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Peristaltic motion of a Micropolar fluid under the effect of a magnetic field in an inclined channel

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ABSTRACT
Mathematical modeling and numerical solution are presented for the flow of a Micro polar fluid under the effect of magnetic field in an inclined channel of half width 'a' under the considerations of low Reynolds number. A long wavelength approximation is used to solve the flow problem. The effect of various parameters on the pumping characteristics is discussed through graphs.

Keywords – *Peristalsis, Micropolar fluid, Magnetic field, inclined channel.*

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I. INTRODUCTION

Peristaltic pumping is a form of fluid transport, generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion which propagates along the length of a tube like structure. Peristalsis occurs naturally as a means of pumping biofluids from one place of the body to another. This mechanism occurs in the gastrointestinal, urinary and reproductive tracts and many other glandular ducts in the living body. The early reviews of Ramachandra Rao and Usha [1] , Jaffrin and Shapiro[2] , Manton [3], Brasseur et al. [4], Srivastava and Srivastava [5], Provost and Schwarz [6], Shukla and Gupta [7], Misra and Pandey [8] ,Rao and Rajagopal [9],Kavitha et.al[10] Vajravelu et al. [11-15], Subba Reddy[16,17], Srinivas [18,19] deal with the peristaltic transport of viscous fluids through tubes and channels having impermeable flexible walls.

Eringen [20,21] reported the theory of micropolar fluids in which the fluid micro elements undergo rotations without stretching. Micropolar fluids are superior to the Navier-Stokes fluids and they can sustain stresses and body couples. Here the micro particles in the volume Δv rotate with an angular velocity about the centre of gravity of the volume in an average sense and is described by the micro rotation vector. The micropolar fluids can support stress and body couples and find their applications in a special case of fluid in which micro rotational motions are important. Airman and Cakmak [22] discussed three basic viscous flows of micropolar fluids. They are Couette and Poiseuille flows between two parallel plates and the problem of a rotating fluid with a free surface. The results obtained are compared with the results of the classical fluid mechanics. Srinivasacharya et al.[23] made a study on the peristaltic pumping of a micro polar fluid in a tube.

Magnetohydrodynamics (MHD) is the science which deals with motion of highly conducting fluids in the presence of a magnetic field. The motion conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise the mechanical forces which modify the flow of the fluid.(Ferraro V C A,[24]). The effect of moving magnetic field on blood flow was studied by Stud et al (Stud et al.[25]), and they observed that the effect of suitable moving magnetic field accelerates the speed of blood.

Krishna Kumari et.al [26] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field. Krishna Kumari et.al [27] studied the peristaltic pumping of a Jeffrey fluid in a porous tube. Ravi Kumar et.al [28] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. However, the peristaltic transport of micropolar fluids in an inclined channel in the presence of magnetic field has not been studied.

In view of this, we considered the peristaltic pumping of a micropolar fluid in an inclined channel under the effect of magnetic field. This mathematical model may be useful to have a better understanding of the physiological systems such as blood vessels.

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II. MATHEMATICAL FORMULATION

Consider the peristaltic pumping of a micropolar fluid in an inclined channel of half-width ‘a’. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity we restrict our discussion to the half-width of the channel as shown in Fig.1. The wall deformation is given by

$$H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where b is the amplitude, λ is the wavelength and c is the wave speed.

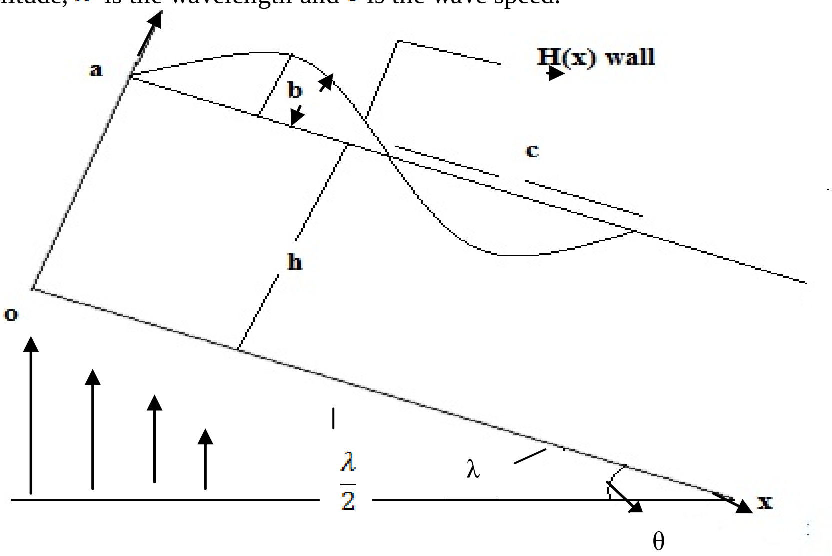

Fig.1 Physical Model

Under the assumption that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y). The transformation between these two frames is given by

$$x = X - ct;\ y = Y;\ u(x,y) = U(X - ct, Y);\ v(x,y) = V(X - ct, Y) \tag{2}$$

where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the flow is inertia-free. Further, we assume that the wavelength is infinite. Using the non-dimensional quantities.

$$\bar{u} = \frac{u}{c};\ \bar{x} = \frac{x}{\lambda};\ \bar{y} = \frac{y}{a};\ \bar{p} = \frac{pa^2}{\lambda c\mu};\ \bar{\Omega} = \frac{\Omega a}{c};\ h = \frac{H}{a}$$

The non-dimensional form of equations governing the motion (dropping the bars) are

$$\frac{\partial^2 u}{\partial y^2} + N\frac{\partial \Omega}{\partial y} - (u-1)M^2 - (1-N)\frac{\partial p}{\partial x} + \eta\sin\theta = 0 \tag{3}$$

$$\frac{2-N}{m^2}\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial u}{\partial y} - 2\Omega = 0 \tag{4}$$

where $N = \frac{k}{\mu + k}$ is coupling number, Ω is the micro rotation velocity, u is the velocity, μ is the viscosity of the fluid, k is the micropolar viscosity, m is the micropolar parameter, p is the fluid pressure, M is the Hartmann number.

The corresponding non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \tag{5}$$

$$\frac{\partial \Omega}{\partial y} = 0 \quad \text{at } y = 0 \tag{6}$$

$$u = -\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \quad \text{at } y = h(x) \tag{7}$$

$$\Omega = -1 - \frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \quad \text{at } y = h(x) \tag{8}$$

where Da is the Darcy number and α is the slip parameter.

III. SOLUTION OF THE PROBLEM

The general solution of (3) and (4) using the boundary conditions (5)-(8) is given by

$$u = L_{10}\cosh(L_3 y)(L_8 L_{14} + L_9)c_1 + \frac{L_4}{L_3}e^{L_3 y}c_1\left(L_{14} - 1\right) + (e^{L_4 y} + e^{-L_4 y})(L_{14} + 1)c_1 + L_{15}\left(L_{10}L_8\cosh(L_3 y)\right.$$
$$\left. + \frac{L_4}{L_3}\left(e^{L_3 y} + e^{L_4 y}\right)\right) - \frac{L_1}{M^2} \tag{9}$$

$$\Omega = L_3 L_{10}\, Sin\,(L_3 y)\left(L_5 - L_3^2\right)\left[(L_6 L_8 L_{14} + L_9)C_1 + L_{15}\right] + L_4\left(L_5 - L_3^2\right)\left[(L_6 L_{14} + 1)C_1 + L_{15}\right]e^{L_3 y}$$
$$+ L_4\left(L_5 - L_3^2\right)\left[L_6(L_{14}c_1 + L_{15})\right]e^{L_4 y} + L_4(L_5 - L_3^2)c_1 e^{-L_4 y} \tag{10}$$

$$L_1 = M^2 + (1 - N)\frac{\partial p}{\partial x} - \eta\sin\theta,\; L_2 = \frac{2 - N}{m^2},\; L_3 = \sqrt{\frac{M^2 + m^2}{2} + \frac{1}{2}\sqrt{\frac{2\left(M^2 - m^2\right)^2 - N\left(M^2 + m^2\right)^2}{2 - N}}};$$

$$L_4 = \sqrt{\frac{M^2 + m^2}{2} - \frac{1}{2}\sqrt{\frac{2\left(M^2 - m^2\right)^2 - N\left(M^2 + m^2\right)^2}{2 - N}}};\; L_5 = M^2 - NL_2;\; L_6 = 1/2NL_2;$$

$$L_7 = L_3(L_3 L_5 - L_3^3);\; L_8 = L_4(L_5(L_3 - L_4) - (L_3^3 - L_4^3));\; L_9 = L_4(L_5(L_3 + L_4) + (L_3^3 - L_4^3));$$

$$L_{10} = L_4 / L_7;\; L_{11} = -\sqrt{Da}/\alpha;$$

$$L_{12} = L_8 L_{10}(\cosh[L_3 h] - L_{11}L_3\sinh[L_3 h]) - L_4((1/L_3) + L_{11})\exp[L_3 h] + (1 - L_{11}L_4)\exp[L_4 h];$$

$$L_{13} = L_9 L_{10}(\cosh[L_3 h] - L_{11}L_3\sinh[L_3 h]) + L_4((1/L_3) + L_{11})\exp[L_3 h] - (1 - L_{11}L_4)\exp[-L_4 h];$$

$$L_{14} = L_{13}/L_{12};\quad L_{15} = L_3 L_{10}(L_5 - L_3^2)(L_8 L_{14} - L_9)(\sinh[L_3 h] - (L_3 L_{11})\cosh[L_3 h]);$$

$$L_{16} = L_4 L_5 L_{14}(\exp[L_3 h] + \exp[L_4 h]);\quad L_{17} = L_4 L_{14}(L_3^2\exp[L_3 h] + L_4^2\exp[L_4 h]);$$

$$L_{18} = L_4 L_5(\exp[L_3 h] + \exp[-L_4 h]);\quad L_{19} = L_4(L_3^2\exp[L_3 h] + L_4^2\exp[-L_4 h]);$$

$$L_{20} = L_3 L_8 L_{10}(L_5 - L_3^2)(L_3 L_{11}\cosh[L_3 h] - \sinh[L_3 h]);$$

$$L_{21} = L_4 L_5(L_{11}(L_3\exp[L_3 h] + L_4\exp[L_4 h]) - (\exp[L_3 h] + \exp[L_4 h]));$$

$$L_{22} = L_4(L_{11}(L_3^3\exp[L_3 h] + L_4^3\exp[L_4 h]) - (L_3^2\exp[L_3 h] + L_4^2\exp[L_4 h]));$$

$$L_{23} = L_4 L_5 L_{11}(L_{14}(L_3\exp[L_3 h] + L_4\exp[L_4 h]) - (L_3\exp[L_3 h] - L_4\exp[-L_4 h]));$$

$$L_{24} = L_4 L_{11}(L_{14}(L_3^3\exp[L_3 h] + L_4^3\exp[L_4 h]) + (L_3^2\exp[L_3 h] - L_4^3\exp[-L_4 h]));$$

$$L_{25} = L_{15} + L_{16} - L_{17} + L_{18} - L_{19} - L_{23} + L_{24};\quad L_{26} = L_{20} + L_{21} - L_{22};$$

$$c_1 = (-1/L_{25}L_6) - (L_{15}L_{26}/L_{25})$$

The volume flux q through each cross section in the wave frame is given by

$$q = \int_0^h u\,dy \tag{11}$$

The pressure gradient is obtained from equation (11)

$$\partial p / \partial x = (M^2 - (1 - N))((q - S_6)/(S_7 - hL_{12}))L_{12} - (M^2 + \eta \sin\theta)/(1 - N); \tag{12}$$

(12) $S_1 = (L_{10}/L_3) Sinh(L_3 h), \quad S_2 = L_4(e^{L_2 h} - 1)/L_3^2, \; S_3 = (e^{L_4 h} - 1)/L_4^2, \quad S_4 = S_1 L_8 - S_2 + S_3,$

$S_5 = S_1 L_9 - S_2 + S_3, \quad S_6 = S_4(1 - L_{14})/L_6 L_{25}, \; S_7 = (L_{26}(S_4(1 - L_{14}) + S_5)/L_{25}$

The time averaged flow rate is

$$\overline{Q} = q + 1 \tag{13}$$

IV. THE PUMPING CHARACTERISTICS

Integrating the equation (12) with respect to over one wave length, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx \tag{14}$$

The dimensionless frictional force F at the wall across one wavelength in the inclined channel is given by

$$F = \int_0^1 h\left(-\frac{\partial p}{\partial x}\right) dx \tag{15}$$

V. RESULTS AND DISCUSSIONS

The variation of pressure rise Δp with time averaged flow rate for different values of α is shown in Fig.2. It is observed that for a given $\overline{Q}$, Δp decreases as the slip parameter α increases in the pumping and free pumping regions. The opposite behavior is observed in co-pumping region. And also for a given Δp the flux $\overline{Q}$ increases with increase in α.

The variation of pressure rise with time averaged flow rate for different values of Hartmann number M is shown in Fig.3. It is observed that for a given $\overline{Q}$, Δp decreases for a decreasing M in pumping and free pumping regions. For a given Δp the flux $\overline{Q}$ depends on M and it increases with increasing M .The variation of pressure rise with time averaged flow rate for different values of micropolar parameter M is shown in Fig.4. It is observed that for a given $\overline{Q}$, Δp increases for a increasing m in pumping and free pumping regions. For a given Δp the flux $\overline{Q}$ depends on m and it increases with increasing m.

The effect of the inclination angle θ on pumping characteristics is shown in Fig.5. It is observed that for a given $\overline{Q}$, Δp increases as the angle of inclination θ increases. Also for a given θ, Δp increases as $\overline{Q}$ increases.The variation of pressure rise with time averaged flow rate for different values of Darcy number *Da* is shown in Fig.6. It is observed that for a given $\overline{Q}$, Δp increases for a increasing in *Da* pumping and free pumping regions. For a given Δp the flux $\overline{Q}$ depends on *Da* and it increases with increasing *Da*.

The effect of coupling parameter on the pumping characteristics is shown in Fig.7. We observed that the large the coupling number N, the pressure rises against which the pumping works. For a given $\overline{Q}$ the pressure difference increases with increase in N .The effect of amplitude ratio on pumping characteristics is shown in Fig.8. It is observed that the large the amplitude ratio, the greater the pressure rise against which the pump works. For a given Δp, the flux $\overline{Q}$ depends on h and it increases with increasing h.

The effect of η on pumping characteristics is shown in Fig.9. It is observed that for a given $\overline{Q}$, Δp increases as η increases. Also for a given η, Δp increases as $\overline{Q}$ increases. Figs.10 to 14 are drawn to study the effect of various parameters on the microrotation velocity. From Fig.10 it is observed that an increase in the slip parameter decreases the microrotation velocity. From Fig. 11 it is noticed that decrease in the darcy number decreases the microrotation velocity. From Fig. 12 it is observed that increase in M increases the microrotation velocity. Similarly increase in coupling parameter increases the microrotation velocity and is shown in figure.The effect of micropolar parameter on the microrotation velocity is shown in Fig.13. It can be seen that the decrease in m decreases the microrotation velocity.

VI. CONCLUSIONS

Mathematical modeling of the peristaltic pumping of a Micropolar fluid under the effect of a magnetic field in an inclined channel is done in this paper. The following are the conclusions drawn from this.

1. Pumping decreases as the slip parameter α increases in the pumping and free pumping regions. The opposite behavior is observed in co-pumping region.
2. For a given time averaged flow rate, the pressure difference decreases for a decreasing magnetic parameter.
3. The increase in micropolar parameter , increases the pumping in all the pumping regions. The same phenomenon is observed for the angle of inclination, Darcy number also.
4. The effect of various parameters on the microrotation velocity is studied. An increase in the slip parameter decreases the microrotation velocity. A decrease in the Darcy number decreases the microrotation velocity. An increase in the magnetic parameter increases the microrotation velocity. Similarly increase in coupling parameter increases the microrotation velocity.

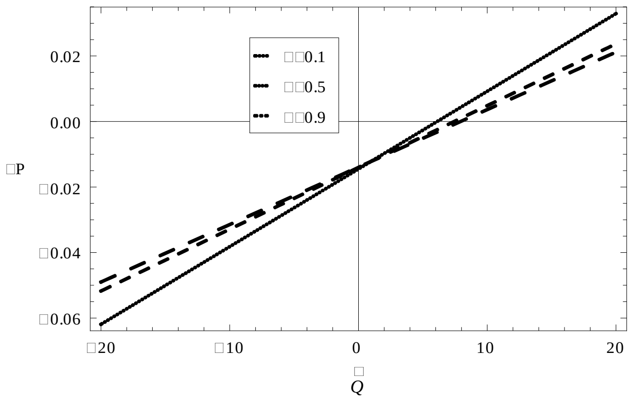

Fig. 2. Variation of Δp with $\overline{Q}$ for different values of α .

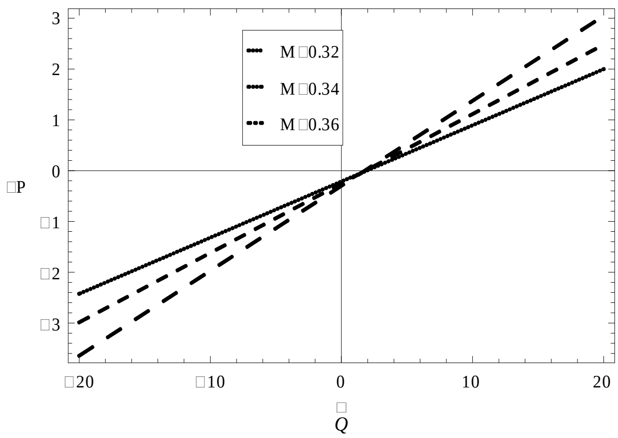

Fig.3. Variation of Δp with $\overline{Q}$ for different values of M .

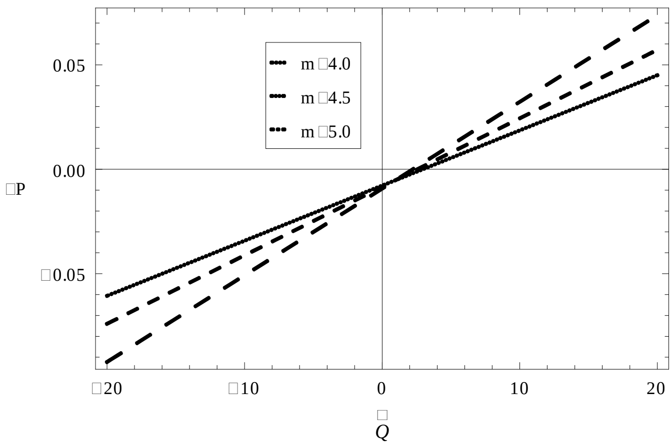

Fig.4. Variation of Δp with $\overline{Q}$ for different values of micropolar parameter m .

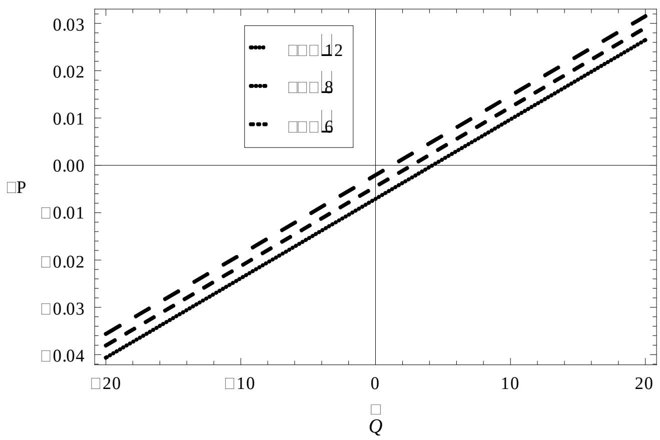

Fig. 5. Variation of Δp with $\overline{Q}$ for different values of angle of inclination θ .

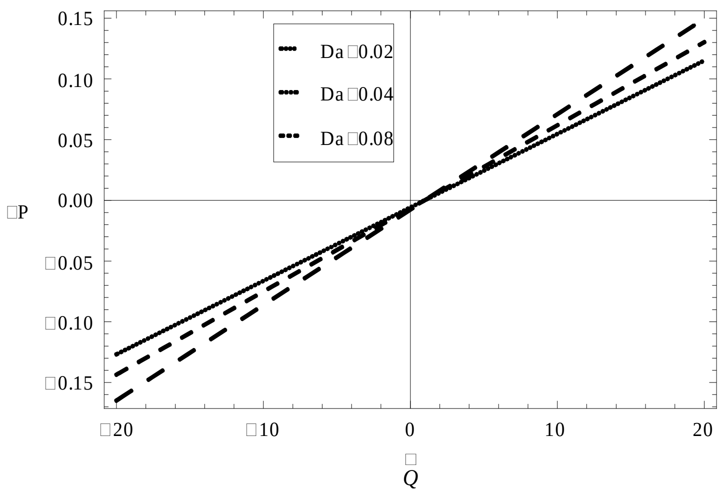

Fig.6. Variation of Δp with $\overline{Q}$ for different values of Darcy number Da .

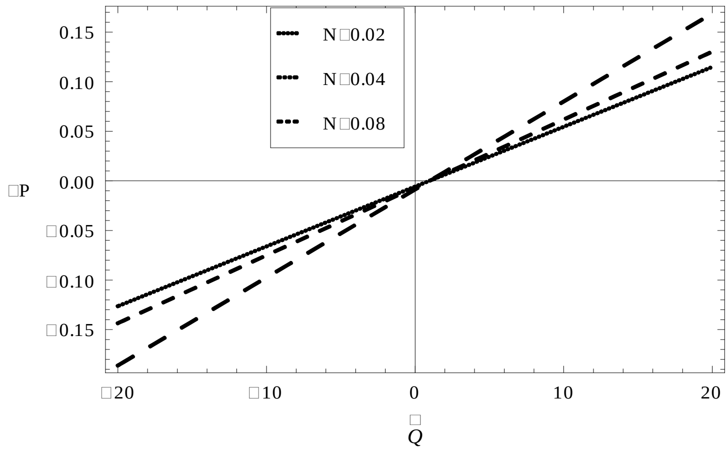

Fig.7. Variation of Δp with $\overline{Q}$ for different values of coupling parameter N .

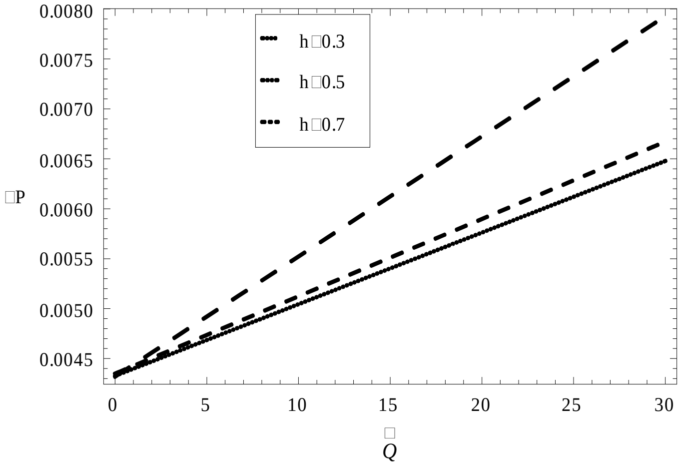

Fig.8. Variation of Δp with $\overline{Q}$ for different values of h .

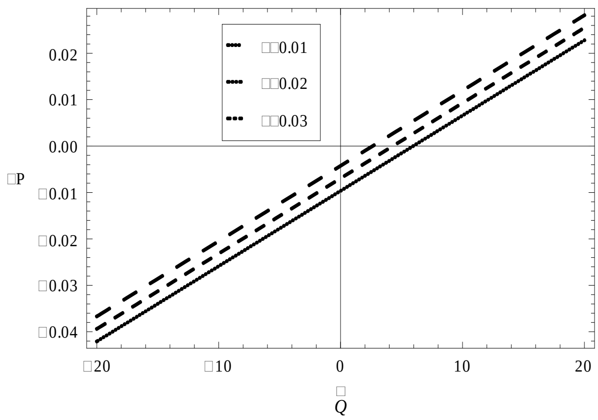

Fig.9. Variation of Δp with $\overline{Q}$ for different values of η .

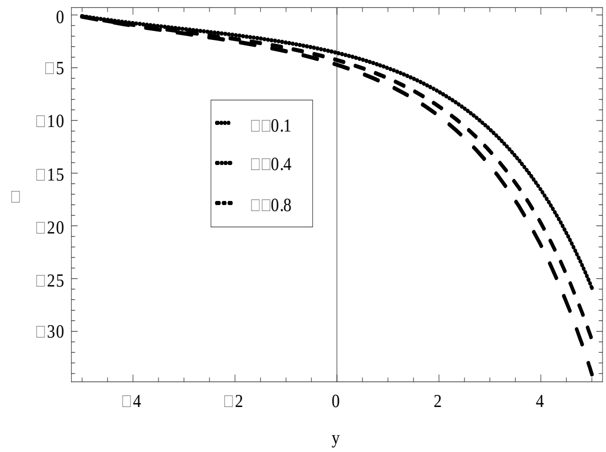

Fig.10. Variation of Ω with y for different values of slip parameter α .

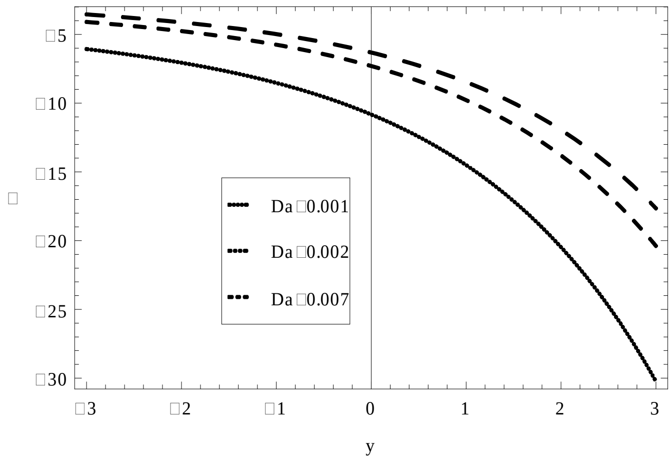

Fig.11. Variation of Ω with y for different values of Darcy number Da .

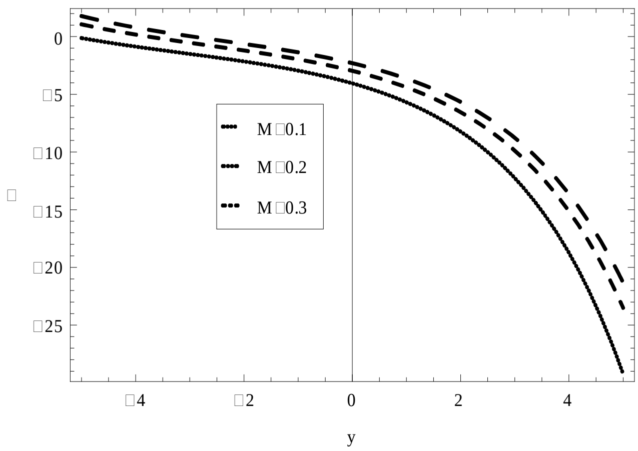

Fig.12. Variation of Ω with y for different values of slip parameter α.

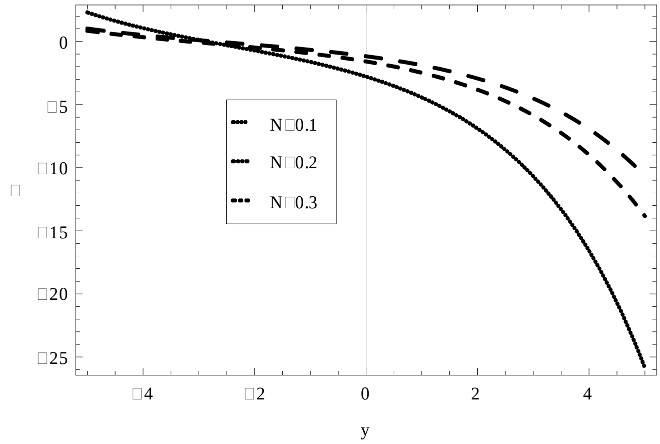

Fig.13. Variation of Ω with y for different values of coupling parameter N .

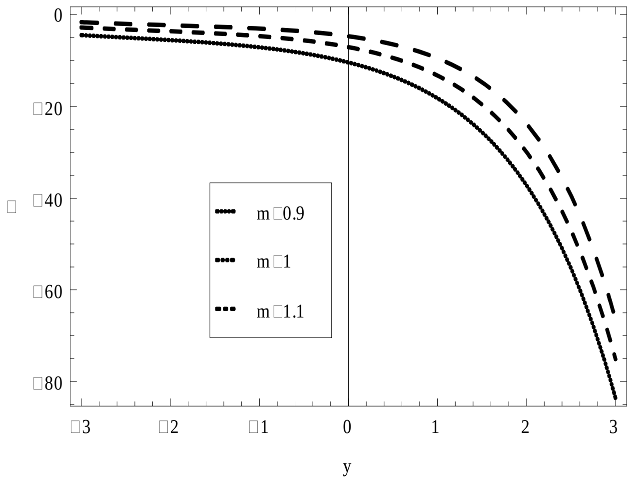

Fig.14. Variation of Ω with y for different values of micropolar parameter m .

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