

Application of Multivariate Multiple Linear Regression Model On Vital Signs and Social Characteristics of Patients

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*--***ABSTRACT***---*

This paper is an attempt to fit a Multivariate Multiple Linear Regression Model on the relationship between three response variables: Systolic Blood Pressure, Temperature, Height and two predictor variables: Age, Sex of patients. Federal Medical Center (F.M.C.), Owerri Imo State, Nigeria is used as a case study. The statistical software used in the data analysis is the "SAS Version 9.2" package. However, the Microsoft Excel office, 2010 played a vital role in the data analysis, especially in obtaining the residual and predicted matrices. The result revealed that the Multivariate Multiple Linear Regression Model was adequate for the relationship between the variables: Systolic Blood Pressure, Temperature and Height of patients on one hand, and the two social characteristics: Age and Sex on the other. A test of significance revealed that Age and Sex have influence on the Vital Signs. Following this result, we recommend that researchers should carry out a similar research work, making the predictor variables up to four to compare result. Consequent upon this finding, Government and private owned hospitals should create a standard statistics/records department, to enable future researchers have access to their data for analysis.

KEY WORD: Vital Signs, Social Characteristics**,** Multivariate Linear Regression, Likelihood Ratio Test, Wilk's Lambda Statistic, Least Squares and Design Matrix.

I. INTRODUCTION

One of the most important and commonly performed tasks of a medical assistant is obtaining and recording patient's vital signs and body measurements. Vital signs, also sometimes referred to as cardinal signs, include temperature, pulse, respiration, and blood pressure, abbreviated TPR B/P. They are indicative of the general health and well-being of a patient and, with regular monitoring, may measure patient's response to treatment. Vital signs in total or in part, are important components of each patient's visit (Palmore and Kivett; 1997). Height and weight measurements, although not considered vital signs, are often a routine part of a patient's visit. However, height will be considered as a vital sign in this paper. Patients will exhibit vital signs readings that are uniquely their own. As a result, baseline assessments of vital signs are usually obtained during the patient's initial visit. These baseline results are used as a reference point for future readings, differentiating between what is normal and abnormal for the patient. Two important habits must be developed by the medical assistant before taking a patient's vital signs: aseptic technique in the form of hand washing and recognition and correction of factors that may influence results of vital signs. Proper hand washing before taking vital signs will assist in preventing cross contamination of patients. Also, emotional factors of patients must be recognized and addressed. Explaining procedure and allowing the patient the opportunity to relax will ease apprehension that may affect vital sign readings.

II. IMORTANCE OF ACCURACY

Vital signs may be altered by many factors. Medical assistants must recognize and correct factors that may produce inaccurate results. For example, patients may exhibit anxiety over potential test results or findings of the provider. They may be angry or may have rushed into the office. A patient may have had something to eat or drink before the visit or may have had a long wait in the reception area. Patient apprehension and mood must always be considered by the medical assistant, because these factors can affect vital signs.

The medical assistant may be required to take vital signs more than once during an office visit to ascertain a baseline and obtain an impression of overall well-being of the patient. Body measurements such as weight may be influenced by what the patient is wearing; height may be influenced by the patient's shoes and how his or her posture is while being measured.

Accuracy in taking vital signs is necessary because treatment plans are developed according to the measurement of the vital signs. Variations can indicate a new disease process or the patient's response to treatment. They may also indicate the patient's compliance with a treatment plan. Although taking vital signs is a task commonly performed by the medical assistant, it is never to be taken casually or lightly, and it should never be rushed or incompletely performed. Concentration and attention to proper procedure will help ensure accurate measurements and quality care of the patient.

III. TEMPERATURE

Body temperature is maintained and regulated by two processes functioning in conjunction with one another: heat production and heat loss. Body heat is produced by the actions of voluntary and involuntary muscles. As the muscles move, they use energy, which produces heat. Cellular metabolic activities, such as the process of breaking down food sugars into simpler components (catabolism), are another source of heat. In a total practice management system, the patient's vital signs are often entered by the medical assistant during clinical care. Many programs have the ability to output vital signs data in a graph format, so a provider can easily note fluctuations in a patient's weight, blood pressure, normal temperature etc.

The delicate balance between heat production and heat loss is maintained by the hypothalamus in the brain. The hypothalamus monitors blood temperature and will trigger either the heat loss or heat production mechanism with as little as $0.04⁰F$ change in blood temperature.

Body temperature is measured in degrees and is influenced by several factors, as follows:

- An increase in temperature may result from a bacterial infection, increased physical activity or food intake, exposure to heat, pregnancy, drugs that increase metabolism, stress and severe emotional reactions, and age. Age becomes a factor in that infants have an average body temperature that is one of two degrees higher than adults.
- Decrease in temperature may result from viral infections, decreased muscular activity, fasting, a depressed emotional state, exposure to cold, drugs that decrease metabolic activities, and age. Age in this instance refers to older adults, in that older adults have decreased metabolic activity resulting in a decrease in body temperature.
- Another factor that can increase or decrease body temperature is time of day. During sleep and early morning, the temperature is at its lowest, whereas later in the day with muscular and metabolic activity, the temperature increases.

Because of the many factors influencing body temperature and the uniqueness of individuals, there is a "normal" temperature. The medical assistant must think of temperatures in terms of the "average", which for an adult is $98.\overline{6}^{\circ}$ F or 37.0 $^{\circ}$ C.

IV. BLOOD PRESSURE

Blood pressure measures cardiovascular function by measuring the force of blood exerted on peripheral arteries during the cardiac cycle or heartbeat. The measurement consists of two components. The first is the force exerted on the arterial walls during cardiac contraction and is called systole. The second is the force exerted during cardiac relaxation and is called diastole. They represent the highest (systole) and lowest (diastole) amount of pressure exerted during the cardiac cycle. Blood pressure is recorded as fraction, with the systolic measurement written, followed by a slash and then the diastolic measurement.

Blood pressure may be affected by many factors, including blood volume, peripheral resistance, vessel elasticity, condition of the muscle of the heart genetics, diet and weight, activity, and emotional state.

V. HEIGHT

To measure a patient's height, a scale with a measuring bar is necessary. A paper towel is placed on the scale because the patient's shoes should be removed for accurate measuring. After the patient is on the platform, the measuring bar is placed firmly on the patient's head, and the line between where the solid bar and sliding bar meet is read.

VI. STATEMENT OF PROBLEM

It is evident that subjective health assessment is a valid indicator of health status in middle-aged populations, and that it can be used in cohort studies and monitoring of a population's health. The use of socioeconomic status has been recommended for screening of majority groups, but not for minority ones. In spite of the fact that self-rated health is such an important factor, little is known about the etiological background to poor perceived health and also less is known about the impact of life satisfaction on health in a primary care practice population. Palmore and Kivett reported in 1997 that life satisfaction at the end of a 4-year period was significantly related to initial levels of self-rated health among subjects aged 46 – 70 years. However, some other studies on the elderly show relations between perceived health and life satisfaction.

VII. SIGNFICANCE OF STUDY

This study is aimed at fitting a Multivariate Multiple Linear Regression Model on the relationship between the three measures of vital signs (Systolic Blood pressure, Temperature and Height) on socio characteristics (Age, and Sex) of the patients. The result of this study may enlighten researchers on how to apply this technique to other fields of life.

VIII. SCOPE OF THE STUDY

This study covers a total of 300 patients that were subjected to measurements on their vital signs (Systolic Blood Pressure, Temperature and Height) and Social Characteristics (Age and Sex) from May 4 – 24, 2013. The Federal Medical Centre, Owerri Imo State, Nigeria was used for the study, with the aim and objective of

 fitting a Multivariate Multiple Linear Regression Model for the three measures of vital signs (Systolic Blood pressure, Temperature and Height) on the two social characteristics (Age and Sex) and

 testing the significance of the parameters of the fitted model in order to draw meaningful conclusion and make helpful recommendation.

IX. LITERATURE REVIEW

Oguebu (2011) carried out a study on a multivariate linear regression study of confectionery products, such as cheese balls produced by Zubix International Company Ogidi between January 2010 and March 2011. He used average Weight and Bulk Density of cheese balls as the dependent variables and oven temperature, moisture content before extrusion and moisture content after extrusion as the predictor variables. The result of the analysis revealed that only oven temperature is significant for the Multivariate Linear Regression Model. In line with this findings, he made the following recommendations to help improve the quality of cheese balls produced by the manufacturer: "oven temperature at 188.57° c should be selected to produce optimum levels of Average weight and bulk density", "the producer should adopt better methods of data collection such that more data would be accessible for future work", "Data should be collected in larger dimensions to accommodate more variables".

Amy (1997) studied the relationship between Body Mass Index (BMI) and Body Fat in Black population samples from Nigeria, Jamaica, and the United States. According to him, this study was undertaken to determine the ability of BMI to predict body fat levels in three populations of West African heritage living in different environments. A total of 54 black men and women were examined in Nigeria, Jamaica, and the United States during 1994 and 1995. A standardized protocol was used to measure height, weight, waist and hip circumferences, and blood pressure at all sites. Percentage of body fat was estimated using bioelectrical impendence analysis. Percentage of body fat and BMI were highly correlated within site-and sex-specific groups and the resulting r^2 ranged from 0.61 to 0.85. The relationship was quadratic in all groups except Nigerian men, in whom it was linear.

Cohn (2000) investigated the relationship between measures of health and eating habits. He collected data on cholesterol, blood pressure and weight. He also collected data on the eating habits of the subjects (that is, how many ounces of red meat, fish-dairy products and chocolate consumed per week). The result of the analysis revealed that only fish-dairy products and chocolate consumed per week related to the cholesterol, blood pressure and weight of the patients.

Onyenawuli (2012) carried out a study on a multivariate linear regression study of measures of health on socio-demographic characteristics of patients using Oguta General Hospital, Imo State Nigeria as a case study. The response variables: Blood Pressure, Body Mass Index (BMI), Pulse and three predictor variables: Age, Sex, Marital Status of patients were used for the study. The analysis revealed that the Blood Pressure of 160mm/Hg, BMI of 20.77Kg/m² and Pulse rate of 73 per min are required for normal health. The following results were revealed: Age, Sex and Marital Status are significantly related to the response variables and the model performed well as a predictive model.

In this paper, Multivariate Multiple Linear Regression Model of Vital Signs on Social Characteristics of patients is applied using Federal Medical Centre, Owerri Imo State Nigeria as a case study.

X. METHODOLOGY

The multiple linear regression with n independent observations on Y and the associated values of Z_i , is the complete model of

$$
Y_{1} = \beta_{0} + \beta_{1}Z_{11} + \beta_{2}Z_{12} + \beta_{2}Z_{13} + \cdots + \beta_{r}Z_{1r} + \varepsilon_{1}
$$

\n
$$
Y_{2} = \beta_{0} + \beta_{1}Z_{21} + \beta_{2}Z_{22} + \beta_{2}Z_{23} + \cdots + \beta_{r}Z_{2r} + \varepsilon_{2}
$$

\n
$$
Y_{3} = \beta_{0} + \beta_{1}Z_{31} + \beta_{2}Z_{32} + \beta_{2}Z_{33} + \cdots + \beta_{r}Z_{3r} + \varepsilon_{3}
$$

\n
$$
\vdots \qquad \vdots \qquad \vdots
$$

\n
$$
Y_{n} = \beta_{0} + \beta_{1}Z_{n1} + \beta_{2}Z_{n2} + \beta_{2}Z_{n3} + \cdots + \beta_{r}Z_{nr} + \varepsilon_{n}
$$

\n(1)

where the error terms are assumed to have the properties:

1.
$$
E(\varepsilon_j) = 0;
$$

\n2. $Var(\varepsilon_j) = \sigma^2 (constan t);$ and
\n3. $Cov(\varepsilon_j, \varepsilon_k) = 0, j \neq k$ (2)

Representing equations (2) in matrix form, we have

$$
\begin{bmatrix}\nY_1 \\
Y_2 \\
Y_3 \\
\vdots \\
Y_n\n\end{bmatrix} =\n\begin{bmatrix}\n1 & Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1r} \\
1 & Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2r} \\
1 & Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3r} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & Z_{n1} & Z_{n2} & Z_{n3} & \cdots & Z_{nr}\n\end{bmatrix}\n\begin{bmatrix}\n\beta_0 \\
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_r\n\end{bmatrix} +\n\begin{bmatrix}\n\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_n\n\end{bmatrix}
$$

or $\sum_{(n \times 1)} = \sum_{[n \times (r+1)]} \beta + \varepsilon$
(n×1) $Y_{\text{x1}} = Z_{[n \times (r+1)]_{[(r+1) \times 1]}} \beta + \varepsilon$

and the specifications in (1) become:

1. $E(\varepsilon) = 0$; and

2.
$$
Cov(\varepsilon) = E(\varepsilon \varepsilon') = \sigma^2 I
$$

A one in the first column of the design matrix Z is the multiplier of the constant term β_0 . It is customary to introduce the artificial variable $Z_{j0} = 1$ so

 $\beta_0 + \beta_1 Z_{j1} + \cdots + \beta_r Z_{jr} = \beta_0 Z_{j0} + \beta_1 Z_{j1} + \cdots + \beta_r Z_{jr}$

Each column of Z consists of the n values of the corresponding predictor variable, while the jth row of Z contains the values for all predictor variables on the jth trial.

Classical Linear Regression Model

$$
\sum_{(n\times l)} = \sum_{[n\times (r+l)]} \beta + \varepsilon \qquad (3)
$$

$$
E(\epsilon)=\underset{\scriptscriptstyle(n\times I)}{0}, \,\mathrm{and}\,\, Cov(\epsilon)=\underset{\scriptscriptstyle(n\times n)}{\sigma^2 I}
$$

where β and σ^2 are unknown parameters and the design matrix Z has jth row $[Z_{j0}, Z_{j1}, \dots, Z_{jr}]$

XI. LEAST SQUARES ESTIMATION

One of the objectives of regression analysis is to develop an equation that will allow the investigator to predict the response for given values of the predictor variables. Thus, it is necessary to "fit" the model in (3) to the observed y_j corresponding to the known values $b_0 + b_1 Z_{j1} + \cdots + b_r Z_{jr}$. That is, we must determine the values for the regression coefficients β and the error variance σ^2 consistent with the available data. Let b be the trial values for β . Consider the difference $y_j - b_0 - b_1 Z_{j1} - \cdots - b_r Z_{jr}$ between the observed responses, y_j , and

the value $b_0 + b_1 Z_{j1} + \cdots + b_r Z_{jr}$ that would be expected if b were the "true" parameter vector. The method of least squares selects b to minimize the sum of squared differences

$$
S(b) = \sum_{j=1}^{n} (y_j - b_0 - b_1 Z_{j1} - \dots - b_r Z_{jr})^2 \dots \dots \tag{4}
$$

= (y - Zb)'(y - Zb)

The coefficient b chosen by the least squares criterion is called least squares estimates of the regression parameters β . They will henceforth be denoted by $\hat{\beta}$ to emphasize their rule as estimates of β .

The coefficients $\hat{\beta}$ are consistent with the data in the sense that they produce estimated (fitted) mean responses,

 $\hat{\beta}_0 + \hat{\beta}_1 Z_{j1} + \cdots + \hat{\beta}_r Z_{jr}$, whose sum of squared differences from the observed y_j is as small as possible. The deviations

$$
\hat{\epsilon} = y_j - b_0 - b_1 Z_{j1} - \dots - b_r Z_{jr} \ j = 1, 2, \dots, n \tag{5}
$$

are called residuals. The vector of residuals $\hat{\epsilon} + y - Z\hat{\beta}$ contains the information about the remaining unknown parameter σ^2 .

XII. MULTIVARIATE MULTIPLE REGRESSION

Let us consider the problem of modeling the relationship between m responses, $Y_1, Y_2, ..., Y_m$, and a single set of predictor variables, $Z_1, Z_2, ..., Z_r$. Each response is assumed to follow its own regression model so that

$$
Y_{1} = \beta_{01} + \beta_{11}Z_{1} + \beta_{21}Z_{2} + \dots + \beta_{r1}Z_{r} + \varepsilon_{1}
$$

\n
$$
Y_{2} = \beta_{02} + \beta_{12}Z_{1} + \beta_{22}Z_{2} + \dots + \beta_{r2}Z_{r} + \varepsilon_{2}
$$

\n
$$
Y_{3} = \beta_{03} + \beta_{13}Z_{1} + \beta_{23}Z_{2} + \dots + \beta_{r3}Z_{r} + \varepsilon_{3}
$$

\n
$$
\vdots
$$

\n
$$
Y_{m} = \beta_{0m} + \beta_{1m}Z_{1} + \beta_{2m}Z_{2} + \dots + \beta_{rm}Z_{r} + \varepsilon_{m}
$$

\n(6)

The error term $\varepsilon = [\varepsilon_1 \varepsilon_2 \varepsilon_3 \cdots \varepsilon_m]'$ has $E(\varepsilon) = 0$ and $Var(\varepsilon) = \varepsilon$. Thus, the error terms associated with different responses may be correlated.

To establish notation conforming to the classical linear regression model, let $[Z_{i0}, Z_{i1}, \dots, Z_{ir}]$ denote the values of the predictor variables for the jth trial, let $Y_j = [Y_{j1}, Y_{j2}, ..., Y_{jm}]'$ be the responses, and let $\varepsilon_j = [\varepsilon_{j1} \varepsilon_{j2} \varepsilon_{j3} ...$ ε_{im} ' be the errors. In matrix notation, the design matrix

$$
Z_{10} = \begin{bmatrix} Z_{10} & Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{20} & Z_{21} & Z_{22} & \cdots & Z_{2r} \\ Z_{30} & Z_{31} & Z_{32} & \cdots & Z_{3r} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ Z_{n0} & Z_{n1} & Z_{n2} & \cdots & Z_{nr} \end{bmatrix}
$$

is the same as that for the single response regression model in (4). The other matrix quantities have multivariate counterparts. Set

$$
\mathbf{Y}_{11} \quad \mathbf{Y}_{12} \quad \mathbf{Y}_{13} \quad \cdots \quad \mathbf{Y}_{1m} \\ \mathbf{Y}_{21} \quad \mathbf{Y}_{22} \quad \mathbf{Y}_{23} \quad \cdots \quad \mathbf{Y}_{2m} \\ \mathbf{Y}_{31} \quad \mathbf{Y}_{32} \quad \mathbf{Y}_{33} \quad \cdots \quad \mathbf{Y}_{3m} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \mathbf{Y}_{n1} \quad \mathbf{Y}_{n2} \quad \mathbf{Y}_{n3} \quad \cdots \quad \mathbf{Y}_{nm} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{(1)} \quad \vdots \quad \mathbf{Y}_{(2)} \quad \vdots \quad \mathbf{Y}_{(3)} \quad \vdots \quad \cdots \quad \vdots \quad \mathbf{Y}_{(m)} \end{bmatrix}
$$
\n
$$
\mathbf{B}_{(11)} \quad \mathbf{B}_{12} \quad \mathbf{B}_{22} \quad \mathbf{B}_{23} \quad \cdots \quad \mathbf{B}_{nm} \\ \mathbf{B}_{21} \quad \mathbf{B}_{22} \quad \mathbf{B}_{23} \quad \cdots \quad \mathbf{B}_{2m} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ \mathbf{B}_{r1} \quad \mathbf{B}_{r2} \quad \mathbf{B}_{r3} \quad \cdots \quad \mathbf{B}_{nm} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{(1)} \quad \vdots \quad \mathbf{B}_{(2)} \quad \vdots \quad \mathbf{B}_{(3)} \quad \vdots \quad \cdots \quad \vdots \quad \mathbf{B}_{(m)} \end{bmatrix}
$$
\nand\n
$$
\mathbf{g}_{11} \quad \mathbf{g}_{21} \quad \mathbf{g}_{22} \quad \mathbf{g}_{23} \quad \cdots \quad \mathbf{g}_{1m} \\ \mathbf{g}_{r1} \quad \mathbf{g}_{r2} \quad \mathbf{g}_{r3} \quad \cdots \quad \mathbf{g}_{1m} \\ \mathbf{g}_{r2} \quad \mathbf{g}_{r3} \quad \cdots \quad \mathbf{g}_{1m} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{(1)} \quad \vdots \quad \mathbf{g}_{(2)} \quad \
$$

The m observations on the jth trial have covariance matrix $\Sigma = \{\sigma_{ik}\}\$, but observations from different trials are uncorrelated. Here β and σ_{ik} are unknown parameters; the design matrix Z has jth row $[Z_{j0},\,Z_{j1},\,Z_{j2},\,\cdots,\,Z_{jr}].$

The ith response $Y_{(i)}$ follows the linear regression model

 $Y_{(i)} = Z\beta_{(i)} + \varepsilon_{(i)}, \quad i = 1, 2, 3, \cdots, m$ … (8) with $Cov(\epsilon_{(i)}) = \sigma_{ii} I$. However, the errors for different responses on the same trial can be correlated. Given the outcomes Y and the values of the predictor variables Z with full column rank, we determine the least squares estimates $\hat{\beta}_{(i)}$ exclusively from the observations, $Y_{(i)}$, on the ith response. Conforming to the singleresponse solution, we take

$$
\hat{\beta}_{(i)} = (Z'Z)^{-1}Z'Y_{(i)}
$$
\n(9)

Collecting these univariate least squares estimates produces

$$
\hat{\beta} = \begin{bmatrix} \hat{\beta}_{(1)} & \vdots & \hat{\beta}_{(2)} & \vdots & \hat{\beta}_{(3)} & \vdots & \cdots & \vdots & \hat{\beta}_{(m)} \end{bmatrix} \\
= (Z'Z)^{-1}Z'\begin{bmatrix} Y_{(1)} & \vdots & Y_{(2)} & \vdots & Y_{(3)} & \vdots & \cdots & \vdots & Y_{(m)} \end{bmatrix}
$$

or

$$
\hat{\beta} = (Z'Z)^{-1}Z'Y
$$
\nNotice of parameters say

\n(10)

For any choice of parameters say

$$
\mathbf{B} = \begin{bmatrix} \mathbf{b}_{(1)} & \vdots & \mathbf{b}_{(2)} & \vdots & \mathbf{b}_{(3)} & \vdots & \cdots & \vdots & \mathbf{b}_{(m)} \end{bmatrix}
$$
, the matrix of errors is

Y – ZB. The error sum of squares and cross-products matrix is \mathbf{y} – \mathbf{y} – \mathbf{y}

$$
= \left[\begin{array}{l} (Y - ZB)(Y - ZB) \\ (Y_{(1)} - Zb_{(1)})' (Y_{(1)} - Zb_{(1)}) \cdots (Y_{(1)} - Zb_{(1)})' (Y_{(m)} - Zb_{(m)}) \\ \vdots & \vdots \\ (Y_{(m)} - Zb_{(m)})' (Y_{(1)} - Zb_{(1)}) \cdots (Y_{(m)} - Zb_{(m)})' (Y_{(m)} - Zb_{(m)}) \end{array} \right] \dots (11)
$$

The selection $\mathbf{b}_{(i)} = \hat{\beta}_{(i)}$ minimizes the ith diagonal sum of squares $(Y_{(i)} - Zb_{(i)})'(Y_{(i)} - Zb_{(i)})$. Consequently, $\text{tr}\left[\left(Y - ZB\right)\left(Y - ZB\right)\right]$ \vert L $tr \left[\left(Y - ZB \right)' (Y - ZB) \right]$ is minimized by the choice $B = \hat{\beta}$. Also, the generalized variance $(Y - ZB)^{'}(Y - ZB)^{'}$ is minimized by the least squares estimates $\hat{\beta}$.

Then, using the least squares estimates $\hat{\beta}$, we can form the matrices of

Predicted values:
$$
\hat{Y} = Z\hat{\beta} = Z(Z'Z)^{-1}Z'Y
$$

Residuals: $\hat{\epsilon} = Y - \hat{Y} = [I - Z(Z'Z)^{-1}Z']Y$ (12)

The orthogonally conditions among the residuals, predicted values, and columns of Z, which hold in classical linear regression, hold in multivariate multiple regression.

They follow form $Z[I - Z(Z'Z)^{-1}Z'] = Z' - Z' = 0$ Specifically,

$$
Z'\hat{\epsilon} = Z'[I - Z(Z'Z)^{-1}Z']Y = 0
$$
 (13)

so the residuals $\hat{\epsilon}_{\scriptscriptstyle (i)}$ are perpendicular to the columns of Z. Also

$$
\hat{Y}'\hat{\varepsilon} = \hat{\beta}Z'[I - Z(Z'Z)^{-1}Z']Y = 0 \qquad \qquad \dots \qquad (14)
$$

Confirming that the predicted values $\hat{Y}_{(i)}$ are perpendicular to all residuals vectors $\hat{\epsilon}_{(k)}$. Because $Y = \hat{Y} + \hat{\varepsilon}$,

$$
Y'Y = (\hat{Y} + \hat{\epsilon})'(\hat{Y} + \hat{\epsilon}) = \hat{Y}'\hat{Y} + \hat{\epsilon}'\hat{\epsilon} + 0 + 0'
$$

or

 $Y'Y = \hat{Y}'\hat{Y} + \hat{\varepsilon}'\hat{\varepsilon}$

 \cdot .) λ $\overline{}$ $\overline{\mathcal{L}}$ ſ $\left.\frac{1}{s}\right|^{1}$ squares and cross-.) λ $\overline{}$ $\overline{\mathcal{L}}$ ſ リ $\left(\right)$ $\overline{}$ \setminus ſ - products) squares and cross-products squares and cross-products residuals(error) sumof squares and cross-products predicted sum of and cross-products total sum of squares $\Big| = \Big(\text{predicted sum of}\Big) + \Big(\text{residuals (error) sum of}\Big) \dots (15)$ The residual sum of squares and cross-products can also be written as

$$
\hat{\varepsilon}' \hat{\varepsilon} = Y'Y - \hat{Y}'\hat{Y} = Y'Y - \hat{\beta}'Z'Z\hat{\beta} \qquad \qquad \dots \qquad (16)
$$

For the least squares estimator

 $\overline{}$ $\frac{1}{2}$ \mathbf{r} L \mathbf{r} $\hat{\beta} = \begin{vmatrix} \hat{\beta}_{(1)} & \cdots & \hat{\beta}_{(2)} \end{vmatrix}$ $\hat{\beta}_{(3)}$ \cdots \cdots $\hat{\beta}_{(m)}$ \vdots \vdots \cdots \vdots \vdots \vdots \vdots \vdots \vdots determined under the multivariate multiple regression

model (3–9) with full rank
$$
(Z) = r + 1 < n
$$

$$
E(\hat{\beta}_{\scriptscriptstyle (i)})=\beta_{\scriptscriptstyle (i) \text{ or } }E(\hat{\beta})=\beta
$$

and

The re

$$
Cov(\hat{\beta}_{(i)}, \beta_{(k)}) = \sigma_{ik} (Z'Z)^{-1}, i, k = 1, 2, \dots, r + 1
$$

siduals $\hat{\epsilon} = \begin{bmatrix} \hat{\epsilon}_{(1)} & \vdots & \hat{\epsilon}_{(2)} & \vdots & \hat{\epsilon}_{(3)} & \vdots & \cdots & \vdots & \hat{\epsilon}_{(m)} \end{bmatrix} = Y - Z\hat{\beta}$

Satisfy $E(\hat{\varepsilon}_{(i)}) = 0$ and

$$
E(\hat{\epsilon}'_{(i)}, \hat{\epsilon}_{(k)}) = (n - r - 1)\sigma_{ik \text{ so}}
$$

$$
E = (\hat{\epsilon}) = 0 \text{ and } E\left(\frac{\hat{\epsilon}'\hat{\epsilon}}{(n - r - 1)}\right) = \Sigma
$$

The mean of the ith response variable is $z'_0 \beta_{(i)}$, and this is estimated by $z'_0 \beta_{(i)}$, the ith component of the fitted regression relationship, collectively,

$$
z'_{0}\hat{\beta} = \begin{bmatrix} z'_{0}\hat{\beta}_{(1)} & \vdots & z'_{0}\hat{\beta}_{(2)} & \vdots & z'_{0}\hat{\beta}_{(3)} & \vdots & \cdots & \vdots & z'_{0}\hat{\beta}_{(m)} \end{bmatrix} \dots (17)
$$

is an unbiased estimator of $Z'_0\beta$ since $E(z'_0\hat{\beta}_{(i)}) = z'_0E(\hat{\beta}_{(i)})$ for each component. Using the covariance matrix for $\hat{\beta}_{(i)}$ and $\hat{\beta}_{(k)}$, the estimation errors, $Z'_{0}\hat{\beta}_{(i)} - Z'_{0}\hat{\beta}_{(i)}$, have covariance

$$
E[z'_{0}(\hat{\beta}_{(i)} - \hat{\beta}_{(i)})(\hat{\beta}_{(k)} - \hat{\beta}_{(k)})'z_{0}] = z'_{0}[E(\hat{\beta}_{(i)} - \hat{\beta}_{(i)})(\hat{\beta}_{(k)} - \hat{\beta}_{(k)})'z_{0}
$$

= $\sigma_{ik}z'_{0}(Z'Z)^{-1}z_{0}$... (18)

 $\boldsymbol{0}$ The related problem is that of forecasting a new observation vector,

 $Y_0 = [Y_{01}, Y_{02}, \dots, Y_{0m}]'$ at z_0 . According to the regression model,

 $Y_{0i} = Z'_0 \beta_{(i)} + \varepsilon_{0i}$ where the "new" error

 $\varepsilon_0 = [\varepsilon_{01}, \varepsilon_{02}, \cdots, \varepsilon_{0m}]'$ is independent of the errors ε and satisfies $E(\varepsilon_{0i}) = 0$ and $E(\varepsilon_{0i}\varepsilon_{0k}) = \sigma_{ik}$. The forecast error for the ith component of Y_0 is

Application Of Multivariate Multiple Linear …

$$
Y_{0i} - z'_0 \hat{\beta}_{(i)} = Y_{0i} - z'_0 \hat{\beta}_{(i)} + z'_0 \hat{\beta}_{(i)} - z'_0 \hat{\beta}_{(i)} = \epsilon_{0i} - z'_0 (\beta_{(i)} - \beta_{(i)})
$$

$$
E(Y_{0i} - z'_0 \hat{\beta}_{(i)}) = E(\epsilon_{0i}) - z'_0 (\beta_{(i)} - \beta_{(i)}) = 0,
$$

Indicating $z'_0 \hat{\beta}$ is an unbiased predictor of Y_{0i} . However, the forecast errors have co-variances

$$
E(Y_{0i} - z'_{0}\hat{\beta}_{(i)})(Y_{0k} - z'_{0}\hat{\beta}_{(k)}) = E[\varepsilon_{0i} - z'_{0}(\hat{\beta}_{(i)} - \beta_{(i)})][(\varepsilon_{0k} - z'_{0}(\beta_{(k)} - \beta_{(k)})]
$$

\n
$$
= E(\varepsilon_{0i}\varepsilon_{0k}) + z'_{0}E(\hat{\beta}_{(i)} - \beta_{(i)})(\hat{\beta}_{(k)} - \beta_{(k)})'z_{0} - z'_{0}E[(\hat{\beta}_{(i)} - \beta_{(i)})\varepsilon_{0k}] - E[\varepsilon_{0i}(\hat{\beta}_{(k)} - \beta_{(k)})']z_{0}
$$

\n
$$
= \sigma_{ij}[1 + z'_{0}(Z'Z)^{-1}z_{0}] \qquad \qquad (19)
$$

It should be noted that $\ E\left[(\hat{\beta}_{_{(i)}} - \hat{\beta}_{_{(i)}}) \varepsilon_{_{0k}} \right] = 0$

So

Since $\beta_{(i)} = (ZZ)^{-1}Z^{\dagger}\varepsilon_{(i)} + \beta_{(i)}$ $\hat{\beta}_{(i)} = (Z'Z)^{-1}Z'\epsilon_{(i)} + \beta_{(i)}$ is independent of ϵ_0 . The same thing is applicable to $E\left[\varepsilon_{0i}(\hat{\beta}_{(k)} - \beta_{(k)})'\right]$.

XIII. LIKELIHOOD RATIO TESTS FOR MULTIVARIATE LINEAR REGRESSION PARAMETERS

The multi-response analog of multiple linear regression, the hypothesis that the responses do not depend on Z_{q+1} , $Z_{\alpha+2}$, …, Z_r becomes

$$
B_{0}: \beta_{(2)} = 0 \text{ where}
$$
\n
$$
\beta = \begin{bmatrix}\n\beta_{(i)} \\
\vdots \\
\beta_{(i-1)\times m} \\
\beta_{(i)} \\
\vdots \\
\beta_{(i-1)\times m}\n\end{bmatrix}
$$
\n
$$
Setting \quad Z = \begin{bmatrix}\nZ_{1}: Z_{2} \\
Z_{1}: Z_{2} \\
\vdots \\
\beta_{(i+1)[N]}\n\end{bmatrix}; the general model can be written as
$$
\n
$$
E(Y) = Z\beta = [Z_{1}: Z_{2}]\begin{bmatrix}\n\beta_{(1)} \\
\vdots \\
\beta_{(2)}\n\end{bmatrix} = Z_{1}\beta_{(1)} + Z_{2}\beta_{(2)}
$$
\n(20)

Under H₀ : $\beta_{(2)} = 0$, Y = Z₁ $\beta_{(1)}$ + ε and the likelihood ratio test of H₀ is based on the quantities involved in the extra sum of squares and cross-products

$$
= (Y - Z_1 \hat{\beta}_{(1)})'(Y - Z_1 \hat{\beta}_{(1)}) - (Y - Z_1 \hat{\beta})'(Y - Z_1 \hat{\beta})
$$

= $n(\hat{\Sigma}_1 - \hat{\Sigma})$
where $\hat{\beta}_{(1)} = (Z_1'Z_1)^{-1}Z_1'Y$ and $\hat{\Sigma}_1 = \frac{(Y - Z_1 \hat{\beta}_{(1)})'(Y - Z_1 \hat{\beta}_{(1)})}{n}$

The likelihood ratio for the test of the hypothesis

 H_0 : $\beta_{(2)} = 0$

can be expressed in terms of generalized variances, so that

$$
\Lambda = \frac{\max_{\beta_{(1)}\varepsilon} L(\beta_{(1)}, \Sigma)}{\max_{\beta,\varepsilon} L(\beta, \Sigma)} = \frac{L(\hat{\beta}_{(1)}, \hat{\Sigma})}{L(\hat{\beta}, \hat{\Sigma})} = \left(\frac{|\hat{\Sigma}|}{|\hat{\Sigma}_1|}\right)^{\frac{n}{2}} \qquad \qquad \dots \qquad (21)
$$

Equivalently, Wilks' Lambda statistic

$$
\Lambda^{\frac{n}{2}} = \frac{\left| \hat{\Sigma} \right|}{\left| \hat{\Sigma}_1 \right|} \tag{22}
$$

can be used.

Let the multivariate multiple regression model of (7) hold with Z of full rank $r + 1$ and $(r + 1) + m \le n$. Let the errors ε be normally distributed. Under H_0 : $\beta_{(2)} = 0$, $n\hat{\Sigma}$ is distributed as $W_{n-r-1}(\cdot|\Sigma)$ independently of $n(\hat{\Sigma}_1 - \hat{\Sigma})$ which, in turn, is distributed as $W_{r-q}(\cdot | \Sigma)$. The likelihood ratio test of H_0 is equivalent to rejecting H_0 for large values of

$$
-2\ln\Lambda = -n\ln\left(\frac{\left|\hat{\Sigma}\right|}{\left|\hat{\Sigma}_1\right|}\right) = -n\ln\frac{\left|n\hat{\Sigma}\right|}{\left|n\hat{\Sigma} + n(\hat{\Sigma}_1 - \hat{\Sigma})\right|}
$$

for n large, the modified statistic

$$
-\left[n-r-1-\frac{1}{2}(m-r+q+1)\right]\ln\frac{|\Sigma|}{|\hat{\Sigma}_1|} \qquad \qquad \dots \tag{23}
$$

 \overline{a}

Has, to a close approximation, a chi-square distribution with $m(r - q)$ d.f. where $n - r$ and $n - m$ are both large.

XIV. DATA COLLECTION

The data for this research work were collected in an arrangement with the nurses of Federal Medical Centre, Owerri Imo State Nigeria. A total number of 300 respondents (patients) were randomly selected and used for this research study. An ethical approval was obtained from the Ethical committee of the hospital, Federal Medical Center, Owerri. The consent of the subjects were sought and obtained and the consenting subjects were used by the patients, using the following inclusion criteria:

1. They were normally balanced (i.e. the subjects were able to answer questions on name, age, sex, date of birth, marital status etc., consciously and correctly.

2. All the patients (both in- and out-patients) within May 4, to May 24. 2013 participated in this exercise, as the nurses did not allow them to know the rationale behind the vital signs measurements.

- 3. Both male and female were recruited.
- 4. No discrimination on the tribe and norms.

XV. SAMPLING PROCEDURE

No sampling procedure was involved in taking the vital signs measurements, as it involves all the patients within the time of study. For the purpose of this study, a simple random sampling of 300 patients was used for this research work, out of over nine hundred (900) patients captured. Table 1 provides the Data Matrix.

XVI. DATA ANALYSIS

The statistical techniques discussed in sections 10 to 13 were used to analyse the data collected for this research work. To make computation less tedious, a statistical software package known as "SAS version 9.2" was used for running the data. The resulting computations are in Appendix 1.

From the **SAS** software output, we have

$$
\hat{Y}_1 = 167.51417 - 0.16401 \text{ Age} - 0.46611 \text{ Sex}
$$

$$
\hat{Y}_2 = 36.18228 - 0.01367 \text{ Age} - 0.01787 \text{ Sex}
$$

$$
\hat{Y}_3 = 1.61038 - 0.00009295 \text{Age} - 0.00074581 \text{Sex}
$$

Test of significance for the Regression parameters

$$
\hat{n}\hat{\Sigma} = \begin{pmatrix} 48826125.63 & 104560.929 & 8857.101 \\ 104560.929 & 81797.214 & -200.325 \\ 8857.101 & -200.325 & 710.883 \end{pmatrix} = \begin{pmatrix} \text{residual sum of} \\ \text{squares and cross} \\ \text{products} \end{pmatrix}
$$

\n
$$
\hat{\Sigma} = \begin{pmatrix} 162753.85210 & 348.53643 & 29.52367 \\ 348.53643 & 272.65738 & -0.66775 \\ 29.52367 & -0.66775 & 2.36961 \end{pmatrix}
$$

The hypothesis that the responses do not depend on Z_2 is H_0 : $\beta_{(2)} = 0$

where
$$
\beta = \begin{bmatrix} \beta_{(1)} \\ \beta_{(2)} \end{bmatrix}
$$

and $\beta = \begin{bmatrix} 167.51417 & 36.18228 & 1.61038 \\ -0.16401 & 0.01367 & -0.00009295 \\ -0.46611 & 0.01787 & -0.00074581 \end{bmatrix}$

Thus, the dimension of $\beta_{(1)}$ is (2 × 3) and that of $\beta_{(2)}$ is (1 × 3)

$$
Z = \left[Z_{(1)} \begin{array}{ccc} | & Z_{(2)} \\ | & | \end{array} \right]
$$

Where the dimension of $Z_{(1)}$ is (300 \times 2) and that of $Z_{(2)}$ is (300 \times 1) The extra sum of squares associated with $\hat{\beta}_{(2)}$ are

$$
n\hat{\Sigma}_{(1)} = (Y - Z_1 \hat{\beta}_{(1)})'(Y - Z_1 \hat{\beta}_{(1)})
$$
\n
$$
= \begin{pmatrix}\n48932145.56 & 106382.521 & 8972.210 \\
106382.521 & 82469.416 & -186.321 \\
8972.210 & -186.321 & 722.883\n\end{pmatrix}
$$
\n
$$
\hat{\Sigma}_{(1)} = \begin{pmatrix}\n163107.1519 & 354.6084 & 29.9074 \\
354.6084 & 274.8981 & -0.6211 \\
29.9074 & -0.6211 & 2.4096\n\end{pmatrix}
$$
\n
$$
n(\hat{\Sigma}_{(1)} - \hat{\Sigma}) = \begin{pmatrix}\n106019.94 & 1821.591 & 115.119 \\
1821.591 & 672.216 & 13.995 \\
115.119 & 13.995 & 11.997\n\end{pmatrix}
$$
\n
$$
|\hat{\Sigma}| = 104.542.012.6
$$

 $\hat{\Sigma}_{1}$ = 107,416,295.4 Using (22), we have $\Lambda^{n} = \frac{1043426120}{1054450054} = 0.97324165$ 107416295.4 $\Lambda^{\frac{2}{n}} = \frac{1045420126}{125445025} =$ Using (23), we have $-\left(300 - 2 - 1 - \frac{1}{2}(3 - 2 + 1 + 1)\right) \ln 0.97324165$ 2 $300 - 2 - 1 - \frac{1}{2}(3 - 2 + 1 + 1)$ $\overline{}$ $\overline{\mathsf{L}}$ $-\left(300 - 2 - 1 - \frac{1}{2}(3 - 2 + 1 + \cdots)\right)$ $= 8.014808685 \approx 8.015$ $\chi^2_{3,0.05} = 7.815$

Since 8.015 > 7.815, we reject the null hypothesis and conclude that sex affects Systolic Blood Pressure, Temperature and Height of patients. Thus, they can be included in the model

Test of significance for β_1

The extra sum of squares associated with $\hat{\beta}_{(1)}$ is

$$
n\hat{\Sigma}_{(2)} = \left[Y - Z_{(2)}\hat{\beta}_{(2)}\right] \left[Y - Z_{(2)}\hat{\beta}_{(2)}\right]
$$
\n
$$
= \begin{pmatrix} 78243675624 & 3456782.521 & 86732.582 \\ 3456782.521 & 923415.362 & 324.321 \\ 86732.582 & 324.321 & 8321.456 \end{pmatrix}
$$
\n
$$
\hat{\Sigma}_{(2)} = \begin{pmatrix} 2608122.5210 & 11522.6084 & 289.1086 \\ 11522.6084 & 3078.0512 & 1.0811 \\ 289.1086 & 1.0811 & 27.7382 \end{pmatrix}
$$
\n
$$
n(\hat{\Sigma}_{(2)} - \hat{\Sigma}) = \begin{pmatrix} 7336106306 & 3352221.591 & 77875.479 \\ 3352221.591 & 841618.146 & 344.3625 \\ 77875.479 & 344.3625 & 7610.577 \end{pmatrix}
$$
\n
$$
|\hat{\Sigma}| = 104,542,012.6
$$
\n
$$
|\hat{\Sigma}_{(2)}| = 2.187445217 \times 10^{11}
$$
\n
$$
\Lambda^{\frac{2}{n}} = \frac{1045420126}{2.187445217 \times 10^{11}} = 0.0004779183122
$$
\n
$$
-\begin{bmatrix} 300 - 2 - 1 - \frac{1}{2} (3 - 2 + 1 + 1) \end{bmatrix} \text{ln } 0.0004779183 & 122
$$
\n
$$
= 2259.413902
$$

There is a very strong rejection since 2259.413902 is greater than 7.815. Hence, we conclude that Age affects Systolic Blood Pressure, Temperature and Height of patients significantly. That is, there is a joint relationship between Systolic Blood pressure, Temperature and Height on one hand and Age on the other hand.

XVII. SUMMARY

This research work is on the Multivariate Linear Regression Model on the relationship between the three measures of Vital Signs of patients and their social characteristics, with a general introduction of the entire research work; thereafter, available literatures were reviewed. The statistical techniques for data analysis were explicitly discussed prior to analysis of the data in details. The statistical software used in this research work for data analysis is the "SAS Version 9.2" package. However, the Microsoft Excel office, 2010 played a vital role in the data analysis, especially in obtaining the residual and predicted matrices. Finally, the results of each analysis were carefully interpreted.

 $\chi^2_{3,0.05} = 7.815$

CONCLUSION

Having carried out the data analysis in the preceding section, the following conclusions can be observed:

- 1. A Multivariate Multiple Linear Regression Model was successfully set up for the relationship between the three measures of Vital Signs (Systolic Blood pressure, Temperature and Height) of patients on one hand, and the two social characteristics (Age, and sex) on the other hand.
- 2. A test of significance revealed that Age and Sex are significant for the Multivariate Multiple Linear Regression Model.

RECOMMENDATIONS

Following the outcome of this study, we recommend that Hospitals should ensure a well standard statistics/records department, to enable researchers have access to data for proper analysis for the good of the system.

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APPENDIX 1

 $\overline{}$ I $\overline{}$ J \setminus \mathbf{I} $\overline{}$ I \setminus ſ $-0.006057224 -0.001004472$ 0.000022943 - $-0.001004472 'Z)^{-1} =$ 0.006057224 -0.000000077 0.013468013 0.001004472 0.000022943 -0.000000077 $0.050038198 - 0.001004472 - 0.006057224$ $(Z'Z)^{-1}$ $\overline{}$ $\overline{}$ $\overline{}$ J \setminus I $\overline{}$ I \setminus ſ \overline{a} $\beta_{1} = \vert - \vert$ 0.46611 0.16401 167.51417 $\hat{\beta}_1 = \begin{vmatrix} -0.16401 \end{vmatrix};$ $\overline{}$ $\overline{}$ $\overline{}$ J \setminus I \mathbf{I} \mathbf{I} \setminus ſ β ₂ = 0.01787 0.01367 36.18228 $\hat{\beta}_2 = | 0.01367 |$ $\overline{}$ $\frac{1}{2}$ \cdot J \setminus I $\overline{ }$ I \setminus ſ \overline{a} $\beta_{3} =$ | -0.00074581 0.00009295 1.61038 $\hat{\bm{\beta}}_{_3}$