The International Journal Of Engineering And Science (IJES)
||Volume||2 ||Issue|| 01 ||Pages|| 220-226 ||2013||
ISSN: 2319 – 1813 ISBN: 2319 – 1805

Generalised Separable Solution of Counter-Current Imbibition Phenomenon In Homogeneous Porous Medium In Horizontal Direction

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Abstract

The phenomenon of counter-current imbibition in double phase immiscible (oil and water) flow through homogeneous porous medium in horizontal direction is discussed. The mathematical formulation of this problem yields a non linear partial differential equation and the generalised separable solution is given in the form of quadratic polynomial. Numerical results are presented graphically using MAT LAB.

Key Words: *Imbibition phenomenon, Immiscible, Homogeneous porous medium, Generalised separable*

Date Of Submission: 21, February, 2013 ⟺ Date Of Publication: 28, February 2013

I. Introduction

In this paper we discusses the imbibition phenomenon in double phase flow during displacement process through homogeneous porous medium in horizontal direction with capillary pressure, due to difference in wetting abilities of the two immiscible fluids flowing in the medium. This phenomenon occurs during secondary recovery process when water is injected to push oil towards oil reservoir. Here water is injected in oil formatted area in horizontal direction. In real, oil formatted area is too large for practical as well as experimental study. Therefore it is necessary to develop mathematical model by selecting its small part as cylindrical porous matrix. For study of one dimensional case we take horizontal cross sectional area which is rectangular shape and instead of real fingers occurs in irregular shape, it has been considered as regular small rectangular fingers. Its average value of saturation of injected fluid is considered for the study. When a porous medium field with some fluid, is brought in contact with another fluid which preferentially wets the medium, there is spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. This phenomenon arising due to difference in wetting abilities of the fluids flowing in the medium is called counter-current imbibition phenomenon. It is the process by which a wetting fluid displaces a non-wetting fluid which initially saturates a porous medium, by capillary forces alone. Suppose a medium is completely saturated with a non-wetting fluid, and some wetting fluid is introduced on its surface. There will be spontaneous flow of wetting fluid into the medium, causing displacement of the non-wetting fluid. This is called imbibitions phenomenon which is shown in figure (1) and (2).

This is one of the most important recovery mechanisms during oil recovery in hydrocarbon reservoir. This phenomenon has been discussed for homogeneous and heterogeneous porous media and also for cracked porous medium by many researchers. This phenomenon has been investigated by many authors like Brownscombe and Dyes (1952), Graham and Richardson (1959), Scheidegger (1960), Verma(1969,1972), Mehta and Verma(1977). Many researchers have discussed this phenomenon with different points of view. Bokserman, Zheltov, and Kocheshkov [1964] have described the physics of oil-water flow in a cracked and heterogeneous porous medium. Mishra and Verma [1973] discussed this phenomenon of ground water replenishment in double phase flow through homogeneous porous media with capillary pressure and transcendental solution has been given using similarity technique. Morrow and Mason [2001] provide an excellent review of the recent developments of spontaneous imbibition. Rangel-German and Kovscek [2002]; Karimaie, Torsaeter, Esfahani, Dadashpour, Hashemi [2006] and Graue, Moe, Baldwin[2000] investigated the effects of injection rate, initial water saturation and gravity on water injection in slightly water-wet fractured porous media. Yildiz, Gokmen, Cesur [2006] examined the effects of shape factor, characteristic length, and boundary conditions on the spontaneous imbibition phenomenon.

II. STATEMENT OF THE PROBLEM

Here it is considered that a finite cylindrical piece of homogeneous porous matrix of length L is fully saturated with native liquid (N). It is completely surrounded by an impermeable surface expect at one end of the cylinder, which is labelled as imbibition surface (x=0) & this end is exposed to an adjacent formation of the injected liquid (I). Oil is assumed as the native liquid and water as the injected liquid. Since water preferentially wets the medium it gives rise to counter-current imbibitions that is spontaneous linear flow of wetting fluid (water) into the medium and a counter flow of the resident liquid (oil) from the medium. This phenomenon is shown in the figure (1) and (2).

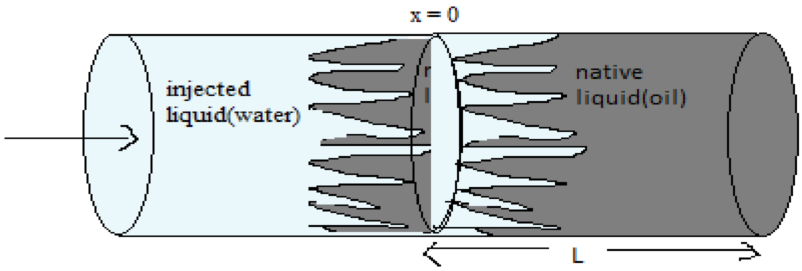

Figure: 1 Representation of linear counter-current imbibition in a cylindrical piece of homogeneous porous media

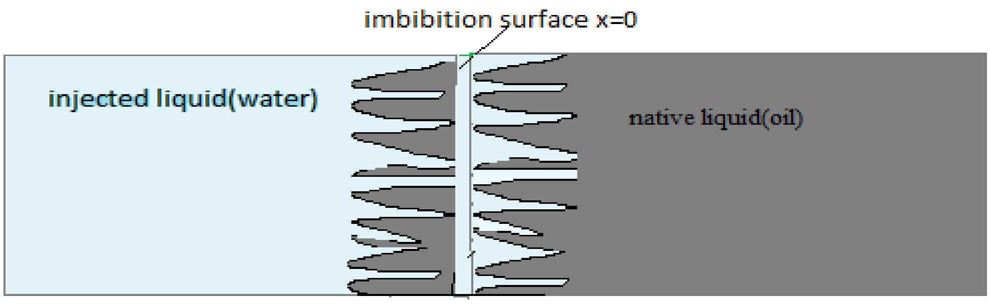

Figure: 2 Schematic presentation of imbibition phenomenon

III. MATHEMATICAL FORMULATION

According to Bear (1972), using Darcy's law, the seepage velocity of injected liquid (water) V_i and native liquid (oil) V_n can be expressed as follows:

$$V_i = -\left(\frac{K_i}{\delta_i}\right) K \left(\frac{\partial P_i}{\partial x}\right) \qquad (3.1)$$

$$V_n = -\left(\frac{K_n}{\delta_n}\right) K \left(\frac{\partial P_n}{\partial x}\right) \qquad (3.2)$$

Where K is the permeability of the homogeneous medium, K_i and K_n are relative permeabilities of water and oil which are functions of saturations S_i and S_n, P_i and P_n are pressure, δ_i and δ_n are constant kinematic viscosity of water and oil respectively. The coordinate x is measured along the axis of the cylindrical medium, the origin being at the imbibition surface.The major difference between instability phenomenon and imbibition phenomenon is that in Instability phenomenon, due to external injecting force the size of fingers can be extend up to the end of porous matrix at x=L while in Imbibition phenomenon it occurs only due to contact of two phase, there is spontaneous flow in counter current direction. Hence saturation of injected phase will increase at very small distance up to x = l which is very near to common interface. In this way there is vast difference between these two phenomenons. In addition to this, for simplification of problem, imbibition condition $V_i = -V_n$ has been used in imbibition phenomenon which does not play any role in instability phenomenon. Also during imbibition, the saturation takes place very near to common interface so it is appropriate to choose linear relation of capillary pressure and saturation as $P_c = -\beta S_i$, where β is constant and $K_i = S_i$For counter-current imbibition phenomenon, the sum of the velocities of injected liquid (water) and native liquid (oil) is zero, therefore the following condition holds true:

$V_i = -V_n$, Scheidegger (1960) (3.3)

From (3.1) and (3.2),

$$\left(\frac{K_i}{\delta_i}\right)\left(\frac{\partial P_i}{\partial x}\right)+\left(\frac{K_n}{\delta_n}\right)\left(\frac{\partial P_n}{\partial x}\right)=0 \quad (3.4)$$

The capillary pressure (P_c) is defined as the pressure difference of the flowing phase across their common interface is a function of phase saturation. It may be written as

$P_c = P_n - P_i$ (3.5)

From equations (3.4) and (3.5)

$$\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)\frac{\partial P_i}{\partial x}+\left(\frac{K_n}{\delta_n}\right)\frac{\partial P_c}{\partial x}=0 \quad (3.6)$$

On substituting the value of $\frac{\partial P_i}{\partial x}$ from equation (3.6) into (3.1), we get

$$V_i = K\frac{\frac{K_i K_n}{\delta_i \delta_n}}{\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}}\frac{\partial P_c}{\partial x} \quad (3.7)$$

The equation of continuity of the wetting phase is

$$P\left(\frac{\partial S_i}{\partial t}\right)+\frac{\partial V_i}{\partial x}=0 \quad (3.8)$$

where P is the porosity of the medium.

On substituting the value of V_i from equation (3.7) to (3.8),

$$P\left(\frac{\partial S_i}{\partial t}\right)+\frac{\partial}{\partial x}\left[K\frac{\frac{K_i K_n}{\delta_i \delta_n}}{\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}}\frac{dP_c}{dS_i}\frac{\partial S_i}{\partial x}\right]=0 \quad (3.9)$$

This equation (3.9) is a non linear partial differential equation which describes the linear counter current imbibitions phenomenon of two immiscible fluids flow through cylindrical homogeneous porous medium with impervious bounding surface on three sides.It is already known that the fictitious relative permeability is function of displacing fluid saturation, so here for definiteness of the mathematical analysis, we assume standard forms of Schiedegger and Johnson(1961) for analytical relationship between relative permeability-phase saturation and capillary pressure-phase saturation as

$K_i = S_i$ (3.10)

$K_n = 1 - \alpha S_i$, $\alpha = 1.11$ (3.11)

As Mehta (1977) suggested that capillary pressure is proportional to saturation of injected liquid in opposite direction,

$P_c = -\beta S_i$, where β is constant of proportionality (3.12)

Since the present investigation involves water and viscous oil, therefore according to Schiedegger (1960), we have

$$\left[\frac{\frac{K_i K_n}{\delta_i \delta_n}}{\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}}\right]\approx\frac{K_n}{\delta_n}=\frac{1-\alpha S_i}{\delta_n} \quad (3.13)$$

On substituting value of P_c from (3.12) into equation (3.9) and using (3.13), we get,

$$P\left(\frac{\partial S_i}{\partial t}\right)=\frac{K\beta}{\delta_n}\frac{\partial}{\partial x}\left[(1-\alpha S_i)\frac{\partial S_i}{\partial x}\right] \quad (3.14)$$

This is the desired non linear partial differential equation describing the linear counter-current imbibitions phenomenon with capillary pressure.

The initial saturation of injected water will very small as parameter ε.

Hence,

$S_i(X,0) = S_0(X)$ (3.15)

Also at common interface and at end, saturation of injected water will be dependent of time T > 0.

Hence a set of boundary conditions will be

$S_i(0,T) = S_{i0}(T)$, $T > 0$ (3.16)

$S_i(l,T) = S_{i1}(T)$, $T > 0$ (3.17)

where $x=l$ is very near to common interface ($x = 0$), S_{i0} and S_{i1} are the saturations at $x = 0$ and $x = l$ respectively.

L being the total length of the cylindrical porous matrix from common interface *x*=0, so that
$0 \leq x \leq l \ , \ l \ll L$.
Since imbibition takes place near to common interface x= 0 for small time and length of fingers are also small. Therefore appropriate boundary conditions will be (3.16) and (3.17). Here $S_{i1}(T)$ is saturation of injected liquid

which is very near to $S_{i0}(T)$ and also depends on time during the imbibition process. Therefore it is considered as function of time T.
We now choose new variables to convert equation (3.14) into dimension less form

$$T = \frac{K\beta}{\propto L^2 P\delta_n} t \ , \quad X = \frac{x}{l} \ \ and \ \ S = 1 - \propto S_i$$

Therefore, the equation (3.14) is converted to

$$\frac{\partial S}{\partial T} = \frac{\partial}{\partial X}\left[S\left(\frac{\partial S}{\partial X}\right)\right] \qquad (3.18)$$

The initial and boundary conditions are converted to

$$S(X,0) = 1 - \propto S_0(X) \qquad (3.19)$$

$$S(0,T) = 1 - \propto S_{i0}(T) \ , \qquad T > 0 \qquad (3.20)$$

$$S(l,T) = 1 - \propto S_{i1}(T) \ , \qquad T > 0 \qquad (3.21)$$

IV. Separable Solution In Form Of Quadratic In X

Equation (3.18) can be rewritten as

$$\frac{\partial S}{\partial T} = S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2 \qquad (4.1)$$

The governing equation is non linear partial differential equation of parabolic type. To find the solution of the problem our basic assumption must be consistent with physical phenomenon. It is well known fact that during imbibition process, the size of fingers is increasing for small distance. The total saturation of all fingers are increasing with respect to distance, hence it is appropriate to choose saturation of injected liquid as a quadratic function of x. Also the process of imbibition depends on time t, hence we choose (4.2) as separable solution. Galaktionav and Posashkov (1989)

$$S(X,T) = \emptyset(T)X^2 + \Psi(T)X + \tau(T) \qquad (4.2)$$

Where the functions $\emptyset(T)$, $\Psi(T)$ *and* $\tau(T)$ are determined by a system of first order ordinary differential equations with variable coefficients as follows:

$$\emptyset'(T) = 6\emptyset^2 \implies \emptyset(T) = -\frac{1}{6T + C_1} \qquad (4.3)$$

$$\Psi'(T) = 6\emptyset\Psi \implies \Psi(T) = \frac{C_2}{6T + C_1} \qquad (4.4)$$

$$\tau'(T) = 2\emptyset\tau + \Psi^2 \implies \tau(T) = \frac{-(C_2)^2}{4(6T + C_1)} + \frac{C_3}{(6T + C_1)^{1/3}} \qquad (4.5)$$

Where C_1, C_2, C_3 are arbitrary constants.
using the condition (3.19),

$$S(X,0) = \emptyset(0)X^2 + \Psi(0)X + \tau(0)$$

$$1 - \propto S_0(X) = \emptyset(0)X^2 + \Psi(0)X + \tau(0) \qquad (4.6)$$

where $\emptyset(0)$, $\Psi(0)$, $\tau(0)$ are non zero constants and $\propto = 1.11$
Let $\emptyset(0) = a$, $\Psi(0) = b$, $\tau(0) = c$ where a, b and c are arbitrary constants.
Equation (4.6) can be written as

$$1 - \propto S_0(X) = aX^2 + bX + c$$

Now to find C_1, C_2 *and* C_3, substituting $\emptyset(0) = a$, $\Psi(0) = b$, $\tau(0) = c$ in (23),(24) and (25) respectively, we get

$$C_1 = -\frac{1}{a} \ , \qquad C_2 = -\frac{b}{a} \ , \qquad C_3 = \left(c - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}$$

Substituting the values of C_1, C_2 *and* C_3 in (4.3),(4.4) and (4.5),

$$\emptyset(T) = -\frac{1}{6T - \frac{1}{a}}$$

$$\Psi(T) = \frac{-\frac{b}{a}}{6T - \frac{1}{a}}$$

$$\tau(T) = \frac{-\left(-\frac{b}{a}\right)^2}{4\left(6T - \frac{1}{a}\right)} + \frac{\left(c - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T - \frac{1}{a}\right)^{1/3}}$$

Substituting values of $\emptyset(T)$, $\Psi(T)$ *and* $\tau(T)$ in (4.2),

$$S(X,T) = -\left(\frac{1}{6T-\frac{1}{a}}\right)X^2 + \left(\frac{-\frac{b}{a}}{6T-\frac{1}{a}}\right)X + \left(\frac{-\left(-\frac{b}{a}\right)^2}{4\left(6T-\frac{1}{a}\right)} + \frac{\left(c-\frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T-\frac{1}{a}\right)^{1/3}}\right) \quad (4.7)$$

By substituting, $S(X,T) = 1 - \propto S_i(X,T)$, we get

$$S_i(X,T) = \frac{1}{\alpha}\left[1 + \left(\frac{1}{6T-\frac{1}{a}}\right)X^2 - \left(\frac{-\frac{b}{a}}{6T-\frac{1}{a}}\right)X - \left(\frac{-\left(-\frac{b}{a}\right)^2}{4\left(6T-\frac{1}{a}\right)} + \frac{\left(c-\frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T-\frac{1}{a}\right)^{1/3}}\right)\right] \quad (4.8)$$

$$where \ \ T = \frac{K\beta}{\propto l^2 P\delta_n} t, \ X = \frac{x}{l}$$

This is the required solution of governing equation (3.18). Numerical values and graphical representation of the solution (4.8) is obtained by Mat Lab coding as follows.

V. Numerical and Graphical Solution

According to Meher et.al.(2010), the initial saturation of injected fluid is $S_0(X) = e^{-X}$ *for any* $X > 0$.
Now substituting $S_0(X) = e^{-X}$ in equation (28), We get,

$1 - \propto e^{-X} = aX^2 + bX + c \quad where \ \propto = 1.11.$

Using the expansion of e^{-X} and equating the coefficients of X^2, X and constant term by neglecting X^3 and higher powers of *X*, we obtain the values of *a*, *b* and *c* as follows:

$a = -0.555, \ b = 1.11, \ c = -0.11$

Numerical and graphical presentations of solution (4.8) have been obtained by using MAT LAB coding. Figure (2.3) shows the graphs of S_i Vs. *X* for time T = 0.5, 0.6, 0.7, 0.8 and Table-2.1 represent the numerical values.

Table showing saturation of injected water (S_i) for different X for fixed T

Time (T) →	0.5	0.6	0.7	0.8
Distance (X) ↓	**Saturation of injected water (Si)**			
0	0.0500	0.0600	0.0700	0.0800
0.1	0.0716	0.0937	0.1151	0.1361
0.2	0.0984	0.1318	0.1642	0.1957
0.3	0.1304	0.1746	0.2173	0.2588
0.4	0.1676	0.2218	0.2743	0.3255
0.5	0.2100	0.2735	0.3353	0.3958
0.6	0.2576	0.3298	0.4003	0.4695

0.7	0.3104	0.3906	0.4693	0.5468
0.8	0.3684	0.4558	0.5422	0.6277
0.9	0.4316	0.5257	0.6191	0.7121
1	0.5000	0.6000	0.7000	0.8000

Table: 1

Graph of saturation of injected water (S_i) vs. distance (x) for different time T > 0

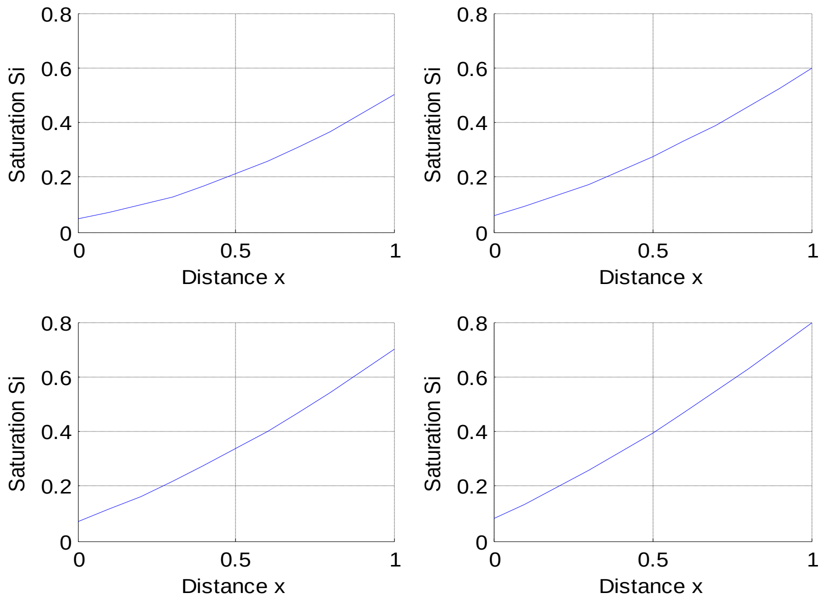

Figure 3

Graph of saturation of injected water (Si) vs. distance(x) for fixed time T > 0

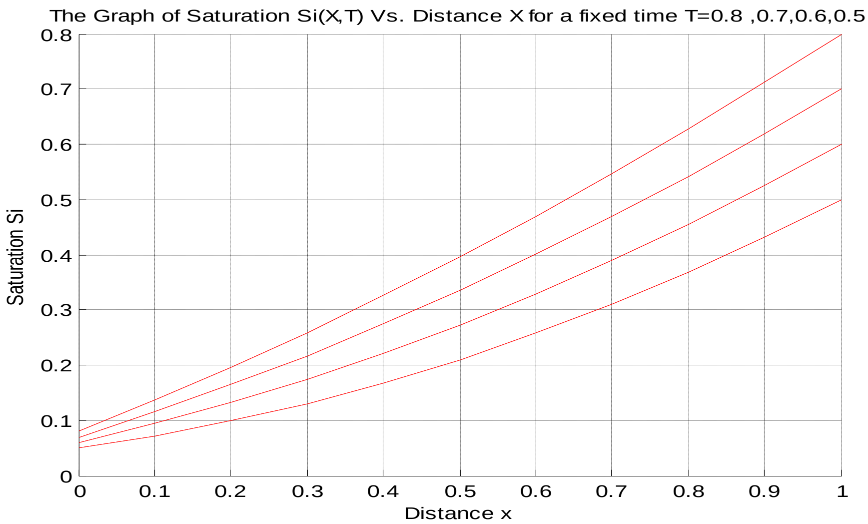

Figure 4

VI. Conclusion

The solution (4.8) represents Saturation of injected water in horizontal direction for any distance X for any time T > 0. When the injected liquid (water) comes into the contact of native liquid (Oil), imbibition takes place at the common interface. The injected liquid enters into the oil formation without any external force up to the distance *x=l*, where 0 < *l* < L. In this analysis we have assumed that the initial saturation at common interface is ε, where $\varepsilon \ll 1$ and it is physically fact that saturation of injected water will increase as distance increases at any time T > 0. The boundary conditions are assumed as a linear function of time. The solution is in the form of quadratic polynomial in X and which satisfies both the boundary conditions (3.20) and (3.21) at X = 0 and at X=L which is parabolic solution. The saturation of water is increasing with respect to distance as well as with respect to time and it is consistent with physical phenomenon. This saturation represents saturation of water in horizontal direction near the common interface maximum up to *x=l*. The graphical representation and numerical values are given using MATLAB coding. The graph of S_i vs. X for any time T is steadily increasing and after some distance X for any time T, it is likely to be constant which is consistent with physical phenomenon.

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