The International Journal Of Engineering And Science (IJES)
||Volume||2 ||Issue|| 9 ||Pages|| 51-57||2013||
ISSN(e): 2319 – 1813 ISSN(p): 2319 – 1805

Free Convective Oscillatory Flow of a Visco-Elastic Fluid Past A Porous Plate In Presence Of Radiation And Mass Transfer

[1]Madhumita Mahanta [2]Rita Choudhury
[1]*Department of Mathematics, Girijananda Chowdhury Institute of Management and Technology, Guwahati-781 017, Assam, India*
[2]*Department of Mathematics, Gauhati University, Guwahati-781 014, Assam, India*

ABSTRACT
The effects of heat and mass transfer on unsteady free convective oscillatory flow of an optically thin fluid past a porous plate in presence of radiation have been investigated. The flow is characterized by the second-order fluid model. Analytical solutions for velocity, temperature and concentration of the governing equations of fluid flow are obtained by using perturbation technique. Also, the coefficients of skin friction, rate of heat transfer and rate of mass transfer at the plate are calculated. The profiles of velocity and skin friction are presented through graphs for various values of physical parameters to discuss the effects of the visco-elastic parameter involved in the solution.

KEY WORDS: *free convection, mass transfer, radiation, oscillatory flow, visco-elastic fluid.*

Date of Submission: 26, August, 2013 Date of Acceptance: 20, September 2013

I. INTRUDUCTION

Heat and mass transfer on unsteady flow past a porous plate in presence of radiation has gained interest owing to its applications in the fields of plasma physics, geophysics, geohydrology, environmental engineering etc. Some examples of such areas are gas turbines, various propulsion devices for aircrafts, missiles, satellites, space vehicles etc. Because of wide applications of such flows, numerous scholars have paid their attention in this field. Seddeek *et al.* [1] have studied the effects of thermal diffusivity on heat transfer over a stretching surface with variable heat flux in the presence of radiation. The effect of radiation on free convective flow of fluid has been investigated by Hossain *et al.* [2, 3] and Raptis *et al.* [4]. Raptis [5], Badruddin *et al.* [6] and Mukhopadhyay *et al.* [7] have considered the flow through a porous medium in presence of radiation. The MHD flow adjacent to a non-isothermal wedge in the presence of heat source or sink has been analyzed by Chamkha *et al.* [8] and Duwairi [9]. England *et al.* [10] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Bestman *et al.* [11] have investigated the hydromagnetic free convection flow with radiation and heat transfer in a rotating and optically thin fluid. The effect of radiation in an optically thin fluid past a vertical plate (when the induced magnetic field is taken into account) has been studied by Raptis *et al.* [12]. The unsteady flow of an optically thin fluid in the presence of convection and mass transfer has been investigated by Raptis *et al.* [13]. The free convection one dimensional flow with radiation and heat transfer for an optically thin fluid past a vertical oscillating plate in the presence of chemical reaction has been studied by Manivannan *et al.* [14]. Vijayalakshmi [15] has investigated the effects of radiation on free convection flow past an impulsively started vertical plate in a rotating fluid. For an optically thin fluid the two-dimensional free convective oscillatory flow and mass transfer past a porous plate in presence of radiation has been investigated by Raptis [16].

In view of the above, the unsteady free convective flow of a visco-elastic fluid past an infinite vertical porous plate in the presence of radiation has been considered in this paper. The constitutive equation for second-order fluid [Coleman & Noll1 (1960)] is

$$\tau_{ij} = -p\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)ik} A_{(1)kj} \tag{1.1}$$

where τ_{ij} are the stress tensor, p the hydrostatic pressure, $A_{(i)}$ are kinematic Rivlin-Erickson tensors; μ_1, μ_2, μ_3 are material constants describing viscosity, elasticity and cross-viscosity where $\mu_2 < 0$ from thermodynamic consideration [Coleman and Markovitz (1964)]. Equation (1.1) is valid for low rates of shear.

II. MATHEMATICAL FORMULATION

We consider the unsteady two-dimensional flow past an infinite vertical porous plate through which suction occurs with constant velocity. Let $\bar{x}$-axis be taken along the direction of the plate and $\bar{y}$-axis be normal to it. All the fluid properties are considered constant except that the influence of density variations with temperature and concentration. The radiation to the $\bar{x}$-direction is considered negligible as compared to the $\bar{y}$-direction. The governing equations of flow are

Equation of Continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.1}$$

i.e. $\bar{v} = -v_0 \ (v_0 > 0)$ (2.2)

Equation of motion:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0 \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{d\bar{U}}{d\bar{t}} + g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty) + \nu_1 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \nu_2\left[\frac{\partial^2}{\partial \bar{y}^2}\left(\frac{\partial \bar{u}}{\partial \bar{t}}\right) - v_0 \frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right] \tag{2.3}$$

Equation of energy:

$$\frac{\partial \bar{T}}{\partial \bar{t}} - v_0 \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{16\, d\bar{\sigma}\bar{T}_\infty^3}{\rho C_p}(\bar{T}_\infty - \bar{T}) \tag{2.4}$$

Equation of concentration:

$$\frac{\partial \bar{C}}{\partial \bar{t}} - v_0 \frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{2.5}$$

subject to boundary conditions

$$\begin{aligned} &\bar{y} = 0: \bar{u} = 0, \bar{v} = -\bar{v}_0, \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{\bar{q}}{\kappa}, \bar{C} = \bar{C}_w \\ &\bar{y} \to \infty: \bar{u} \to \bar{U} = U_0(1 + \varepsilon e^{i\bar{\omega}\bar{t}}),\ \bar{T} \to \bar{T}_\infty, \bar{C} \to \bar{C}_\infty \end{aligned} \tag{2.6}$$

Here $\bar{u}$ and $\bar{v}$ are components of the velocity in the directions of $\bar{x}$ and $\bar{y}$-axis, $\bar{t}$ is the time, $\bar{U}$ is the free stream velocity, g is the acceleration due to gravity, $\beta, \bar{\beta}$ are the coefficients of volume expansion for heat and mass transfer, $\bar{T}$ and $\bar{C}$ are the fluid temperature and concentration, ν_1, ν_2 are the kinematic coefficients of viscosity and elasticity, ρ is the density of the fluid, C_p is the specific heat at constant pressure, κ is the thermal conductivity, d is the absorption coefficient, $\bar{\sigma}$ is the Stefan-Boltzman constant, $\bar{T}_\infty$is the fluid temperature far away from the plate, $\bar{C}_w$and $\bar{C}_\infty$ are the molar concentration at the plate and far away from the plate, D is the chemical diffusivity, $\bar{q}$ is the radiative heat flux, $U_0$is the mean free stream velocity, $\bar{\omega}$ is the frequency of vibration of the fluid and v_0 is the constant suction velocity.

Let us introduce the following non-dimensional quantities

$$\begin{aligned} &y = \frac{\bar{y}v_0}{\nu_1}, t = \frac{\bar{t}v_0^2}{4\nu_1}, T = \frac{\bar{T} - \bar{T}_\infty}{\nu_1\bar{q}/\kappa v_0}, C = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \\ &u = \frac{\bar{u}}{U_0}, \omega = \frac{4\bar{\omega}\nu_1}{v_0^2} \end{aligned} \tag{2.7}$$

Introducing the non-dimensional quantities (2.7), the equations (2.3) to (2.5) reduce to

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU}{dt} + G_r T + G_c C + \frac{\partial^2 u}{\partial y^2} + \alpha\left[\frac{1}{4}\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}\right) - \frac{\partial^3 u}{\partial y^3}\right] \tag{2.8}$$

$$P_r\left(\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} - ST \tag{2.9}$$

$$S_c\left(\frac{1}{4}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y}\right) = \frac{\partial^2 C}{\partial y^2} \tag{2.10}$$

subject to boundary conditions

$$\begin{aligned} &y = 0: u = 0,\ \frac{\partial T}{\partial y} = -1, C = 1 \\ &y \longrightarrow \infty: u \to U(t) = 1 + \varepsilon e^{i\omega t},\ T \to 0, C \to 0 \end{aligned} \tag{2.11}$$

where $G_r = \frac{g\beta v_1^2 q}{\kappa U_0 v_0^3}$ is the Grashof number for heat transfer, $G_c = \frac{g\bar{\beta} v_1 (C_w - C_\infty)}{U_0 v_0^2}$ is the Grashof number for mass transfer, $P_r = \frac{\rho v_1 C_p}{\kappa}$ is the Prandtl number, $S = \frac{16 d\sigma T_\infty^2 v_1^2}{\kappa v_0^2}$ is the radiation parameter, $S_c = \frac{v_1}{D}$ is the Schmidt number and $\alpha = \frac{v_2 v_0^2}{v_1^2}$ is the visco-elastic parameter.

III. METHOD OF SOLUTION

To solve the equations (2.8) to (2.10) subject to boundary conditions (2.11), we assume the solutions for $\varepsilon_c \approx \varepsilon \ll 1$ as follows

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon_c u_1(y) e^{i\omega t}, \\ T(y,t) &= T_0(y) + \varepsilon_c T_1(y) e^{i\omega t} \\ C(y,t) &= C_0(y) + \varepsilon_c C_1(y) e^{i\omega t} \end{aligned} \tag{3.1}$$

Substituting (3.1) into equations (2.8) to (2.10) and equating the harmonic and non-harmonic terms, we get

$$\alpha u_0''' - u_0'' - u_0' = G_r T_0 + G_c C_0 \tag{3.2}$$

$$\begin{aligned} &\alpha u_1''' - \left(1 + \alpha \frac{i\omega}{4}\right) u_1'' - u_1' - \frac{i\omega}{4} u_1 \\ &= -G_r T_1 - G_c C_1 - \frac{i\omega}{4} \end{aligned} \tag{3.3}$$

$$T_0'' + P_r T_0' = S T_0 \tag{3.4}$$

$$T_1'' + P_r T_1' - \frac{i\omega}{4} P_r T_1 = -S T_1 \tag{3.5}$$

$$C_0'' + S_c C_0' = 0 \tag{3.6}$$

$$C_1'' + S_c C_1' - \frac{i\omega}{4} S_c C_1 = 0 \tag{3.7}$$

where the primes denote the differentiation with respect to *y*.
The corresponding boundary conditions are

$$\begin{aligned} &y = 0: u_0 = 0, u_1 = 0, \frac{\partial T_0}{\partial y} = -1, \frac{\partial T_1}{\partial y} = 0, C_0 = 1, C_1 = 0 \\ &y \longrightarrow \infty: u_0 = 1, u_1 = 1,\ T_0 = 0 = T_1, C_0 = 0 = C_1 \end{aligned} \tag{3.8}$$

Substituting the solutions of equations (3.2) to (3.7) under the boundary conditions (3.8), we obtain

$$T(y,t) = \frac{1}{a_1} e^{-a_1 y} \tag{3.9}$$

$$C(y,t) = e^{-S_c y} \tag{3.10}$$

where $a_1 = \frac{1}{2}[P_r + \sqrt{P_r^2 + 4S}]$

and

$$u(y,t) = 1 + A_1 e^{-y} - \frac{G_r}{a_1^2(a_1-1)}e^{-a_1 y} - \frac{G_c}{S_c(S_c-1)}e^{-S_c y}$$
$$+ \alpha\left[A_2 e^{-y} - \frac{A_1}{a_1^2 - a_1}e^{-a_1 y} + \frac{G_r}{(a_1-1)^2}e^{-a_1 y} + \frac{G_c S_c}{(S_c-1)^2}e^{-S_c y}\right]$$
$$+ \varepsilon(\cos\omega t + i\sin\omega t)\left[1 - e^{-\frac{1}{32}\omega^2 y} + \alpha\left(\frac{1}{4} - \frac{\omega^2}{16}\right)\frac{1}{\sqrt{1+\omega^2}}ye^{-\frac{1}{32}\omega^2 y}\right] \quad (3.11)$$

where $A_1 = \frac{G_r}{a_1^2(a_1-1)} + \frac{G_c}{S_c(S_c-1)} - 1$ and $A_2 = \frac{A_1}{a_1^2 - a_1} + \frac{G_r}{(a_1-1)^2} + \frac{G_c S_c}{(S_c-1)^2}$.

Separating real and imaginary parts and taking only the real part, we obtain the velocity field for $\omega t = \pi$ in the form

$$u(y,t) = 1 + A_1 e^{-y} - \frac{G_r}{a_1^2(a_1-1)}e^{-a_1 y} - \frac{G_c}{S_c(S_c-1)}e^{-S_c y}$$
$$+ \alpha\left[A_2 e^{-y} - \frac{A_1}{a_1^2 - a_1}e^{-a_1 y} + \frac{G_r}{(a_1-1)^2}e^{-a_1 y} + \frac{G_c S_c}{(S_c-1)^2}e^{-S_c y}\right]$$
$$- \varepsilon\left[1 - e^{-\frac{1}{32}\omega^2 y} + \alpha\left(\frac{1}{4} - \frac{\omega^2}{16}\right)\frac{1}{\sqrt{1+\omega^2}}ye^{-\frac{1}{32}\omega^2 y}\right]$$

IV. SKIN FRICTION, RATE OF HEAT AND MASS TRANSFER

The non-dimensional skin friction τ_w at the plate ($y=0$) is given by

$$\tau_\omega = [u_0'(0) - \alpha u_0''(0)] + \varepsilon e^{i\omega t}[(1 - \alpha i\omega)u_1'(0) - \alpha u_1''(0)] \quad (4.1)$$

The dimensionless heat transfer co-efficient at the plate is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = 1 \quad (4.2)$$

Similarly, the mass transfer co-efficient S_h at the plate is given by

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = S_c \quad (4.3)$$

V. RESULTS AND DISCUSSION

The aim of the problem is to bring out the effects of the visco-elastic parameter on the flow characteristics. The visco-elastic effect is exhibited through the non-dimensional parameter α. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\alpha = 0$. Figures 1 to 3 illustrate the variations of velocity profile against y for various values of visco-elastic parameter $(\alpha = 0, -0.25, -0.4)$along with various values of Prandtl number (P_r), Grashof number for heat transfer (G_r), Grashof number for mass transfer (G_c), radiation parameter (S), Schmidt number (S_c) and some fixed values ω=0.1, ε=0.02 and ωt=π. It is noticed from the figures that the velocity of the fluid enhances at each point of the flow field in both Newtonian and non-Newtonian cases. Again, the velocity profile accelerates with the increase of the absolute values of visco-elastic parameter in comparison with that of Newtonian fluid flow phenomenon. Also, it is seen that the velocity increases during the rising behaviour of the Grashof number for mass transfer

(Fig. 1 and Fig. 2) but reverse behaviour is observed in case of Schmidt number (Fig. 1 and Fig. 3) in both Newtonian and non-Newtonian cases.The variations of shearing stress at the plate (y=0) against various values of flow parameters are presented in figures 4, 5 and 6. The figures reveal that the shearing stress at the plate decreases due to the increase of the absolute values of visco-elastic parameter along with the increase of the Prandtl number (Fig. 4), Grashof number for heat transfer (Fig. 5) and frequency of oscillation (Fig. 6).

The temperature and concentration fields are not significantly affected by visco-elastic parameter.

VI. CONCLUSION

The problem of unsteady two-dimensional free convective flow past an infinite vertical porous plate is studied analytically. The results of investigation may be summarized in the following conclusions:

- The velocity distribution accelerates and then decelerates in both Newtonian and non-Newtonian cases.
- The rising trend of the absolute values of the visco-elastic parameter depicts the increase of velocity profile in comparison to that of Newtonian fluid.
- The growth of the absolute values of the visco-elastic parameter diminishes the nature of the skin friction at each point of the fluid region with the increase of the Prandtl number, Grashof number for heat transfer and frequency of oscillation respectively.
- The temperature and the concentration fields are unaffected due to the variation of visco-elastic parameter.

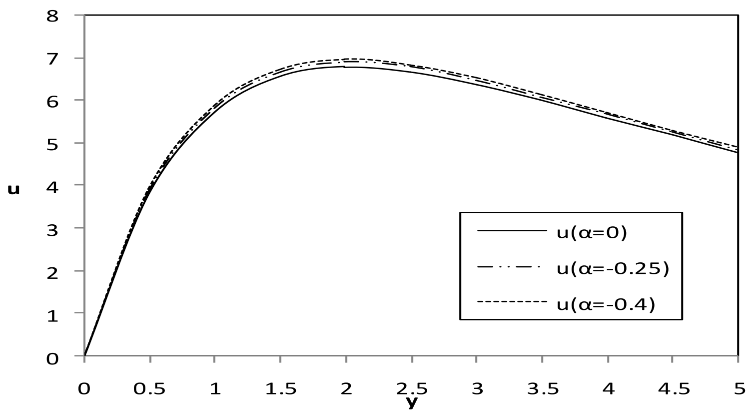

Fig. 1 Variation of velocity u against y for
Pr=5, G_r=5, G_c=2, S_c=0.22, S=0.2, ω=0.1, ε=0.02

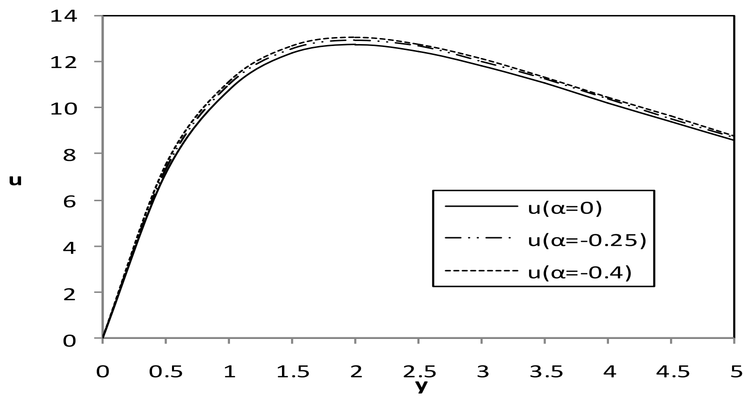

Fig. 2 Variation of velocity u against y for
Pr=5, G_r=5, G_c=4, S_c=0.22, S=0.2, ω =0.1, ε=0.02

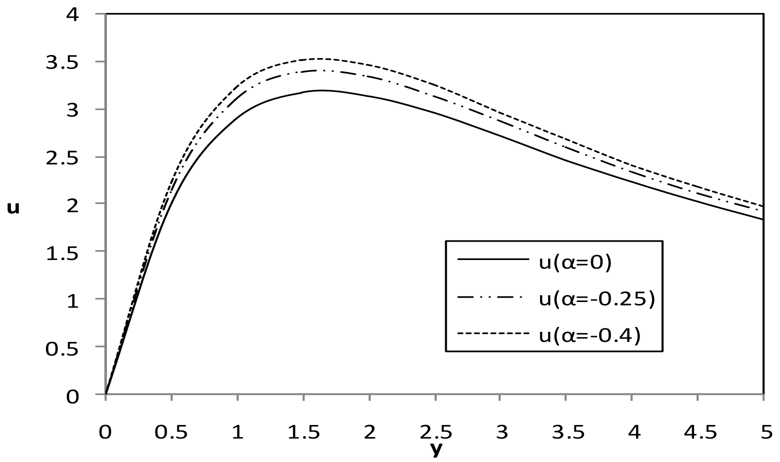

Fig. 3 Variation of velocity u against y for
Pr=5, G_r=5, G_c=2, S_c=0.44, S=0.2, ω =0.1, ε=0.02

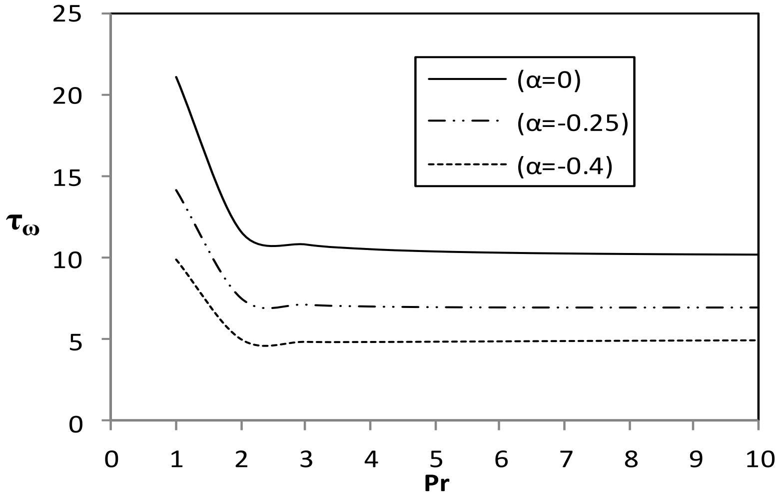

Fig. 4 Variation of skin friction τ_ω against P_r for
G_r=5, G_c=2, S_c=0.44, S=0.2, ω =0.1, ε=0.02

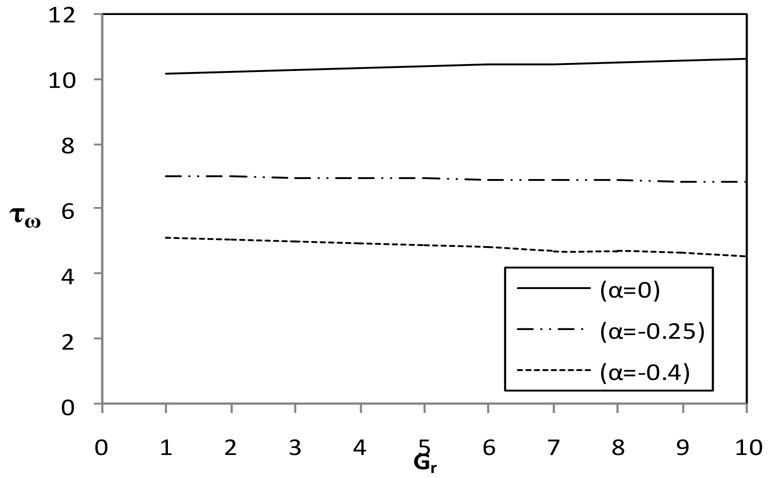

Fig. 5 Variation of skin friction τ_ω against G_r for
P_r=5, G_c=2, S_c=0.44, S=0.2, ω =0.1, ε=0.02

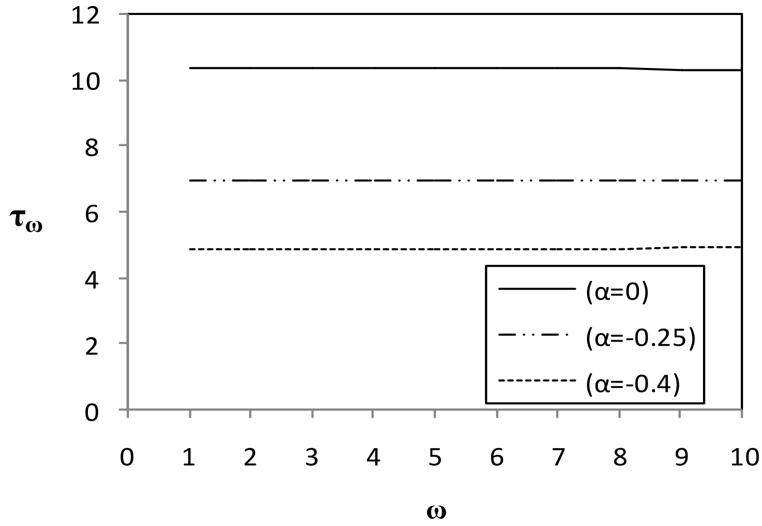

Fig. 6 Variation of skin friction τ_ω against ω for P_r=5, G_r =5, G_c=2, S_c=0.44, S=0.2, ε=0.02

REFERENCES

[1] Seddeek, M. A., Abdelmeguid, M. S., Effects of Radiation and Thermal Diffusivity on Heat Transfer over a Stretching Surface with Variable Heat Flux, *Phys Letters A, 348 (2006), 3-6, 172-179.*

[2] Hossain, M. A., Alim M. A., Rees, D. A. S., The Effect of Radiation on Free Convection from a Porous Vertical Plate, *Int. J. Heat Mass Trans, 42 (1999), 1, 181-191.*

[3] Hossain, M. A., Khanafer, K., Vafai, K., The Effect of Radiation on Free Convection Flow of a Fluid with Variable Viscosity from a Porous Vertical Plate, *Int. J. Therm. Sci., 40 (2001), 2, 115-124.*

[4] Raptis, A., Toki, C. J., Thermal Radiation in the Presence of Free Convective Flow Past a Moving Vertical Porous Plate-An Analytical Solution, *Int. J. Appl. Mech. Engg., 14 (2009), 4, 1115-1126.*

[5] Raptis, A., Radiation and Free Convection Flow Through a Porous Medium, *Int. Comm. Heat Mass Trans., 25 (1998), 2, 289-295.*

[6] Badruddin, I. A., Zainal, Z. A., Awastha Narayan, P. A., Seetharamu, K. N., Siew, L. W., Free Convection and Radiation for a Vertical Wall with Varying Temperature Embedded in a Porous Medium, *Int. J. Therm. Sci., 45 (2006), 5, 487-493.*

[7] Mukhopadhyay, S., Layek, G. C., Radiation Effect on Forced Convective Flow and Heat Transfer over a Porous Plate in a Porous Medium, *Meccanica, 44 (2009), 5, 587-597.*

[8] Chamkha, A. J., Mujtaba, M., Quadri, A., Issa, C., Thermal Radiation Effects on MHD Forced Convection Flow Adjacent to a Non-Isothermal Wedge in the Presence of a Heat Source or Sink, *Heat Mass Trans., 39 (2003), 4, 305-312.*

[9] Duwairi, H. M., Viscous and Joule Heating Effects on Forced Convection Flow from Radiate Isothermal Porous Surfaces, *Int. J. Numer. Methods Heat Fluid Flow, 15 (2005), 5-6, 429-440.*

[10] England, W. G., Emery, A. F., Thermal Radiation Effects on the Laminar Free Convection Boundary Layer of an Absorbing Gas, *J. Heat Trans., 91 (1969), 1, 37-44.*

[11] Bestman, A. R., Adiepong, S.K., Unsteady Hydromagnetic Free Convection Flow with Radiative Heat Transfer in a Rotating Fluid, *Astrophys. Space Sci., 143 (1988), 1, 73-80.*

[12] Raptis, A., Perdikis, C., Leontitsis, A., Effects of Radiation in an Optically Thin Gray Gas Flowing Past a Vertical Infinite Plate in the Presence of a Magnetic Field, *Heat Mass Trans., 39 (2003), 8-9, 771-773.*

[13] Raptis, A., Perdikis, C., Thermal Radiation of an Optically Thin Gray Gas, *Int. J. Appl. Mech. Engg., 8 (2003), 1, 131-134.*

[14] Manivannan, K., Muthucumaraswamy, R., Thangaraj, V., Radiation and Chemical Reaction Effects on Isothermal Vertical Oscillating Plate with Variable Mass Diffusion, *Therm. Sci. 13 (2009), 2, 155-162.*

[15] Vijayalakshmi, A. R., Radiation Effects on Free Convection Flow Past an Impulsively Started Vertical Plate in a Rotating Fluid, *Theoret. Appl. Mech., 37 (2010), 2, 79-95.*

[16] Raptis, A., Free Convective Oscillatory Flow and Mass Transfer Past a Porous Plate in the Presence of Radiation for an Optically Thin Fluid, *Therm. Sci. 15 (2011), 3, 849-857.*