

## Construction of Operating Characteristics Curve for Acceptance Sampling Plan by Using MATLAB Software.

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### -----Abstract-----

This paper reveals that too many procedures yield too many rather different solutions, thus confusing the user in choosing a plan. In this paper a procedure is developed to draw an O.C.Curve by using hypergeometrical distribution method. The research problems under consideration are relationships between sampling risks and other elements inbuilt in a single acceptance sampling plan. Two levels of quality are considered: first, average quality level desired by the consumer AQL and, second, quality level called lot tolerance percent defective LTPD, or the worst level of quality that the consumer may tolerate. The producer's risk  $\alpha$  is the risk of incorrect rejection is the risk that the sampling plan will fail to verify an acceptable lot's quality set by AQL and, thus, reject it. The probability of acceptance a lot with LTPD quality is the consumer's risk  $\beta$  or the risk of incorrect accepting. Operating characteristic (OC) curve describes how well an acceptance plan discriminates between good and bad lots. Acceptance sampling plan consists of a sample size  $n$ , and the maximum number of defective items that can be found in the sample  $c$ . The OC curve pertains to a specific plan, i.e. to a combination of the sample size  $n$  and the acceptance criterion or level  $c$ . Moreover, some fundamental considerations concerning sampling inspection and process control lead to the conclusion that acceptance sampling was introduced many years ago might be replaced by efficient computerized sampling control tools.

**Key-Words:** Statistical quality control, decision making, acceptance sampling plan, consumer's risk, producer's risk, acceptable quality level, lot tolerance percent defective.

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### I Introduction

Acceptance sampling is a method of measuring random samples of populations called "lots" of materials or products against predetermined standards. Acceptance sampling is a part of operations management or of accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management. Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. Acceptance sampling is a major field of statistical quality control. A typical application of acceptance sampling is as follows. A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot and some quality characteristics of the units in the sample inspected.

On the basis of the information in this sample a decision is made regarding lot disposition usually this decision is either to accept or to reject the lot. Sometimes we refer to this decision as lot sentencing. Accepted lots are put into productions; rejected lots may return to the vendor or may be subjected to some other lot disposition action. While it is customary to think of acceptance sampling as a receiving inspection activity, there are other uses of sampling methods for example; frequently a manufacturer will sample and inspect its own product at various stages of production. Lots that are accepted are sent forward for further processing while rejected lots may be reworked or scrapped. Acceptance sampling is most likely to useful in the following situation.

1. When testing is destructive.
2. When the cost of 100% inspection is extremely high.
3. When 100% inspection is not technologically feasible or would require so much calendar time that production scheduling would be seriously impacted.
4. When there are many items to be inspected and the inspection error rate is sufficiently high that 100% inspection might cause a higher percentage of defective units to be passed than would occur with the use of sampling plan.
5. When the vendor has an excellent quality history, and some reduction in inspection from 100% is desired, but the vendor's process capability ratio is sufficiently low to make no inspection an unsatisfactory alternative.
6. When there are potentially serious product liability risks and although the vendor's process is satisfactory a program for continuously monitoring the product is necessary. The products may be grouped into batches or lots or may be single pieces from a continuous operation. A random sample is selected and could be checked for various characteristics. For lots, the entire lot is accepted or rejected in the whole. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test. In the case of continuous production process, a decision may be made to continue sampling or to check subsequent product 100%.

## II . Review of Related Literature:

An acceptance-sampling plan is best described in graphical terms on an operating-characteristic curve (OC curve). An OC curve is a plot of the actual number of nonconforming units in a lot (expressed as a percentage) against the probability that the lot will be accepted when sampled according to the plan. The shape of an OC curve is determined primarily by sample size,  $n$ , and acceptance number,  $c$ , although there is a small effect of lot size,  $N$ . (Acceptance number,  $c$ , is the largest number of nonconforming units, or non-conformance, that may be found in the sample without causing rejection of the lot.)

## III Mathematical Model

The probability of observing exactly  $d$  defectives is;

$$P\{d \text{ defectives}\} = f\{d\} = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

The probability of acceptance is just the probability that  $d$  is less than or equal to  $c$ .

$$P(\alpha) = P\{d \leq c\} = \sum_{d=0}^c \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

## IV Operating Characteristic (OC) Curve:

Probability of acceptance is determined with respect to the lot percent defective. Table 1 indicates the different values.

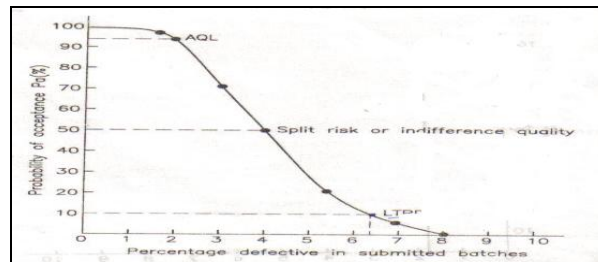
**Table -1**

<b>n</b>	<b>np'</b>	<b>p'</b>	<b>Pa</b>	<b>Pap'</b>
150	2.0	0.0133	0.983	0.0131
150	2.6	0.0173	0.951	0.0164
150	4.5	0.03	0.703	0.02109
150	5.7	0.038	0.495	0.0188
150	8.0	0.053	0.191	0.01013
150	10.6	0.071	0.048	0.00338
150	12.0	0.080	0.02	0.0016

An operating characteristics curve for above values can be drawn.

### 4.1 Acceptable Quality Level (AQL):

AQL is defined as the maximum percentage of defects (or maximum number of defects per hundred units) that, for the purposes of sampling inspection, can be considered satisfactory. (Note: percentage defective  $C$  for (sated) probability of acceptance).



It is usual to lay down a quality level, which the customer will consider acceptable in most instances. It is common practice to set the AQL at 0.95 probability of acceptance, which means that if batches of the minimum acceptable quality are submitted, 95% will be accepted on average. There is, of course, a chance that sometimes batches of a worse quality may be accepted, as the ideal of curve is unattainable.

#### 1.5 Lot Tolerance Per Cent Defective (LTPD):

LTPD is defined as the incoming fraction defective (or number of defects per 100 units) in a lot that the consumer is willing to accept with a very small probability of occurrence. This point is shown at the lower end of the OC curve. It is often chosen for 10% consumer risk; that is, there is a 10% chance that the consumer will accept batches whose quality is worse than the acceptable level. From Fig, it can be seen that the consumer has 10% chance of accepting batches worse than 6.2% defective.

**4.2 Indifference Quality (Split Risk or Point of Control):** The indifference quality is the quality level, which has an even chance of acceptance or rejection, the producer and consumer taking the same risk in the application of the plan.

**4.3 Average Outgoing Quality Limit (AOQL):** The AOQL is the maximum average outgoing quality level, which, in the long run, will not become worse whatever the quality of the incoming items. It is the limit to the average quality of batches after inspection. The existence of the AOQL depends on the following conditions: 1. All rejected batches must be re-inspected 100%. 2. Every defective in the re-inspected batches must be detected. 3. Every defective item found must be replaced by a good item so that batch size remains constant. Since conditions 2 will not often be met the AOQL is an approximation; but it is a useful concept. Values of AOQL are found by multiplying together the incoming quality (per cent defective) and the proportion of batches expected to be accepted. Remember that the word 'average' is important because individual batches getting through inspection may be far worse than the AOQL and it is only when, a large number of batches is averaged that the quality cannot, in the long run, be worse than the AOQL.

#### 4.4 Risks in Acceptance Sampling:

Neither sampling, nor 100% inspection can guarantee that every defective item in a lot shall be located. Sampling involves a risk that the sample may not adequately reflect the condition of the lot. i.e., it may not represent the lot correctly. 100% inspection has the risk that monotony and excess amount of inspection work will result in inspectors missing some of the defective components. Sampling risks are of two kinds, (1) Producer's risk (2) Consumer's risk. The operating characteristics (OC) curve for a sampling plan quantifies these risks.

#### 4.5 Producer's Risk:

Assume that the lot produced is good, but unluckily the sample selected out of this lot did not represent the lot faithfully. The sample had a higher proportion of defectives than the lot as a whole. Hence the (otherwise) good lot got rejected, resulting in loss for the producer. This is known as producer's risk. Producer's risk is designated as the alpha risk. Producer hopes to keep this risk low, say at 1 to 5% if a good lot is rejected; it is referred to as type I error.

#### 4.6 Consumer's Risk:

Assume that the lot is bad, but the sample selected out of this lot did not represent the lot truly. In other words, the sample had a higher proportion of good components than the lot as a whole. Hence the bad lot got selected, resulting in loss for the consumer. This is known as consumer's risk. Consumer's risk is designated as the Beta risk. Consumer wants to keep this risk low. If a bad lot is accepted, it is referred to as type II error. The

a risk at the AQL level and the (3 risk at the LTPD level establish two points from which the sample size,  $n$ , and acceptance number,  $c$  are determined. Given these two points, the OC curve can be drawn to describe the risk characteristics of the specific sampling plan.

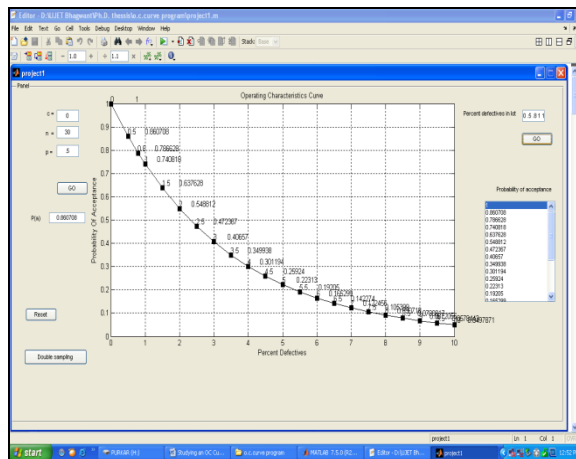
**V The computer software:**

In previous discussion acceptance sampling procedure has been adapted by constructing o.c. curve by manually. It is noticed that procedure for calculations of the probability of acceptance is very much time consuming. Hence it is decided to develop computer software, which can help in determination of probability and will construct operating characteristics curve automatically. Computer software by using MATLAB has been successfully developed for the procedure of acceptance sampling in both single and double sampling plan. Some curves drawn by using the software are as shown below;

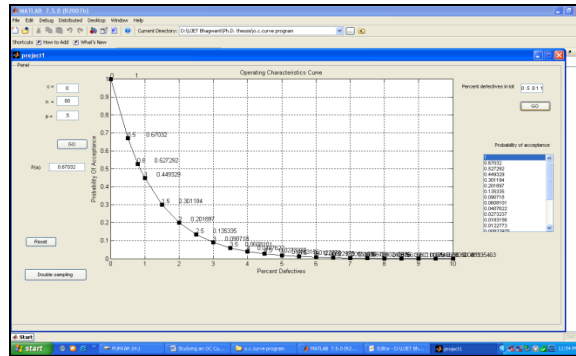
**Table.2**

<b>C=0</b>	<b>n=30</b>	
<b>Sr.No.</b>	<b>Percent Defective</b>	<b>Probability of Acceptance (%)</b>
1	0	100
2	0.5	86.0
3	0.8	78.6
4	1	74.0
6	2	54.8
8	3	40.63
10	4	30.11
12	5	22.31
14	6	16.52
16	7	12.24
18	8	9.07
20	9	6.72
22	10	4.97

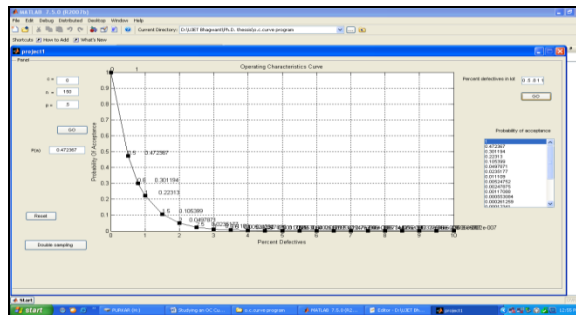
**Fig.1 OC Curve for  $n=30, c=1, AQL=1\%$ ,  
LTPD=5%,  $\alpha =0,0361, \beta =0,5535$ , and  
Probability of Acceptance  $= (1-\alpha) =0,9639$ .**



**Fig.2 OC Curve for  $n=80, c=0, AQL=1\%$ ,  
LTPD=5%,  $\alpha =0,1908, \beta =0,0861$ , and  
Probability of Acceptance  $= (1-\alpha) =0,8092$ .**



**Fig.3** OC Curve for  $n=150$ ,  $c=0$ ,  $AQL=1\%$ ,  $LTPD=5\%$ ,  $\alpha=0.4430$ ,  $\beta=0.0041$ , and Probability of Acceptance =  $(1-\alpha)=0.557$ .



## VI Conclusion:

The design of the acceptance sampling process includes decisions about sampling versus complete inspection, attribute versus variable measures, AQL,  $\alpha$ , LTPD,  $\beta$ , and sample size. By making computer software most of the things became easier and less time consuming. Values of Producer's and consumer's risk can easily determined at specific AQL and LTPD.

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