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Analysis of Gamma Factor in (1+1)-Dimensional External Trapping Potential Applied For BEC

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Abstract
This paper will focus on theoretical analysis of Gamma Factor in (1+1) external trapping potentials which are usually used in experimental that lead to produced Bose-Einstein condensation BEC in ultra cold gases. Two mixed types of trapping potentials are used in this analysis. The first one is a harmonic oscillator potential (HOP) assume to be applied normal to the propagation axis, and the second is a double well potential DWP assume to be applied Parallel to the propagation axis. The cases of slowly and rapidly varying in gamma factor for HOP are considered. Although these analyses give us the overall view of the region of confinement that the external trapping potentials have employed but also shows that the gamma factor in the mathematical formula of HOP play a major part in term of values and shape of the trapping external potential of confinement region.

Keywords: Laser cooled atom, BEC atom, Trapping, Atom laser, Condensation

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1. Introduction

The recognition of Bose-Einstein condensates (BECs) in dilute quantum gases has strained a great deal of attention to the dynamics of nonlinear excitations in matter waves, such as dark [1] and bright solitons [2], vortices [3, 4], supervortices [5], etc. For the detail discussions see also [6-7]. In these verifications, theoretical exploration of characteristic of trapped potential needs a mathematical model describing those potentials which are used experimentally to produce BEC at very low temperatures. Many different shape of Bose-Einstein condensation has been achieved by using different type of trapping potential. External parabolic potential in (highly anisotropic) of the axial symmetry has been used to develop BEC see for example [8-13]. In some literatures, many authors investigated the effect of gravitation [14] by adding the gravitational potential as an external interaction. In this paper, we analyze the effect of gamma (γ)-factor on (1+1) dimensional harmonic oscillator potential which propagates along y-axis plus double well trapping potential along the x-axis

2. Theory

The time dependent many-body Hamiltonian describing N interacting bosons confined by an external potential is given in second quantization by

$$\hat{H} = \int \hat{\psi}^{\dagger}(r,t)\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\boldsymbol{r},t)\right)\hat{\psi}(r,t)\,d\boldsymbol{r} + \frac{1}{2}\iint \hat{\boldsymbol{\psi}}^{\dagger}(\boldsymbol{r},\boldsymbol{t})\,\hat{\boldsymbol{\psi}}^{\dagger}(\boldsymbol{r}',\boldsymbol{t})V_{int}(\boldsymbol{r}'-\boldsymbol{r})\hat{\boldsymbol{\psi}}^{\dagger}(\boldsymbol{r},\boldsymbol{t})\hat{\boldsymbol{\psi}}^{\dagger}(\boldsymbol{r}',\boldsymbol{t}) \qquad (1)$$

Where: $V_{ext}(\boldsymbol{r},t)$ is the external trapping potential and $V_{int}(\boldsymbol{r}'-\boldsymbol{r})$ is the two-body inter-atomic interacting potential. $\hat{\psi}^{\dagger}(r,t) = \sum_{\alpha}\psi_{\alpha}(\boldsymbol{r},t)a_{\alpha}$ is the field operator; $\psi_{\alpha}(\boldsymbol{r},t)$ is the single particle wave function; a_{α} is the annihilation/creation operators:

$$a_{\alpha}|n_0, n_1, n_2, \ldots, n_{\alpha} \ldots\rangle = \sqrt{n_{\alpha}}|n_0, n_1, n_2, \ldots, n_{\alpha} - 1 \ldots\rangle$$

$$a_{\alpha}a_{\alpha}^{\dagger}|n_0, n_1, n_2, \ldots, n_{\alpha} \ldots\rangle = \sqrt{n_{\alpha}+1}|n_0, n_1, n_2, \ldots, n_{\alpha} + 1 \ldots\rangle$$

And the ground state wave function will be $\psi_0 = 1/\sqrt{V}$. In a Bose Einstein condensation macroscopic occupation of the ground state will be approximate to:
$N_0 \pm 1 \approx N_0$ Thus $a_0 = a_0^{\dagger} = \sqrt{N_0}$, the Approximation of the field operator at very low temperature will take the form: $\hat{\psi}^{\dagger}(r,t) = \sqrt{N_0/V} + \hat{\psi}'(r,t)$ and $\hat{\psi}'(\boldsymbol{r},t)$ represent small perturbation [1]. In general $\hat{\psi}(\boldsymbol{r},t) = \Phi(r,t) + \hat{\psi}'(r,t)$ With $\Phi(r,t) = \langle\hat{\psi}(\boldsymbol{r},t)\rangle$ represent the classical field. And $n_0(r,t) = |\Phi(r,t)|^2$.

At zero temperature, all anomalous terms and the non-condensate part can be neglected. This is equivalent to replacing

the quantum electrodynamics field $\hat{\psi}(\boldsymbol{r},t)$ in (1) by the classical electromagnetism field $\psi(\boldsymbol{r},t)$. It gives rise to a nonlinear Schrodinger equation, the well-known Gross-Pitaevskii equation (GPE),

$$i\hbar\frac{\Phi(r,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\boldsymbol{r}) + g|\Phi(r,t)|^2\right]\Phi(r,t) \quad (2)$$

for the Bose-Einstein condensed system where $\Phi(r,t)$ is analogous to E and B of the Maxwell theory. So the condensate wave function represents the classical limit of the de Broglie waves, where the corpuscular aspect of matter does not matter. The external trapping potential $V_{ext}(\boldsymbol{r})$ is taken to be time-independent. The GPE "which is a self-consistent mean field nonlinear Schrodinger equation (NLSE)" describes BECs in traps that are non-uniform, was first developed independently by Gross [15] and Pitaevskii [16] in 1961 to describe the vortex structure in superfluid. The macroscopic wave function/order parameter is normalized to the total number of particles in the system, which is conserved over time, i.e.

$$\int|\psi(\boldsymbol{r},t|^2 d\boldsymbol{r} = 1 \quad (3)$$

For ideal (non-interacting) gas, all particles occupy the ground state at $T = 0\ K^0$ and $\psi(\boldsymbol{r},t)$. in the GPE describes the properties of all N particles in the system. For interacting gas, owing to the inter-particle interaction, not all particles condense into the lowest energy state even at zero temperature. This phenomenon is called the quantum depletion. In a weakly interacting dilute atomic vapor, which is the main concern in this paper, the non-condensate fraction is very small. The mean field theory can be successfully applied and the quantum depletion can be neglected at zero temperature, assuming a pure BEC in the system. The External Potential in equation (2) assist an important part to bring the condensation to a reality. In early BEC experiments, quadratic harmonic oscillator well was used to trap the atoms. Recently more advanced and complicated traps have been applied for studying BECs in laboratories [17, 18, 19, 20]. In this section, we will discuss the one dimensional harmonic oscillator potential which is widely used in current experiments. Assume the harmonic oscillator potential is applied along the y-axis, in this case the mathematical formula will be read as: $V_{hop}(\boldsymbol{r}) = V_{hop}(y)$ $\quad V_{hop}(y) = \frac{m}{2}\omega_y^2 y^2$, where, ω_y, is the trapping frequencies in y-direction. The other trapping used in this analysis is the double well potential dwp [19] (Type I) along the x-axis which is read as: $V_{dwp}^I(r) = \frac{m}{2}\nu_x^4(x^2-\hat{a}^2)^2$, this formula of dwp will replace by the formula which seem to be closer to the experimental arrangement of the BEC, this will called it (Type II) of dwp and read as: $V_{dwp}^{II}(r) = \frac{m}{2}\nu_x^2(x-\hat{a})^2$ where, $\pm\hat{a}$ are the double well centers along the x-axis, ν_x is a given constant with physical dimension $1/[\mathrm{m\,s}]^{1/2}$. The choices for the scaling parameters t_0 and x_0, the dimensionless potential $V(r)$ with $\gamma_y = t_0\omega_y$, the energy unit $E_0 = \frac{\hbar}{t_0} \Rightarrow E_0 = \frac{\hbar^2}{mr_0^2}$, and the interaction parameter $\beta = \frac{4\pi a_s N}{r_0}$ for harmonic oscillator potential along y-axis is: $t_0 = \frac{1}{\omega_r}$, $r_0 = \sqrt{\frac{\hbar}{m\omega_r}}$ $\quad V(y) = \frac{1}{2}\gamma_y^2 y^2$, and for double well potential along the x-axis will be: $t_0 = \left(\frac{m}{\hbar\nu_x^4}\right)^{1/3}, r_0 = \left(\frac{\hbar}{m\nu_x^2}\right)^{1/3}, a = \frac{\hat{a}}{r_0}, V(x) = \frac{1}{2}\gamma_x^2(x^2-a^2)$.

3. Result and Discussion

Since we concerned in harmonic oscillator potential in term of value and the shape and in more precisely, the effect of γ_y on this potential, we will fixed first the shape and the distribution of double well Potential along the x-axis and study the distribution of HOP along y-axis. This potential presented in figure (1a). One can conclude from this figure that γ_y has no effect as long as the shape of the potential concern where its preserve the parabola open right like function, however the values of the potential change with γ_y. Another remarkable point in this figure that is the distribution is symmetric just about the centre of propagation and the point of intersection of these figures is y = 0. The Double well Trapping Potential presented in figure (1b). The distribution of this potential along X-axis is shown in this figure for different value of centre of trap (1., 2., 3., 4., 5.). The shape of this distribution is Parabola open up like function and is symmetrical around the centre of propagation. One can observe that there is no specific point of intersection when the centre of trapping potential is change. To make a comprehensives view of the interaction of two potentials in XY plane a counters level are presented in figure (2) for difference values of γ_y (0.5,1.0,1.4,2.0, 3.0,4.0,5.0,7.0) as long as the harmonic oscillator potentials, and for fixed value of the centre of propagation of double well potential (a = 5.). A quick view of these figures, one can conclude that for $\gamma_y = 1$, their exist a complete mirage and symmetry between the HOP and DWP which is analogous with the experimental result for this type of potential. Additionally a concentrated circle around the centre of propagation shown in figure (2b), this means that the condensation can be confine around the centre under an equal forces exert on this condensation from all direction by an applied external potential. If the experimental set up in such a way that the condensation need to be propagate along the y-axis, in this case we should use $\gamma_y < 1$ in the theory figure (2a), and the value of γ_y can be scan to fit the experimental result. Likewise for $\gamma_y > 1$ the condensation preserve its symmetry and confinement around the centre of propagation, but in this case tend to parallel to the x-axis as shown in figures (2c-2h). This means that the HOP will control the shape and values of the condensation and the DWP begin gradually loss his effectiveness on the region of confinement of BEC.

4. Conclusion

Although the theories which describe the harmonic oscillator and double well potential are working very well to some extent, but this result shows that care must be taken in to account in order to explain the experimental results more accurately. The limitation of different factor in theory need more study in parallel with the available of experimental data. It is necessary to balance between the HOP and DWP to confine the condensate matter in to the centre of propagation

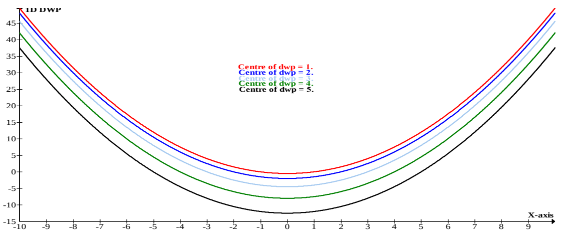

Figure (1b) the distribution of DWP along the X-axis

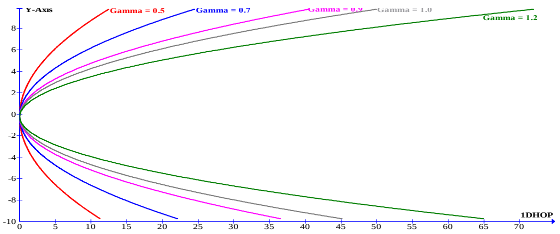

Figure (1a) the distribution of HOP along the Y-axis

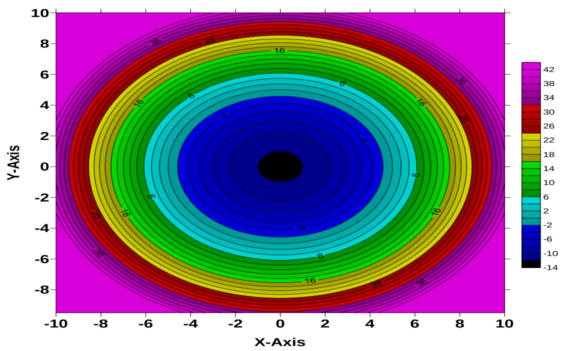

Figure (2b) $\gamma_y = 1$.

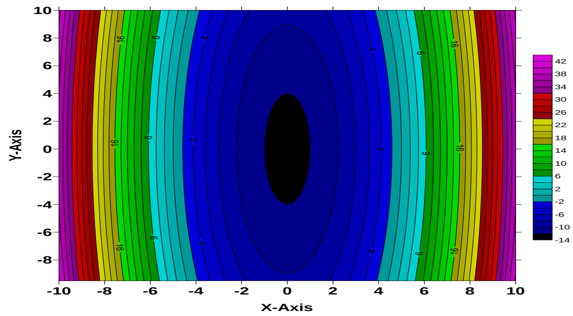

Figure (2a) 1D Harmonic Oscillator Potential +1D Double well Potential when $\gamma_y = 0.5$.

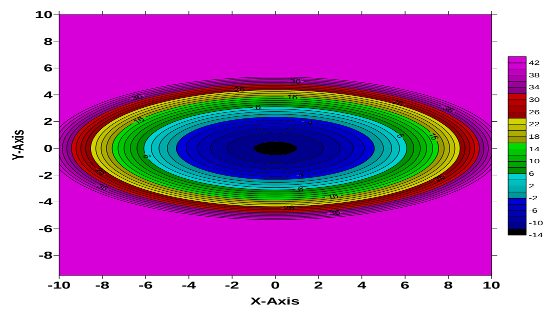

Figure (2c) $\gamma_y = 1.4$

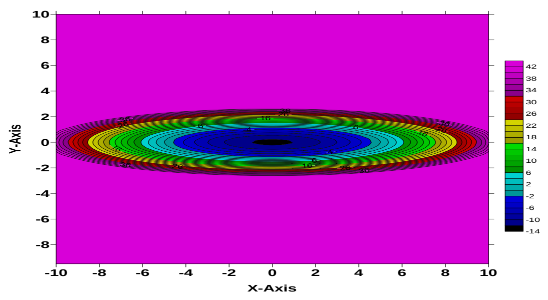

Figure (2d) $\gamma_y = 2.0$

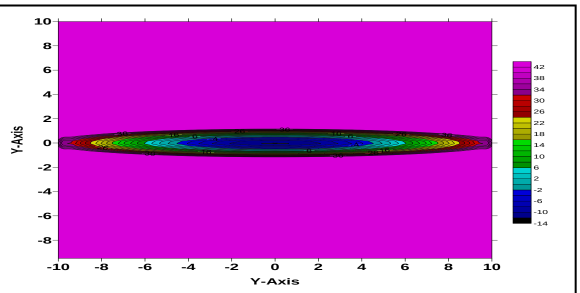

Figure (2e) γ_y= 3.0

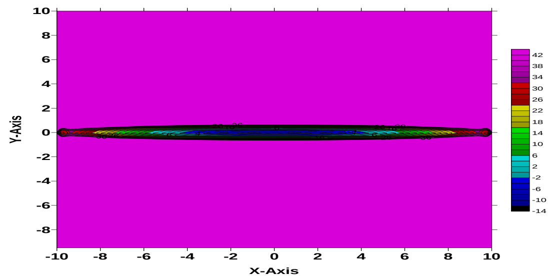

Figure (2f) γ_y= 4.0

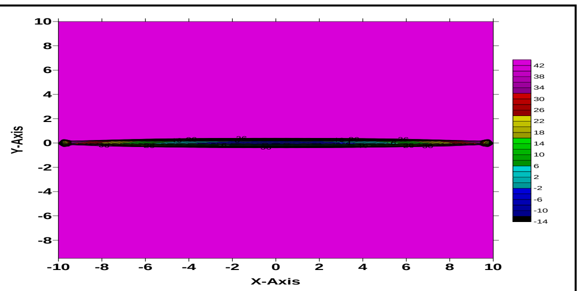

Figure (2g) γ_y= 5.0

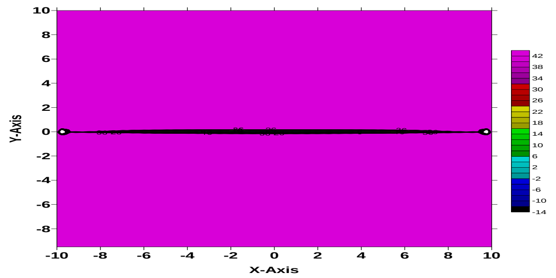

Figure (2h) γ_y= 7.0

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