071	0	0.9695	0.8008
	0.5	0.2	8.9088	0.8924	0.9695	0.2002
		0.4	8.2398	0.8304	0.9695	0.4004
		0.6	7.0583	0.7868	0.9695	0.6006
		0.8	6.7980	0.7544	0.9695	0.8008
	1.0	0.2	5.5050	2.7583	0.9695	0.2002
		0.4	5.1224	2.5667	0.9695	0.4004
		0.6	4.8532	2.4319	0.9695	0.6006
		0.8	4.6534	2.3318	0.9695	0.8008

TABLE II: Value of C_f, C_m, $\theta^1(0)$ and $C^1(0)$ for various values of Pr With $t=1$, $\varepsilon=0.001$, $\beta=0.1$, $\delta=0.01$, $A=0.0$, $n=0.5$, $M=2.0$, $Sc=0.4$, $R=0.5$, $K=1.0$, $Gr=2.0$, $Gc=2.0$ and $U_p=0.5$.

Pr	C_f	C_m	$-\theta^1(0)$	$-C^1(0)$
0.71	6.5454	1.5615	0.9695	0.4004
0.5	6.3726	1.5203	1.2088	0.4004
1.0	5.9379	1.4167	2.1214	0.4004
2.0	5.3912	1.2863	5.0577	0.4004

TABLE IIII: Value of C_f , C_m , $\theta^1(0)$ and $C^1(0)$ for various values of R with $t = 1.0$, $\varepsilon = 0.001$, $\beta = 0.1$, $\delta = 0.01$, $A = 0.0$, $n = 0.5$, $M = 2.0$, $Sc = 0.4$, $\Pr = 0.71$, $K = 1.0$, $Gr = 2.0$, $Gc = 2.0$, and $U_p = 0.5$.

R	C_f	C_m	$-\theta^1(0)$	$-C^1(0)$
0.0	6.7795	1.6173	0.7112	0.4004
0.5	6.5454	1.5615	0.9695	0.4004
1.0	6.2466	1.4903	1.4178	0.4004
2.0	5.8512	1.3961	2.3888	0.4004

V. CONCLUSIONS

In this work the problem of combined heat and mass transfer flow of a viscous incompressible electrically conducting micropolar fluid past a steadily moving infinite vertical plate under the action of a uniform magnetic field with thermal radiation field is investigated. The resulting governing equations are solved by perturbation scheme. Numerical results are presented to illustrate the details of the MHD convective flow and mass transfer characteristics and their dependence on the fluid properties and flow conditions. We may conclude that the translational velocity across the boundary layer and the magnitude of microrotation at the wall are decreased with increasing values of M , Sc and $\Pr$, while they show opposite trends with increasing values of n , Gr and Gc . Also, we found that C_f and C_m decreased as R increases, but it increased due to the absolute values of the heat transfer rate.

APPENDIX

$$m_1 = \frac{1}{2\left(1+\beta\right)}\left[1+\sqrt{1+4N(1+\beta)}\right], \quad m_2 = \frac{1}{2\left(1+\beta\right)}\left[1+\sqrt{1+4(N+\frac{\delta}{4})(1+\beta)}\right],$$

$$m_3 = \frac{\Pr}{2}\left[1+\sqrt{1+\frac{4R^2}{\Pr^2}}\right], \; m_4 = \frac{\Pr}{2}\left[1+\sqrt{1+\frac{4(R^2+\Pr^2)}{\Pr^2}}\right],$$

$$m_5 = \frac{\eta}{2}\left[1+\sqrt{1+\frac{\delta}{\eta}}\right], \quad m_6 = \frac{Sc}{2}\left[1+\sqrt{1+\frac{\delta}{Sc}}\right]$$

$$a_1 = U_p - 1 - a_2 - a_3 - a_4, \; a_2 = \frac{-Gr}{\left(1+\beta\right)m_3^2 - m_3 - N},$$

$$a_3 = \frac{-Gc}{\left(1+\beta\right)Sc^2 - Sc - N}, a_4 = \frac{2\beta\eta}{\left(1+\beta\right)\eta^2 - \eta - N} \; k_1 = \theta_1 k_1, b_1 = \frac{Am_1 a_1}{\left(1+\beta\right)m_1^2 - m_1 - (N+\frac{\delta}{4})},$$

$$b_2 = -(1+b_1+b_3+b_4+b_5+b_6+b_7+b_8),$$

$$b_3 = \frac{Am_3\left(a_2+\frac{4Gr}{\delta}\right)}{\left(1+\beta\right)m_3^2 - m_3 - (N+\frac{\delta}{4})}, \quad b_4 = \frac{-Gr\left(1+\frac{4Am_3}{\delta}\right)}{\left(1+\beta\right)m_4^2 - m_4 - (N+\frac{\delta}{4})},$$

$$b_5 = \frac{2\beta m_5}{\left(1+\beta\right) m_5^2 - m_5 - (N + \frac{\delta}{4})} k_2 = \theta_2 k_2 , \quad b_6 = \frac{-Gc\left(1 + \frac{4ASc}{\delta}\right)}{\left(1+\beta\right) m_6^2 - m_6 - (N + \frac{\delta}{4})} ,$$

$$b_7 = \frac{ASc\left(a_3 + \frac{4Gc}{\delta}\right)}{\left(1+\beta\right) Sc^2 - Sc - (N + \frac{\delta}{4})} , \quad b_8 = \frac{A\left(a_3\eta - \frac{8\beta\eta^2 k_1}{\delta}\right)}{\left(1+\beta\right)\eta^2 - \eta - (N + \frac{\delta}{4})} ,$$

$$k_1 = \frac{n}{1 - n\theta_1\left(\eta - m_1\right)}\left\{\left(U_p - 1\right) m_1 + a_2\left(m_3 - m_1\right) + a_3\left(Sc - m_1\right)\right\} \; k_2 = \frac{k_3 + nk_4 m_2}{1 + n\theta_2\left(m_2 - m_5\right)} ,$$

$$k_3 = k_1\frac{A\eta}{\delta} + n\left(b_1 m_1 + b_3 m_3 + b_4 m_4 + b_6 m_6 + b_7 Sc + b_8\eta\right) \; k_4 = -(1 + b_1 + b_4 + b_5 + b_6 + b_7 + b_8) .$$

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