

Dynamics of Small Hydro-Power Station (SHS) Turbine for Slow Moving Water Body

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-----ABSTRACT-----

This article presents an analytical technique for theoretical determination of the optimal impact angle of the incident force of the slow moving water bodies on the turbine blade of a small hydro-power station to yield maximum electric power to ensure optimal turbine blade designs for impact angle enhanced efficiency. The process (dynamics) of converting the energy of flowing water bodies to electricity and the quantum of the derivable power depends largely on the head, the speed and the impact angle of the incident force of the water body on the turbine blades. It therefore follows that the determination of the optimal impact angle of the incident force of the water body on the turbine blades for small hydro-power stations (SHS) is of major engineering interest in slow moving water bodies where the head and the speed are relatively 'low'. It also investigated the variation of impact angle with the power output so as to determine the optimal impact angle for maximum power output. This SHS can easily be deployed by small and cottage firms in slow moving waters without elaborate cost and technology, and the electricity generated can be sold to the neighboring consumers thereby reducing their dependency on fossil fuel generators and national grid for electricity thus reducing the carbon foot print of such benefiting communities.

KEYWORDS - Climate change; Electricity generation; Optimal impact angle; Small hydro-power stations.

I. INTRODUCTION

Climate change due to carbon emissions from fossil fuel is already a major environmental concern for everybody. There is a global concern about the fossil fuels environmental impact and degradation such as global warming; therefore there is need for sustainable alternative energy sources that are affordable and environmentally friendly. The world is now moving towards "clean energy sources" for electricity generation such as solar, wind, small hydro-power stations (SHS) to mention a few.

In hydro power generation, the quantum of energy generation depends largely on the head, the speed and the impact angle of the incident water body on the turbine blade. But in slow moving water bodies such as streams and lagoons, the head and speed are relatively lower than in the water can be used for large hydro stations. Therefore, for SHS the impact angle of the incident force from the moving water body on the turbine blade to a large extent determines the quantum of derivable power from the system. It therefore follows that determination of the optimal impact angle of the force of water on the turbine blades for SHS is of major engineering interest. This research effort intends to study the dynamics of SHS deployed for electricity generation and theoretically estimate the optimal impact angle of the human force on the pedal to yield maximum electric power. The results will be applied to the analysis of the energy conversion process in SHS to determine optimal turbine designs for impact angle enhanced efficiency.

SHS can be used by individuals and small companies to generate electricity and sell to the local people in their surroundings thereby reducing dependence on the National Grid and fossil fuel electricity. A miniature model for electricity generation from slow flowing waters such as lagoons and streams can be easily replicated anywhere. Several works has been carried out on small hydropower stations and turbines [1 – 5] but there is need to investigate the impact angle and how it affects the power output for maximal power conversion in small hydro turbines. The main objective of this research is to investigate the variation of impact angle with

the power output so as to determine the optimal impact angle for maximum power output of a typical SHS energy conversion process and predict optimal impact for various turbine configurations.

II. NOMENCLATURE

a	area of the turbine blade
$\mathbf{F}, \mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z, \mathbf{F}_r$	force, force in the x, y, z , directions and result force respectively
h	length of the turbine blade
i, j, k	unit vectors along x, y , and z , axes of the rotating frame
I, J, K	unit vectors along X, Y , and Z , axes of the inertia frame
i, \dot{i}, \ddot{i}	distance, first and second time derivates of distance in x direction
j, \dot{j}, \ddot{j}	distance, first and second time derivates of distance in y direction
k, \dot{k}, \ddot{k}	distance, first and second time derivates of distance in z direction
\dot{m}	mass flow rate of water
\mathbf{M}_o	moments about the centre of turbine blade shaft
P	power developed by the turbine blade
$P0(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 0°
$P10(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 10°
$P20(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 20°
$P30(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 30°
$P40(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 40°
$P50(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 50°
$P60(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 60°
$P70(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 70°
$P80(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 80°
$P90(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 90°
$P45(\text{watts})$	power developed by the turbine when inclination angle to the vertical θ is 45°
$\mathbf{r}_p, \mathbf{v}_p, \mathbf{a}_p$	position vector, velocity and acceleration derivatives respectively of the water particle in the control volume about to hit the turbine blade
u	velocity of water particle immediately after hitting the turbine blade
\bar{v}	velocity of the water body
x, y, z	rotating frame axes
X, Y, Z	inertia frame axes
x, \dot{x}, \ddot{x}	distance, first and second time derivates of distance in x direction
y, \dot{y}, \ddot{y}	distance, first and second time derivates of distance in y direction
z, \dot{z}, \ddot{z}	distance, first and second time derivates of distance in z direction
$\varphi, \dot{\varphi}, \ddot{\varphi}$	angle, angular velocity and angular acceleration of the turbine blade
$\theta, \dot{\theta}, \ddot{\theta}$	angle, angular velocity and angular acceleration of the turbine blade inclination to the vertical
ρ	density of water at atmospheric temperature and pressure

III GOVERNING EQUATION DERIVATION AND ANALYSIS

If the turbine blade is inclined to the vertical at angle θ and it is impinged by the flowing water body with velocity v , Fig. 1. The turbine blade will rotate counter clockwise with angular velocity $\dot{\varphi}$. If two frames, inertia and rotating frames, Fig. 2, are assigned to the turbine blade. If the inertia frame coordinates are XYZ and the rotating frame coordinates are xyz , and the Z and z axes of the two frames coincide along the vertical, the angle between the rotating frame and inertia frame is gotten by rotating the rotating frame counter-clockwise through angle φ about $Z(z)$ axes. Therefore, the unit vectors attached to the rotating frame are:

$$i = \cos \varphi I + \sin \varphi J \quad (1)$$

$$j = \cos \varphi J - \sin \varphi I \tag{2}$$

$$k = K \tag{3}$$

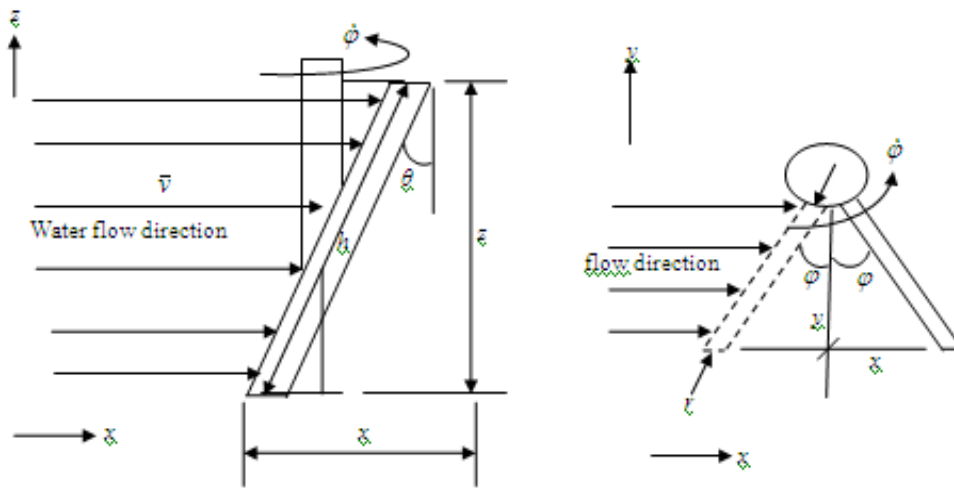


Fig 1(a): Front view of the Turbine impacted by water body, 1(b): The Plan.

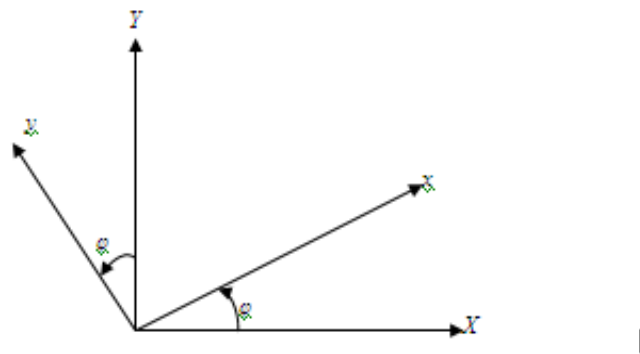


Fig 2: The two coordinate system for the Turbine/water dynamics

If the unit vectors above were differentiated with respect to time, yielding rate of change of these unit vectors, as equation 4, equation 5, and equation 6,

$$\dot{i} = -\dot{\varphi} \sin \varphi I + \dot{\varphi} \cos \varphi J = \dot{\varphi} j \tag{4}$$

$$\dot{j} = -\dot{\varphi} \cos \varphi I - \dot{\varphi} \sin \varphi J = -\dot{\varphi} i \tag{5}$$

$$\dot{k} = 0 \tag{6}$$

The unit vectors are differentiated the second time to obtain equation 7, equation 8, and equation 9,

$$\ddot{i} = \ddot{\varphi} j + \dot{\varphi} \dot{j} = \ddot{\varphi} j - \dot{\varphi}^2 i \tag{7}$$

$$\ddot{j} = -\ddot{\varphi} i - \dot{\varphi} \dot{i} = -\ddot{\varphi} i - \dot{\varphi}^2 j \tag{8}$$

$$\ddot{k} = 0 \tag{9}$$

The rotation of the moving frame attached to the shaft is $\dot{\varphi}$, this equals the angular velocity of the turbine shaft.

The position vector associated with a water particle that is about to hit the turbine blade is

$$\mathbf{r}_p = \mathbf{r}_{p/B} = x i + y j + z k \tag{10}$$

The velocity and the acceleration derivatives of the position vectors are:

$$\begin{aligned} \mathbf{v}_p &= \dot{x} \mathbf{i} + \dot{y} \mathbf{j} + \dot{z} \mathbf{k} + x \dot{\mathbf{i}} + y \dot{\mathbf{j}} + z \dot{\mathbf{k}} \\ &= \dot{x} \mathbf{i} + \dot{y} \mathbf{j} + \dot{z} \mathbf{k} + x(\dot{\varphi} \mathbf{j}) + y(-\dot{\varphi} \mathbf{i}) + z(0) \end{aligned} \quad (11)$$

and

$$\begin{aligned} \mathbf{a}_p &= \ddot{x} \mathbf{i} + \ddot{y} \mathbf{j} + \ddot{z} \mathbf{k} + x \ddot{\mathbf{i}} + y \ddot{\mathbf{j}} + z \ddot{\mathbf{k}} + 2(\dot{x} \dot{\mathbf{i}} + \dot{y} \dot{\mathbf{j}} + \dot{z} \dot{\mathbf{k}}) \\ &= \ddot{x} \mathbf{i} + \ddot{y} \mathbf{j} + \ddot{z} \mathbf{k} + x(\ddot{\varphi} \mathbf{j} - \dot{\varphi}^2 \mathbf{i}) + y(-\ddot{\varphi} \mathbf{i} - \dot{\varphi}^2 \mathbf{j}) + 2(\dot{x} \dot{\varphi} \mathbf{j} + \dot{y} (-\dot{\varphi} \mathbf{i})) \end{aligned} \quad (12)$$

From the Fig 1 (a & b) above, x, y, z can easily be determined thus:

$$x = h \sin \theta + r \sin \varphi \quad (13)$$

$$y = r \cos \varphi \quad (14)$$

$$z = h \cos \theta \quad (15)$$

The first derivatives of equations (13), (14) and (15) give equations (16), (17) and (18)

$$\dot{x} = h \dot{\theta} \cos \theta + r \dot{\varphi} \cos \varphi \quad (16)$$

$$\dot{y} = -r \dot{\varphi} \sin \varphi \quad (17)$$

$$\dot{z} = -h \dot{\theta} \sin \theta \quad (18)$$

While the second derivatives give equations (19), (20), and (21)

$$\ddot{x} = h(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + r(\ddot{\varphi} \cos \varphi - \dot{\varphi}^2 \sin \varphi) \quad (19)$$

$$\ddot{y} = -r(\ddot{\varphi} \sin \varphi + \dot{\varphi}^2 \cos \varphi) \quad (20)$$

$$\ddot{z} = -h(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \quad (21)$$

Expression for the forces generated by the moving water body is obtained by considering the momentum change of a control volume of water jet striking the turbine plate in Fig. 3. The control volume is attached to the turbine blade and it is fixed relative to the blade, therefore, the control volume will be moving with the blade.

$$\begin{aligned} \dot{m} &= \left\{ \begin{array}{l} \text{Mass flow rate} \\ \text{outside the control} \\ \text{volume} \end{array} \right\} - \left\{ \begin{array}{l} \text{Mass flow rate required to fill} \\ \text{the control volume due to the} \\ \text{velocity of the turbine blade} \end{array} \right\} \\ &= \rho a \bar{v} - \rho a u = \rho a (\bar{v} - u) \end{aligned} \quad (22)$$

Initial component of water velocity relative to turbine blade in x direction is

$$v_{x_{in}} = (\bar{v} - u)(\cos \theta + \cos \varphi) \quad (23)$$

Final component of water velocity relative to turbine blade in x direction $v_{x_{out}} = 0$ (24)

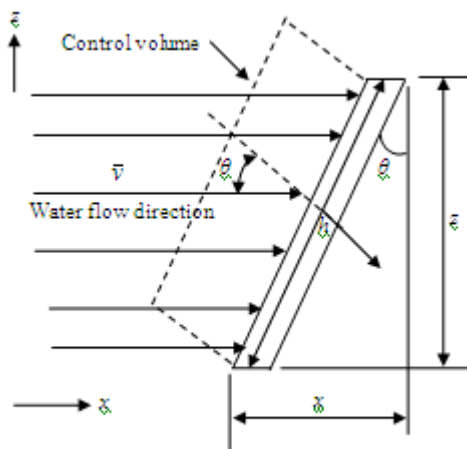


Fig 3: Force exerted on the Turbine blade

Initial component of water velocity relative to turbine blade in y direction

$$v_{yin} = (\bar{v} - u) \sin \varphi \tag{25}$$

Final component of water velocity relative to turbine blade in y direction $v_{yout} = 0$ (26)

Initial component of water velocity relative to turbine blade in z direction

$$v_{zin} = (\bar{v} - u) \sin \theta \tag{27}$$

Final component of water velocity relative to turbine blade in z direction $v_{zout} = 0$ (28)

The force exerted on the turbine blade by the fluid in any direction is

$$\mathbf{F} = -\dot{m}(v_{out} - v_{in}) = \dot{m}(v_{in} - v_{out}) \tag{29}$$

$$\dot{m} = \rho a (\bar{v} - u) \tag{30}$$

$$\mathbf{F}_x = \dot{m}(v_{xin} - v_{xout}) \tag{31}$$

$$\mathbf{F}_y = \dot{m}(v_{yin} - v_{yout}) \tag{32}$$

$$\mathbf{F}_z = \dot{m}(v_{zin} - v_{zout}) \tag{33}$$

$$\mathbf{F}_r = \sqrt{\mathbf{F}_x^2 + \mathbf{F}_y^2 + \mathbf{F}_z^2} \tag{34}$$

The free body diagram of the turbine blade is shown in Fig. 4, and it is being acted upon by gravitational moment and water moment. The moment developed is

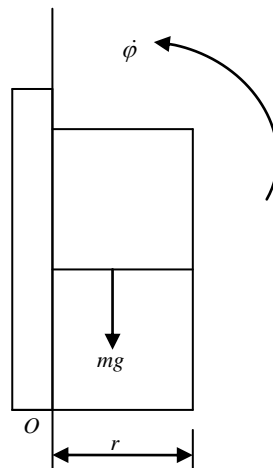


Figure 4: A turbine blade

$$\mathbf{M}_o = \sum M_o = I_o \alpha = -\frac{r}{2}(mg) + \mathbf{F}_r \cdot y \tag{35}$$

Power developed by the turbine shaft is

$$P = \mathbf{F}_r \cdot \mathbf{V}_p + \mathbf{M}_o \cdot \dot{\varphi} \tag{36}$$

IV. SIMULATION AND RESULTS

The basic values used for the simulations were tabulated in Table 1 below.

S/N	DESCRIPTION	SYMBOLS	VALUES
1	Turbine blade width	r	2m
2	Turbine blade length	h	1.2m
3	Stream velocity	\bar{v}	1, 3, 5 m/s
4	Blade inclination to the vertical	θ	$0^0 - 90^0$
5	Water density	ρ	1000kg/m ³
6	Estimated mass of one turbine blade	m	2kg
7	Acceleration due to gravity	g	9.81 m/s ²
8	Turbine blade swept angle	φ	$-90^0 - +90^0$

Table 1: Simulation Basic Values

Fig. 5 -7 shows the result of the simulations at different velocities, 1m/s Fig. 5, 3m/s Fig. 6 and 5m/s Fig. 7 for a particular configuration of the SHS. It was discovered that the peak power was produced at between 15° and 20° inclination of the blade to perpendicular axis of the flow. It is also observed that the inclination of the blade to the vertical axis gives maximum power at angle 90° followed by 50°.

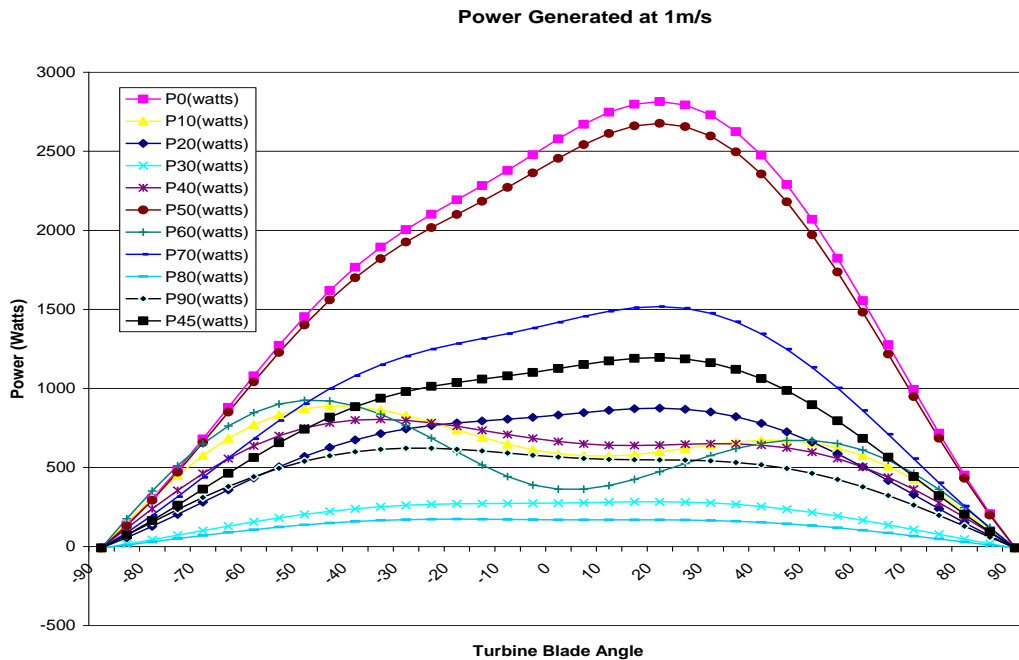


Fig 5: Power generated at different Turbine Angle combinations at 1 m/s

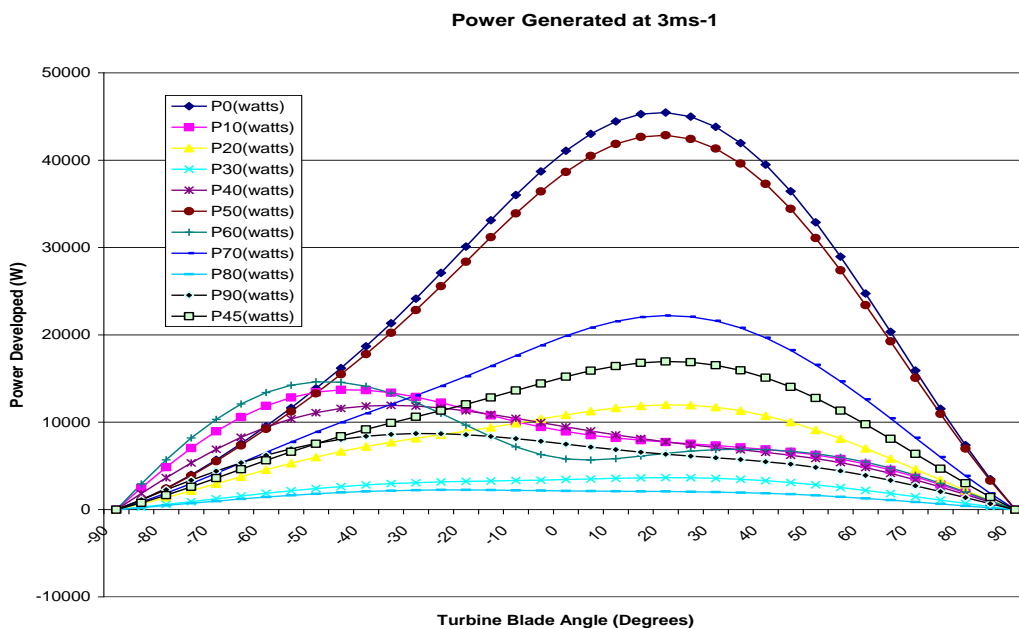


Fig 6: Power generated at different Turbine Angle combinations at 3 m/s

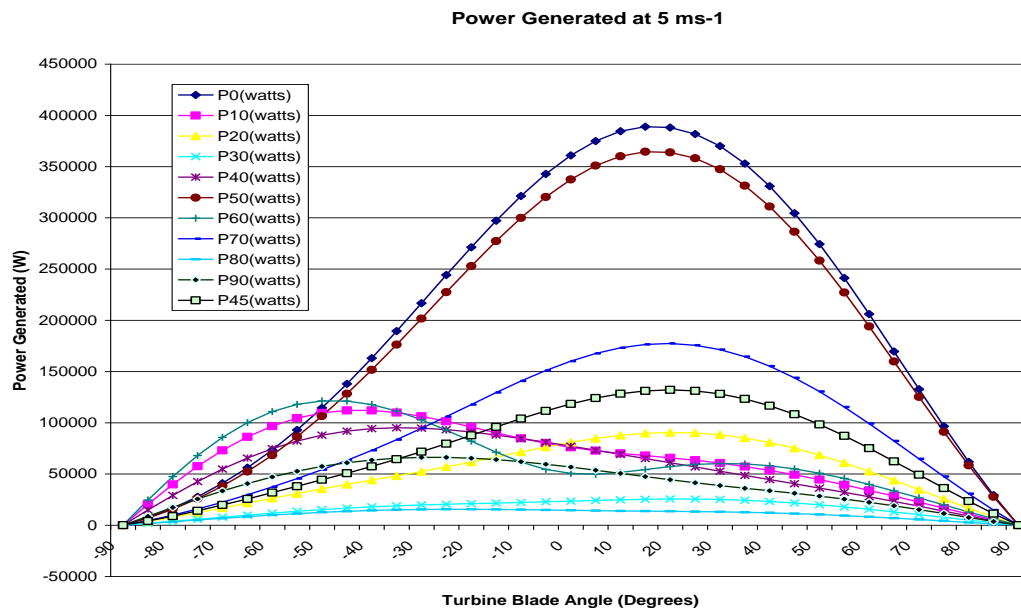


Fig 7: Power generated at different Turbine Angle combinations at 5 m/s

V. CONCLUSION

The analysis of the optimal impact angle of the water body on the turbine blade has been presented. The necessary equations were first derived and used for simulation in order to determine the optimal impact angle. It was discovered that the optimal impact angle will be 50° inclination of the turbine blade to the vertical in the $x-z$ plane, when the blade is at $15^\circ - 20^\circ$ inclination in the $x-y$ plane. The maximum power is determined from the graph as the peak of the graph. It is also discovered that for this configuration, three blades will give the maximum power output. This model can be used for any water body once the flow velocity is known.

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